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A Strategy for Multiparticle Dynamics *†

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Abstract

The strategy consists in introducing a few characteristic quantities and express all observables, in particular fluctuations of multiplicities, in terms of these parameters. By comparing with experiment the phenomenological laws of multiparticle dynamics are derived. Quantum statistics is used to define these characteristic quantities. They are (besides the mean multiplicity) the chaoticity p and the correlation length ξ. It turns out that both p and ξ increase with energy.

1 Introduction

Multiparticle production is one of the most characteristic properties of strong interactions because it reflects the strong coupling nature of the underlying field theory which is assumed to be quantum chromodynamics (QCD). A theoretical approach to this chapter of physics must be phenomenological not only because QCD can be applied only in the perturbative regime but primarily because of the very nature of the problem which involves very many degrees of freedom. The situation is analogous to condensed matter physics where the underlying theory is well known (QED) but where microscopic concepts cannot replace macroscopic variables. Thus, far from being a deficiency and a provisional characteristic of multiparticle dynamics, the phenomenological nature of this theory is an intrinsic quality of it.

As in any phenomenology, the strategy is to find a few important physical quantities in terms of which most, if not all, observables can be expressed and to study these quantities - parameters - under various physical conditions, i.e. at various c.m. energies, for various particle reactions. The regularities which can be abstracted from such a study constitute in some sense the "laws" which govern this field and should in principle be derivable from a microscopic theory, much in the same way as certain "matter" constants and their dependence on state variables like temperature can be interpreted in terms of QED.

In the case of multiparticle dynamics the observables are of two types: average quantities like mean multiplicity, mean energy per particle etc. on the one hand and the fluctuations

*This lecture is dedicated to the 70th anniversary of my friend, collaborator and teacher G.N.Fowler in remembrance of our long standing collaboration. It is an attempt to summarize the newest developments in the realization of the program ~~outlined in our joint paper [1]~~.

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around these quantities on the other. These fluctuations are described by distributions like the multiplicity distribution $P(n)$ or energy (inelasticity) distribution $\chi(K)$, K being the inelasticity.

Fluctuations are, of course, a quite general phenomenon observed in many different chapters of physics and much experience and knowledge has been accumulated so far on this subject. It is then clear that any attempt to describe multiparticle dynamics has to use this knowledge, if it wants to avoid repetition.[1]

A particularly simple and powerful method for the description of fluctuations is represented by quantum statistics and is based on the density matrix approach. In this approach fluctuations are related to directly observable correlations. This method lies also at the basis of quantum optics including laser physics and it will be shown in the following that it can provide an economical and useful description also for phenomena observed in multiparticle dynamics.

As to the mean quantities, here, too, experience from other fields is available and has been used. In particular hydrodynamics and statistical thermodynamics may provide a framework for the description of these quantities in terms of temperature, which in its turn is related by an equation of state to the energy of the system. If hydrodynamics is not applicable, then either other more involved theoretical methods have to be applied or one just uses the experimental mean values. This last approach is used in the following.

The distributions which describe the fluctuations can be derived from the density matrix ρ. In particular

$$P_n = < n|\rho|n > . \tag{1}$$

It turns out that under certain circumstances the parametrization of the density matrix is easier than that of other fundamental quantities like the S matrix and this explains the wide applications the density matrix approach (i.e. the quantum statistical approach) has found among other things in optics. For identical particles this formalism can be taken over into hadron multiparticle production provided the energies are high enough so that the rest masses of particles and conservation laws can be neglected.

An important tool in this approach is the coherent state formalism which is particularly useful when coherence effects are present. Such coherence effects which, up to a certain point, correspond to classical fields and/or condensates exist in present theories of particle physics, although the experimental evidence for these effects is still disputed. A program for multiparticle dynamics based on the above ideas was outlined in ref. [1] where effects, characteristic for this type of approach, were predicted. Some of these effects were later found experimentally, others still await their confirmation, in particular a more direct evidence of coherence in Bose-Einstein correlations.

