PRINT-92-0253 (CARLETON)
Fermilab Library
0 1160 0013765 7

A Test of Optimized Perturbative QCD to $O(\alpha_s^2)$ Using Measurements of Jet Rates and Energy Correlations in e^+e^- Annihilations at the Z^0 Peak*

Sukhpal Sanghera
Department of Physics
Carleton University
Ottawa, Canada, K1S 5B6

JUL 1 1992

ABSTRACT

A QCD analysis of the published LEP data on jet rates and energy correlations is performed in order to probe the renormalization scale optimization of perturbative QCD to $O(\alpha_s^2)$. The explicit dependence of the fundamental QCD parameter $\Lambda_{\overline{MS}}$ on the renormalization scale μ^2 is extracted from the data. The theoretical predictions for μ^2 optimized to $O(\alpha_s^2)$ are calculated using various QCD inspired procedures, and are compared to the experimental results. The general conclusion is that the theoretical predictions of the optimized μ^2 agree with the experimental results and that $\mu^2 = E_{cm}^2$ is not necessarily a good choice in $O(\alpha_s^2)$ QCD.

1. Introduction

The perturbative QCD contains the renormalization scale μ^2 used to regularize the infinities in the loop corrections. This is often called an unphysical parameter of which the physical quantities, calculated to all orders in the perturbative expansion, should be independent. But, the analytical calculations of most of the observables are available only at the most to second order and the perturbative expansion truncated at a finite order may depend upon the choice of the scale. QCD itself does not predict at which value of the scale the second order formulae should be evaluated. In the absence of a unique and commonly accepted way of treating the scale, all the four LEP experiments differ in their treatment of the scale in presenting the final results on $\alpha_s(M_{z^0})$. Obviously, the scale issue is unresolved and needs further investigation.

2. Experimental Optimization of QCD

We use the published LEP data on jet rates[1–3] and energy correlations[4,5] to perform this study. The data were corrected for hadronization using Jetset shower Monte Carlo[6] at parton shower cutoff mass of 1 GeV. Like all the four experiments at LEP we also, in our analysis, use the cluster jet algorithm introduced by the JADE collaboration.[7] Resolvable jets of particles in an event are defined by requiring that the scaled jet pair masses y_{ij} exceed a threshold value $y_{cut} : y_{ij} = M_{ij}^2/E_{vis}^2 > y_{cut}$, where M_{ij} is the jet pair mass and E_{vis} is the visible energy of the event. The different approaches to combine the four momenta of the pair gives rise to different recombination schemes, namely E0-,P0-,P- and E- schemes.[3,8] For statistical reasons, it is the derivative of 2-jet rates, $D_2(y)$, which is compared to the theory. The energy-energy correlations (EEC) are defined[8] as the histogram of the angle χ_{ij} between all combinations of pairs of particle

*A talk presented at the general meeting of the Division of Particles and Fields of the American Physical Society (DPF91), University of British Columbia, Canada, August 1991.

tracks in hadronic events, weighted by their energies, suitably normalized , and averaged over all events. The asymmetry of the energy-energy correlations (AEEC) is defined by the expression AEEC(χ)= EEC(π-χ) - EEC(χ). In second order (O(α_s^2)) perturbative QCD, an observable O may be written as a power series in α_s ie $O \propto C_1.\alpha_s + C_2.\alpha_s^2$. The coupling constant $\alpha_s(\mu, \Lambda)$ can be written as a function of ln($\frac{\mu^2}{\Lambda^2}$),[9] where Λ is the fundamental QCD parameter to be determined by the experiment. Next-to-leading order coefficients C_2 are recombination scheme dependent and exhibit an explicit dependence on the renormalization scale $\mu^2(= \mathbf{f}.E_{cm}^2)$.

We determine $\Lambda_{\overline{MS}}$ in a one parameter fit of the analytical calculations of Ellis, Ross and Terrano[10] as parameterized by Kunszt et al.[8] with the data. The fits were repeated for different values of **f** in its entire range in which the χ^2 of the fit was reasonable. The $\Lambda_{\overline{MS}}$ thus determined is plotted as a function of **f** in Fig. 1.

