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# Theoretical Interpretation of the Kreuzer Experiment and General Relativity

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## Abstract

In 1966, the Kreuzer experiment set an upper limit of 5 parts in $10^5$ on the difference in the ratio of active to passive mass between fluorine and bromine. In 1976, this experiment was interpreted by Will as an upper limit on his PPN parameter combination which is unrelated to nuclear energy. Moreover, Will's formula actually suggests violation of mass equivalence. These results are, however, originated from the unphysical nature of Will's nucleus model, and is obtained with questionable calculations. From the viewpoint of general relativity, to determine the constraints on gravity generated from different forms of matter, the mass equivalence is simply not yet adequate. Experimental measurement for the coupling constant to electromagnetism is recommended.

Theoretical Interpretation of the Kreuzer Experiment

and

General Relativity

## I. Introduction.

In 1966, the Kreuzer experiment [1] set an upper limit of 5 parts in $10^5$ on the difference in the ratio of active to passive mass between fluorine and bromine. Because of the electromagnetic structure of matter, naturally attempts were made to interprete the Kreuzer experiment as a test of the manner in which different forms of matter and energy generate gravity. In 1971, a primitive interpretation was made by Thorne et al. [2] by using the perfect-fluid Parametrized Post-Newtonian (PPN) formalism. However, C. M. Will believes that, based on mass equivalence alone, it is possible to have a reliable study of the generation of gravity by different atoms and nuclei. In this paper, we shall show that Will's nucleus model and his theoretical interpretations are <u>not</u> valid.

Using a PPN framework based on point particles, in 1976 Will [3] extended Nordvedt's formalism [4] to include the effects of interactions between the point particles (i.e. nucleons) via electric Coulomb fields. Then, in addition to Nordtvedt's point-mass potentials, this extended formalism includes three new "electric" gavitational potentials and their associated PPN parameters $\varepsilon_1$, $\varepsilon_2$ and $\varepsilon_3$. Thus, there are 11 parameters -- $\gamma$, $\beta$, $\zeta_1$, $\zeta_2$, $\zeta_w$, $\alpha_1$, $\alpha_2$, $\alpha_3$, $\varepsilon_1$, $\varepsilon_2$ and $\varepsilon_3$. Using this new PPN metric, for a body in which the distribution of particles is spherical, Will claims that the active mass $m_A$ and inertial mass $m_I$ are related by

$$m_A = m_I + (4\beta - \gamma - 3 - 2\zeta_2 - 2\zeta_w - \frac{1}{2}\alpha_3 - \frac{1}{3}\zeta_1)\Omega^* + (\varepsilon_1 + \frac{1}{3}\varepsilon_2 - \frac{1}{2}\alpha_3 - \frac{1}{3}\zeta_1)\Omega_E^*,$$

where

$$\Omega^* = -\frac{1}{2}\sum_{i\neq j}\frac{m_i m_j}{|\mathbf{x}_i - \mathbf{x}_j|}, \quad \Omega_E^* = \frac{1}{2}\sum_{i\neq j}\frac{e_i e_j}{|\mathbf{x}_i - \mathbf{x}_j|}, \tag{1}$$

where $m_i$, $e_i$, and $\mathbf{x}_i$ are the rest mass, charge, and position of the ith particle. In the above formula, the speed of light and Newtonian gravitational constant are chosen as unity. In a nucleus, the self-gravitational energy $\Omega*$ is negligible compared to the electromagnetic energy $\Omega_E^*$ [3]. It follows that, in comparison with experiment, Will obtains an upper limit on the PPN parameter combination:

$$|\varepsilon_1 + \frac{1}{3}\varepsilon_2 - \frac{1}{2}\alpha_3 - \frac{1}{3}\zeta_1| < 3 \times 10^{-2} . \tag{2}$$

In deriving eq. (2), Will [3] assumed that the passive mass $m_P = m_I$ justified by Eötvös experiments [5].

However, the validity of eq. (1) should be examined because the coefficient of $\Omega*$, which includes the number 3, actually suggests violation of mass equivalence. Moreover, there is no term related to nuclear energy although the nucleons are considered as point particles. Examination of Will's derivation reveals that the nuclear force and energy energy are simply ignored without sufficient justification (see § II). To this end, he <u>implicitly</u> uses a number of unverified assumptions. In addition, the Newtonian virial theorem is not correctly used. In his model, the bounding energy for a nucleus is actually positive. In summary, Will's nucleus model and related calculations are problematic.