[1] Nevertheless, as we shall see in the following, sometimes such "rediscoveries" occured.

2 Quantum Statistical Approach to Correlations and Fluctuations of Coherent and Chaotic Fields

There are two extremes in the QS approach to fluctuations: "order" represented by coherent fields π_c and "disorder" represented by chaotic fields π_{ch}. In most cases known in optics, all fields $\pi(y)$ can be represented by a linear superposition of these two fields:

$$\pi(y) = \pi_c(y) + \pi_{ch}(y). \tag{2}$$

It appears reasonable to assume, as is done in most applications discussed below that the same superposition principle holds also in multiparticle production. An exception to this is discussed in section 4.5. y stands for several variables which characterize the field, either in momentum or configuration space. The difference between chaotic and coherent fields lies in the correlations of these fields. Thus for chaotic fields we have

$$< \pi^\dagger_{ch}(y_1)\pi_{ch}(y_2) > = Tr\left[\rho\pi^\dagger(y_1)\pi(y_2)\right] = \gamma(|y_1 - y_2|/\xi) \tag{3}$$

where ξ is the correlation length is in y space.

Eq. (3) expresses stationarity of the (chaotic) fields. If this condition is not satisfied in the variable y, then another variable, in terms of which (3) is satisfied has to be looked for. The simplest procedure in this case is to "average" out the non-stationarity or, equivalently, to consider narrow enough $|y_1 - y_2|$ intervals (cf. below).

For coherent fields, on the other hand, one has

$$< \pi^\dagger_c(y_1)\pi_c(y_2) > = < \pi^\dagger_c(y_1) > < \pi_c(y_2) >, \tag{4}$$

i.e. factorization.

The fields π are related to the multiplicities via

$$< |\pi|^2 > = < n > . \tag{5}$$

Then it is useful to characterize the chaoticity by the parameter

$$p = < n_{ch} > / < n >, \tag{6}$$

where

$$< n > = < n_c > + < n_{ch} >, \tag{7}$$

and the indices c and ch refer to coherence and chaos as defined above.

The distribution P_n or rather its moments can be expressed as integrals over the correlation function. Thus e.g. the second factorial moment is

$$\begin{aligned} < n(n-1) > &= \int \rho_2(k_1, k_2)dk_1 dk_2 = \\ &= \int C_2(k_1, k_2)\rho_1(k_1)\rho_1(k_2)dk_1 dk_2, \end{aligned} \tag{8}$$

where C_2 is the two particle correlation function:

$$C_2 = \frac{\rho_2(k_1 k_2)}{\rho_1(k_1)\rho_1(k_1)}. \tag{9}$$

ρ_1 and ρ_2 are one and two particle inclusive cross sections. Therefore it is reasonable to start our discussion with the correlation functions. In the following we shall review the developments in this field since 1989. As will be seen, some quite new and surprising facts emerged in this short period.

3 Correlations of identical bosons

Bose-Einstein Correlations (BEC)

3.1 Surprises due to the space-time approach in quantum statistics

The space-time approach has certain advantages as compared with the momentum (rapidity) approach because it does not assume stationarity in momentum space but rather in configuration space which is expected always to hold, as long as $|x_1 - x_2| < R$. Here x_i denote space-time coordinates and R is the radius (lifetime) of the source. We write thus the correlator for the fields

$$\begin{aligned} < \pi^\dagger_m(x_1)\pi_n(x_2) > &= \delta_{mn}\, C(x_1 - x_2), \qquad m,n = 1,2,3 \\ C(x_1 - x_2) &= C(0)\exp\left[-(x_1 - x_2)_0^2 L^2 - (\vec{x}_1 - \vec{x}_2)^2/L^2\right], \end{aligned} \tag{10}$$