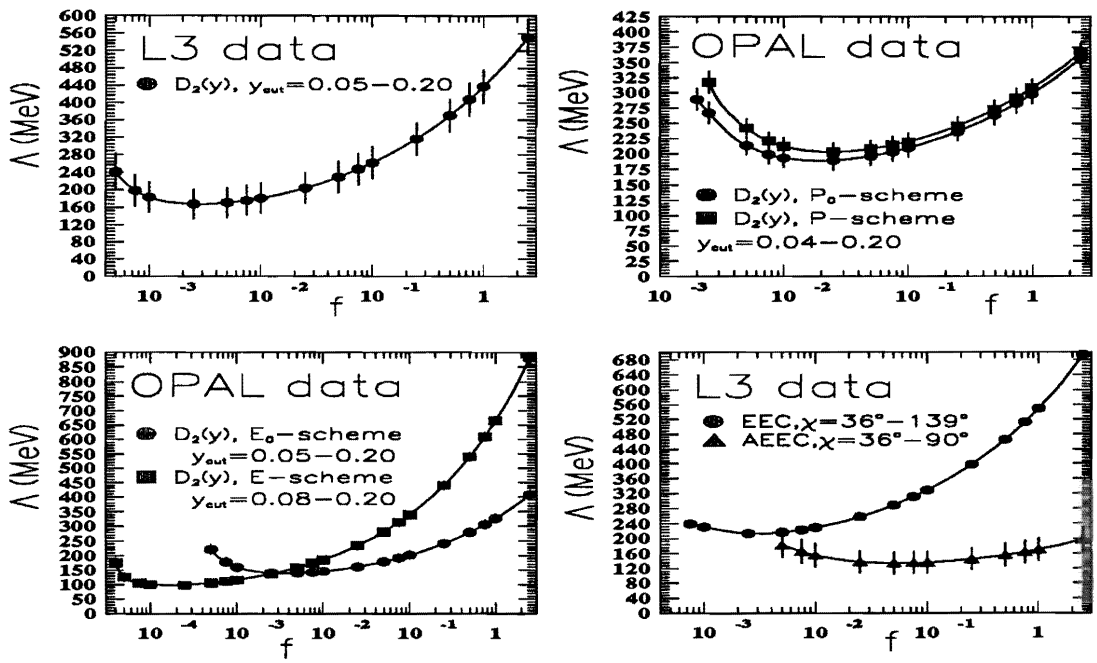

Fig. 1 Λ determined from experimental data on jet rates and energy correlations, as a function of the renormalization scale factor $\mathbf{f}=\mu^2/E_{cm}^2$.

The errors shown on $\Lambda_{\overline{MS}}$ are the percentage statistical errors. The region of y_{cut} and χ in which the fits were made are shown in the diagrams. These regions were chosen to exclude the 4-jet rates because they are calculated only to leading order, and to exclude the unphysical region where the predicted cross sections are negative and hadronization corrections are unacceptably high. For all the points shown in the plots, χ^2 of the fit per degree of freedom was ≤ 3 which ascends sharply for smaller f values.

From these plots, the following conclusions may be drawn: 1. $\Lambda_{\overline{MS}}$ strongly depends on **f** around **f**=1, while it reaches a relatively stable region at smaller values of **f** which we shall call the optimized region of **f**, 2. The optimized region of **f** may, in general, be different for different observables.

3. Theoretical Optimization of QCD

Although QCD itself does not predict the value of μ^2, various QCD inspired theoretical procedures to optimize μ^2 have been proposed and are summarized here. Stevenson's principle of minimal sensitivity (PMS)[11] proposes:Since the true result is completely independent of μ^2, approximants should be least sensitive to small changes in the scale. Therefore, the calculated observable can only be a good approximation in the scale region where it is flat ie the derivative

of a given observable with respect to μ^2 should vanish. We relax this condition to demand that the observable has a moderate scale dependence (MSD) but the first derivative of the observable with respect to μ^2 is minimal and hence the second derivative should vanish. The fastest apparent convergence (FAC), proposed by Grunberg,[12] is based on absorbing the higher order terms into the strong coupling constant α_s. In second order theory, this means that the scale is chosen in such a way that the coefficient of the next to leading order correction vanishes. Another procedure, proposed by Brodsky, Lepage and Mackenzie (BLM) ,[13] is based on absorbing terms which depend upon the number of active fermions n_f, into the strong coupling constant α_s. In second order theory this implies that the coefficient of next to leading order correction is independent of n_f.

We apply these procedures to the QCD formulae[8] for jet rates and energy correlations to calculate the optimized scale factor **f**. The results obtained from PMS, FAC and MSD are displayed in Fig. 2.