## II. Questions Related to Nuclear Energy.

For a probem related to the structure of nuclei, it is inconceivable that the nuclear energy is not considered. On the other hand, due to our limited knowledge in nuclear physics, one should expect that little tangible result would be obtained if the nuclear force has to be accurately accounted for. In view of this, natually one would attempt to circumvent our ignorance concerning nuclei.

Will [3] claimed that his model of a nucleus is a gas of electromagnetically interacting particles in a square-well nuclear potential. However, in reality he simply ignored the nuclear energy. This is evident since there is no "nuclear" PPN potentials in his metric [3],

$$g_{00} = 1 - 2U^* + 2\beta U^{*2} - (2\gamma+1+\alpha_3+\zeta_1)\Phi_1^* - 2(1-2\beta+\zeta_2)\Phi_2^* + 2\zeta_w\Phi_w^* + \zeta_1 A^* - (1+\gamma+\varepsilon_1)E_1^* + \varepsilon_2 E_2^* + \frac{1}{2}(1+\gamma+\varepsilon_3)E_3^* ,$$

$$g_{0\alpha} = \frac{1}{2}(4\gamma+3+\alpha_1-\alpha_2+\zeta_1)V_\alpha^* + \frac{1}{2}(1+\alpha_2-\zeta_1)W_\alpha^*,$$

$$g_{\alpha\beta} = -\delta_{\alpha\beta}(1+2\gamma U^*), \qquad (3a)$$

where

$$U^* = \sum_i \frac{m_i}{r_i} , \quad \Phi_1^* = \sum_i \frac{m_i v_i^2}{r_i} , \quad \Phi_2^* = \sum_i \frac{m_i}{r_i} \sum_{j\neq i} \frac{m_j}{r_{ij}} ,$$

$$\Phi_w^* = \sum_{ij\neq i} \frac{m_i m_j r_i}{r_i^3} \cdot \left(\frac{r_{ij}}{r_j} - \frac{r_j}{r_{ij}}\right), \quad A^* = \sum_i \frac{m_i [v_i \cdot r_i]^2}{r_i^3}$$

$$E_1^* = \sum_i \frac{e_i}{r_i} \sum_{j\neq i} \frac{e_j}{r_{ij}} \ , \quad E_2^* = \sum_{ij\neq i} \frac{e_i e_j \mathbf{r}_{ij}\cdot \mathbf{r}_i}{r_{ij}^3 r_i} \ , \quad E_3^* = [\sum_i \frac{e_i}{r_i}]^2 \ , \qquad (3b)$$

where $v_i^\alpha$ is the velocity of the *i*th particle and where

$$\mathbf{r}_i = \mathbf{x} - \mathbf{x}_i, \ r_i = |\mathbf{r}_i|, \ \mathbf{r}_{ij} = \mathbf{x}_i - \mathbf{x}_j \ , \ r_{ij} = |\mathbf{r}_{ij}|. \qquad (3c)$$

Therefore, a necessary <u>implicit</u> assumption is that the nuclear energy-stress tensor has no (or at least negligible) gravitational effects.

This implicit assumption is manifested again in the following stress-energy tensor for matter and nongravitational fields [3],

$$T^{ab} = (-g)^{-\frac{1}{2}} \sum_i m_i \delta(\mathbf{x}-\mathbf{x}_i) u^a u^b / u^0 - \frac{1}{4\pi}(F^a{}_c F^{bc} - \frac{1}{4} g^{ab} F_{cd} F^{cd}). \qquad (4)$$

Note that, according to general relativity, a square-well nuclear potential should have an energy-stress tensor since a source tensor must be consistent with the equation of motion [6]. As a consequence of eq. (4), the equation of motion includes only the Lorentz force and the gravitational force as follows:

$$m_i \ u_i^b \ u_i^a{}_{;b} = - \ e_i \ F^a{}_b(\mathbf{x}_i) \ u_i^b \ , \qquad (5)$$

Using the Newtonian virial theorem superficially, Will [3] obtained a non-physical equation as follows,

$$\sum_i m_i v_i^2 + \Omega^* + \Omega_E^* = 0 \ . \qquad (6)$$

The validity of this equation is clearly very unlikely if one considers the numerical ratio between protons and neutrons in a nucleus and the coupling strength ratio between gravitational and electromagnetic interactions. The root of the above problem is that, without the nuclear force, the virial theorem [7] does not imply eq. (6) since the Coulomb force could have disintegrated the nucleus.