where we have assumed a Gaussian parametrization and defined the configuration space correlation length L. It is clear that the correlation length L, as defined above, is different from, but related to, the correlation length ξ introduced in eq. (3) which refers ussually to rapidity space. The indices m, n refer to the isospin components $1,2,3$ of the pion iso-triplet:

$$\pi_\pm = (\pi_1 \pm i\pi_2)/\sqrt{2} \qquad \pi_0 = \pi_3. \tag{11}$$

The space-time distribution of the source is for reasons of mathematical convenience also parametrized by a Gaussian:

$$\rho(x) \sim \exp(-x^2/R^2). \tag{12}$$

Define now the pion correlator in momentum space:

$$F(k_1, k_2) = (2\pi)^3\sqrt{4E_1E_2} < \pi^\dagger_m(k_1)\pi_m(k_2) >, \tag{13}$$

and

$$F(-k_1, k_2) = -(2\pi)^3\sqrt{4E_1E_2} < \pi_m(k_1)\pi_m(k_2) > . \tag{14}$$

In terms of the correlator (10) and the space-time distribution of the source (12) F can be written:

$$F(k_1, k_2) = \int \frac{d^4k}{(2\pi)^4} f^*(k_1 + k)C(k)f(k + k_2) \quad k_r^0 = E_r, r = 1,2. \tag{15}$$

The advantage of the configuration space approach can now be seen immediately: With the standard assumption about f and C, it follows from (15) that not only (13) but also (14) does not vanish. This is at a first look surprising because in the assumption of stationarity in momentum space, (14) would vanish. This explicitly proves that this last assumption cannot hold in general and it leads to observable, although small, new effects (cf. ref. [2]).
Indeed from eqs. (13)-(15) follows:

$$\begin{aligned} C_2^{--} = C_2^{++}(\vec{k}_1, \vec{k}_2) = 1 + |d_{12}|^2; \quad C_2^{+-}(\vec{k}_1, \vec{k}_2) = 1 + |\tilde{d}_{12}|^2 \\ C_2^{-0} = C_2^{+0}(k_1, k_2) = 1; \quad C_2^{00}(\vec{k}_1, \vec{k}_2) = 1 + |d_{12}|^2 + |\tilde{d}_{12}|^2. \end{aligned} \tag{16}$$

Here $+, -, 0$ refer to positive, negative and neutral pions and d_{rs}, $\tilde{d}_{rs}$ are the normalized correlators

$$d_{rs} = \frac{F(k_r, k_s)}{[F(k_r, k_r)F(k_s, k_s)]^{\frac{1}{2}}} \quad \tilde{d}_{rs} = \frac{F(k_r, -k_s)}{[F(k_r, k_r)F(k_s, k_s)]^{\frac{1}{2}}} \tag{17}$$

Eqs. (16) contain a few "surprises":

- The (BEC) for identical charged pions is different from those of neutral pions. In particular $MaxC_2^{00} = 3$ while $MaxC_2^{--} = 2$. [2]
- There exists a QS correlation C_2^{+-} between positive and negative pions.

With the parametrizations (10),(12) we have

$$\begin{aligned} d_{rs} &= \exp\left(-\frac{(\vec{k}_1 - \vec{k}_2)^2 R^4}{8(R^2 + L^2)}\right)\exp\left(-\frac{(E_1 - E_2)^2 R_0^4}{8(R_0^2 + L^2)}\right) \\ \tilde{d}_{rs} &= \exp\left(-\frac{(\vec{k}_1 + \vec{k}_2)^2 R^4}{8(R^2 + L^2)}\right)\exp\left(-\frac{(E_1 + E_2)^2 R_0^4}{8(R_0^2 + L^2)}\right). \end{aligned} \tag{18}$$

Here R_0 is the lifetime of the source of radius R.
It results that the "new" terms $\tilde{d}_{rs}$ are in general suppressed as compared with the "old" ones d_{rs} and that this suppression can be partially overcome for soft pions. The importance of the new terms $\tilde{d}$ lies in the fact that (i) they present two particle coherent states $< aa >$, i.e. squeezing [3] in the same way as terms of the form $< a >$ represent single particle coherent states; (ii) they prove explicitly that Bose-Einstein correlations are more complex than would be expected from the two-particle symmetrized wave function approach which does not lead to particle-antiparticle correlations; (iii) the normalization procedure in terms of $(+-)$ pairs which is used in experimental BEC studies has to be qualified for soft pions.