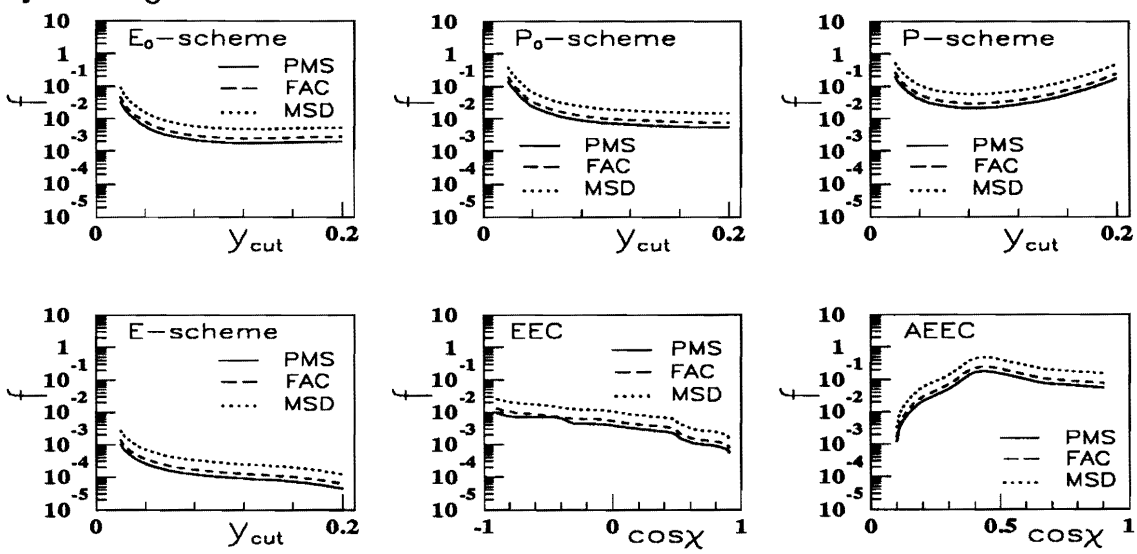

Fig. 2 Theoretical predictions of the renormalization scale factor $\mathbf{f}=\mu^2/E_{cm}^2$ for jet rates and energy correlations.

Applying the BLM procedure to jet rates yielded similar results. One can verify by comparing Fig. 2 with Fig. 1 that the theoretical and the experimental results for optimized **f** are very much in agreement with each other. The analysis performed on the published data on jet rates by DELPHI[1] and on energy correlations by OPAL[5] yielded similar results.

4. Conclusions

A general conclusion of this study is that **f** =1 (ie. $\mu^2 = E_{cm}^2$) is not necessarily a good choice in $O(\alpha_s^2)$ perturbative QCD. The theoretical predictions for the optimized scale are in good agreement with the experimental fit results and are, in general, different for different observables. These differences may be understood in terms of the difference in the size of the known second order and unknown higher order corrections for different observables. From Fig. 1, we estimate the optimized region of **f** as 0.0025 to 0.01 for EEC and 0.025 to 0.25 for AEEC. For jet rates, the optimized region of **f** is estimated to be 0.0025 to 0.01 in E_0 - scheme, 0.0075 - 0.10 in P_0- scheme, 0.010 to 0.10 in P-scheme, and 0.000075 to 0.0010 in E-scheme. In these ranges of **f** the $\Lambda_{\overline{MS}}$ values determined in our analysis, from all these observables, fall in the range of 165±60 MeV which corresponds to $\alpha_s(M_{z^0}) = 0.113\pm0.006$; an excellent agreement with the QCD prediction of $\alpha_s(M_{z^0}) = 0.11\pm0.01$[14] based on the lower energy experimental results.

I am grateful to S. Bethke of Heidelberg, talking to whom has always resulted in learning something about QCD. I am thankful to my teachers at Carleton: Hans Mes, R. J. Hemingway, R. K. Carnegie, M. Dixit and P. Estabrooks for useful comments and suggestions. Thanks are also due to the department of physics at Carleton for its financial support.

References

1. DELPHI Collab., P. Abreu et al., CERN-EP/90-89, 22 June 1990; Phys. Lett. B247 (1990) 167.
2. L3 Collab., B. Adeva et al., Phys. Lett. B248 (1990) 464.
3. OPAL Collab., M.Z. Akrawy et al., Zeit. Phys. C49 (1991) 375.
4. L3 Collab., B. Adeva et al., L3 preprint 023 (1990).
5. OPAL Collab., M.Z. Akrawy et al., Phys. Lett B252 (1990) 159. Zeit. Phys. C49 (1991) 375.
6. T. Sjostrand, Comp. Phys. Comm. 43 (1987) 367.
7. JADE Collab., W. Bartel et al., Z. Phys. C, 33 (1986) 23;
   JADE Collab., S. Bethke et al., Phys. Lett., B213 (1988) 235.
8. Z. Kunszt et al. in "Z Physics at LEP1", CERN 89-08, 21 Sept. 1989.
9. Review of Particle Properties, Phys Lett B239 (1990)III.51.
10. R.K. Ellis, D.A. Ross and A.E. Terrano, Nucl. Phys. B178 (1981)421.
11. P. M. Stevenson, Nucl. Phys. B231 (1984)65.
12. G. Grunberg, Phys. Lett. B95 (1980) 70.
13. S. J. Brodsky, G. P. Lepage and P. B. Mackenzie, Phys. Rev. D28(1983) 228.
14. G. Altareli, CERN-TH-5290/89; Ann. Rev. of Nuclear and Particle Science, Vol. 39 (1989) 357.