In order to be formally consistent in mathematics, Will assumes that the inertial mass $m_I$ takes the following form

$$m_I = \sum_i m_i + \frac{1}{2} \sum_i m_i v_i^2 + \Omega^* + \Omega_E^* \tag{7}$$

where

$$\Omega^* = -\frac{1}{2} \sum_{i \neq j} \frac{m_i m_j}{|x_i - x_j|} , \quad \Omega_E^* = \frac{1}{2} \sum_{i \neq j} \frac{e_i e_j}{|x_i - x_j|} ,$$

Now, although it is well-known that the bounding energy for a nucleus is negative, according to eq. (7), this also does not seem to be possible. Note that eqs. (6) and (7) are crucial for eq. (1).

In short, there is actually no nuclear force in Will's nucleus model, and Will's calculation is also not valid.

It should be pointed out that Will's "square-well" model is also not supported by nuclear physics. Such a model requires at least the nuclear energy is independent of the charges. A completely isospin independence of nuclear energy is not supported by nuclear physics [8]. Since the nuclear energy is much larger in magnitude than the electromagnetic energy, a small isospin dependence would imply a validity problem for

Will's square-well model.

In an attempt to defend the absence of nuclear forces, Will argued that nuclear forces could be incorporated properly by introducing "Nuclear" PPN potentials into the metric (3). He mentioned that he could use potentials generated by Yukawa forces. However, first, it is not trivial to develop meaningful "nuclear" PPN potentials. Note that general relativity implies that gravity depends not only on the energy but also the energy form since the source is a tensor in Einstein equation. More important, Yukawa forces are not exact, but approximations. Now, it is clear that mass equivalence alone is inadequate. Given the vast magnitude differences between nuclear and electromagnetic energies, it is doubtful that the so developed formula is also the same eq. (2). Note that a crucial argument in deriving eq. (2) from eq. (1) is the fact that the self-gravitational energy $\Omega_*$ is negligible. But, in comparison with electromagnetic energy, nuclear energy is certainly not negligible.

## III. Conclusion and Discussion.

It has been pointed out that the derivations of eqs. (1) and (2) are problematic and therefore they cannot be used in applications. Will's calculation illustrates that without a thorough understanding of all the interactions involved, it would be impossible to determine the constraints due to mass equivalence. To obtain the gravitational effects due to electromagnetism, an experiment due to only electromagnetic energy would be indispensable. For example, one can measure the accompanying gravitational wave of an electromagnetic wave.

Theoretically, there are unsettled problems in general relativity [6,9]. Pauli [10] pointed out that general relativity does not provide a physical interpretation for the sign and numerical value of the gravitational coupling constant, but takes these data from experiment. Thus, the assumption of extending universal coupling to electromagnetism remains to be verified. In Will's model, universal coupling is implicitly extended to electromagnetism, but not to the nuclear energy. Although, I believe, Will's model is unlikely valid, however such a possibility, in principle, remains open.

General relativity is, in principle, incompatible with a massive point-particle because it implies an infinitely concentrated mass. Therefore, from the viewpoint of general relativity, a point-particle model should be the limit of a dust model. (One might argue that a contiuum picture no longer applies at atomic and nuclear dimension. But, this does not imply that a continuum model for a particle is not valid. Note that Einstein [11], among other physicists, studied a continuum model for the electron.) However, a dust model implies that there are little interactions among different parts of a particle. This is incompatible with the fact that in nature stable massive particles are strongly bounded together. On the other hand, a perfect fluid model for a charged particle would mean a violation of extending universal coupling to electromagnetism. Although this may not affect Nordvedt's formalism, this <u>does</u> put an additional validity problem on Will's extension. Now, it is even more obvious that, from the view point of general relativity, it is premature to determine the constraints due to mass equivalence.

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