[2] It appears as if neutral pions would be "more equal" than charged ones.

Another consequence of the space-time approach (eq.(18)) is the fact that it provides a clear relation between the correlation length L and the geometrical radius R. Furthermore it appears that (BEC) do not measure R (as was thought so far) but rather a combination of R and L, i.e.

$$r_L^2 = \frac{R^4}{R^2 + L^2}. \tag{19}$$

The effects discussed above refer to chaotic fields. In the presence of coherence supplementary "surprises" emerge. Indeed it can be shown that the presence of coherence manifests itself not only through the modification of the intercept $C_2(0)$ but also in the dependence of C_2 on k_1, k_2.

While almost all experimental studies before 1989 considered an one exponential structure of C_2 and tried to estimate the chaoticity by extrapolating C_2 to the origin, QS in fact predicts in the presence of coherence a two-exponential structure, the two exponents being related in a very definite way. This result had in fact been pointed out already in 1978 by Biyajima, Miyamura, Nakai [4] but its relevance for the experimental determination of chaoticity was emphasized only much later [5]. Its practical importance lies among other things in the fact that the determination of the intercept $C_2(0)$ is hampered by the experimental difficulty of measuring very small values of $|\vec{k_1} - \vec{k_2}|$. The two-exponent structure shifts in part the burden of measurements to larger values of $|\vec{k_1} - \vec{k_2}|$, where these difficulties are present to a lesser degree. The first experimental attempt to detect the two-exponent structure has been reported in ref. [6]. Because of the limited statistics, however, no conclusive results could be obtained. Progress along these lines may be facilitated by the most recent development on this subject when it has been realized that the two-exponential structure may be observable only for soft pions [7].

Indeed, by using eqs. (3)-(4) we get

$$\begin{aligned} C_2(k_1,k_2) &= 1 + 2p(0)[1-p(0)]\exp\left[-\frac{1}{4}(k_1^2+k_2^2)r_L^2\right] \cdot \\ &\quad \cdot \exp\left[-\frac{1}{8}(k_1-k_2)^2 r_L^2\right] / [A(k_1)A(k_2)] \\ &+ p^2(0)\exp\left[-\frac{1}{4}(k_1-k_2)^2 r_L^2\right] / [A(k_1)A(k_2)]. \end{aligned} \tag{20}$$

The metric is Euclidian, i.e. $k^2 = \vec{k}^2 + k_0^2$ and

$$A(k_i) = p(0) + [1-p(0)]\exp\left[-\frac{1}{2}k_i^2 r_L^2\right]. \tag{21}$$

Here

$$p(0) = \frac{R_L^4 C(0)}{R_L^4 C(0) + R^4|\pi_0|^2}, \quad R_L^2 = \frac{R^2 L^2}{R^2 + L^2}, \quad \pi_0 = \pi_\alpha(0). \tag{22}$$

For simplicity in (20) we assumed $R = R_0$.

Eq. (20) contains the new elements mentioned above: There are two exponentials which differ by a factor of 2 and the weights of which are in the ratio $2p(0)[1-p(0)]/p^2(0)$. The radius and the correlation length appear again in the combination r_L^2 and, last but not least, the first exponential, which contains the interference between the coherent field $(1-p(0))$ and the chaotic field $p(0)$ is multiplied by a suppression factor $exp(-\frac{1}{4}(k_1^2+k_2^2)r_L^2)$.

3.2 Multiplicity dependence of BEC due to quantum statistical constraints

Another application of QS to BEC lies in the dependence of BEC on the multiplicity [8]. As is well known, the operators for the field (intensity) and number of particles do not commute. This means that measurements of "true" BEC can be performed only when no restriction on the multiplicity n, which fluctuates from event to event, is made. In practice, however, such restrictions are imposed, either simply because to measure a 2 identical particle correlation, one has to select events with at least 4 charged particles, or because of theoretical prejudices (centrality). This introduces a bias which has been estimated in [8] and we refer the reader to this reference for technical details. We quote here only some of the results (cf. Fig. 1):

1. C_2 decreases noticeably with the cut in n, n_{min}. This effect is more pronounced the larger n_{min} is.

2. C_2 at fixed n and at a given value of $|y_1 - y_2|$ is always smaller than the corresponding unbiased C_2.

These results illustrate again the importance of a consistent QS treatment of BEC. The "old fashioned" two-particle wave function approach cannot yield similar results.

4 Multiplicity Distributions and Quantum Statistics

4.1 Rapidity dependence - "Beta" scaling

One of the straightforward consequences of QS applied to multiplicity distributions P_n is the prediction contained already in [1] of the broadening of P_n with the decrease of the width of the (symmetric) rapidity window δy. The surprise connected with the discovery of this effect in subsequent experiments at the CERN collider by the UA-5 collaboration was just due to the fact that the QS result was practically unknown to the high energy physics community and new attempts to interpret it were obviously condemned to repetition of old derivations. Indeed, the observed rapidity dependence follows from the dependence of the factorial cumulants μ_r on δy

$$\mu_r = \frac{< n >^r}{\nu^{r-1}}\left[(r-1)!p^r B_r + r!p^{r-1}(1-p)\bar{B}_r\right]. \tag{23}$$

In this relation derived in quantum optics by Jaiswal and Mehta in 1970 and proposed for the description of multiparticle dynamics already in [1] ν is the number of "cells", $\nu = mf$, where m is the number of independent charge states and f the number of independent

sources, p is the chaoticity (the momentum dependence of p is not considered here) and $B, \tilde{B}$ are analytical functions of $\beta \simeq \delta y/\xi$. The form of $B, \tilde{B}$ determines the δy dependence of μ and the fact that μ does not depend on δy and ξ separately but rather on β was called "beta" scaling [9].
The functions B and $\tilde{B}$ can be calculated from the formulae:

$$B_r = \frac{1}{(\delta y)^r} \prod_{j=1}^{r} \int_0^{\delta y} dy_j \gamma(y_1, y_2) \cdots \gamma(y_{r-1}, y_r) \gamma(y_r, y_1) \tag{24}$$

$$\tilde{B}_r = \frac{1}{(\delta y)^r} \prod_{j=1}^{r} \int_0^{\delta y} dy_j \gamma(y_1, y_2) \cdots \gamma(y_{r-1}, y_r) \tag{25}$$

where the functions γ are defined by eq. (3).
Eq. (23) assumes stationarity in y and it is understood that the other variables (e.g. p_T) are averaged out. Eq. (23) reduces for $p = 1$ and $\beta = 0$ to
$\mu_r = < n >^r (r-1)!/\nu^{r-1}$ which is the negative binomial result, while for $p = 0$, $\mu_r = 0, r \geq 2$ we get the Poisson distribution. We see thus that the negative binomial (n.b.) distribution, which made headlines in the mid-eighties is a particular case of the distribution (23) put forward in 1978. Moreoever the QS formalism appears superior to the n.b. approach not only because it is more general but also because the theoretically not understood δy dependence of the n.b. parameter k has a very clear and definite meaning in QS being just a manifestation of the two-particle correlations γ which enter the functions B (linked-pair correlations) and ($\tilde{B}$) (ring-pair correlations).
A particular case of the rapidity dependence of μ_r is the forward-backward (F-B) correlation. It could be shown [9] that a consistent description of the experimental data of hadronic, semihadronic and e^+e^--reactions involving both the δy dependence and the F-B correlation is possible within this QS frame.

4.2 Quantum statistics and intermittency

The rapidity dependence of moments of multiplicity distributions has become in the last five years a testing ground for another interesting phenomenon, namely intermittency [10]. Intermittency leads to a power law dependence of the normalized factorial moments

$$\Phi_r = F_r / < n >^r \sim (\delta y)^{-\alpha}, \tag{26}$$

where α is an increasing function of r. These are related to the factorial cumulants μ_r by the relation

$$\exp\left[\sum_{n=1}^{\infty} \frac{\mu_n S^n}{n!}\right] = \sum_{k=0}^{\infty} \frac{F_k S^k}{k!} \tag{27}$$

which is to be understood as an identity in S. Given the fact that QS predicts qualitatively the same behaviour as that contained in (26), i.e. a decrease with δy which becomes more pronounced the higher the order r of the moment, it was interesting to check up to which point the predictions of QS and intermittency coincide. It is clear, however, that for very

small values of δy eqs. (26) and (24) must differ because μ_r contains a finite scale in rapidity, ξ, while (26) assumes the absence of such a scale. The presence of an ultimate scale leads to a saturation of $ln\Phi_r$ as a function of $-ln\delta y$ (cf. Figs. 2,3). In ref. ([11]) a comparison of the prediction of QS represented by eq. (24) and data was performed for hadronic and nuclear reactions. Before discussing these results it is important to note that eq. (24) assumes among other things: (i) identical particles, (ii) stationarity in rapidity. On the other hand the data available in 1988, when this comparison was performed, referred to charged particles and the condition of stationarity was fulfilled only for the p-p data in the very central region. Nevertheless, as can be judged from Figs. 2,3, the agreement between eq. (23) and data is good, including the tendency of saturation. This was the first conventional explanation of the intermittency effect, and it was substantiated further in [12] where the importance of a three-dimensional analysis was emphasized. However, in [12] an apparently obvious "test" of the QS mechanism in intermittency was proposed and this test turned out to be negative. The test assumes that the QS mechanism is based entirely on Bose-Einstein correlations and therefore the intermittency slopes for identical particles α_{id} should be twice as large as those for charged ones α_{ch}. Experimental data on the other hand showed that $\alpha_{id} \simeq \alpha_{ch}$. This apparent failure of the QS interpretation of the intermittency data may have several explanations:

(i) it could mean that the intermittency phenomenon is real and that the hadronic scale ξ is, for reasons not yet understood, absent in the multiplicity data. However, before reaching this conclusion, other more conventional explanations have to be considered, within the QS framework. These are:

(ii) there exist correlations due to strong interactions between the charged particles (pions) which are similar to those between identical particles. This theoretical possibility was discussed already in ref. [9] in connection with the observation by the Na22 collaboration that the effective negative binomial parameter for charged data k_{ch} is related to the effective k of identical particle data k_{id} by the relation

$$k_{ch} = \frac{1}{2} k_{id} \tag{28}$$

rather than the naive expectation $k_{ch} = 2k_{id}$ which assumes independent charges. Support for this possibility comes from a recent investigation by Shih [13] which will be discussed in the next section.

(iii) the quantum statistical correlations between $\pi^{+\prime}s$ and $\pi^{-\prime}s$ discussed in section 3.1 (ref. [2]) contribute to the intermittency effect and thus the naive expectation

$$\alpha_{id} = 2\alpha_{ch} \tag{29}$$

does not hold. Indications in this direction come from the observation that the intermittency effect is strongly enhanced for soft pions [14] where also the QS $\pi^+\pi^-$ correlation is enhanced. A quantitative investigation of this possibility is under way [15].

4.3 Charge correlations and quantum statistics

The difference between quantum statistics as applied to optics and that specific for multiparticle dynamics lies in the nature of strong interactions. One of the consequences of strong interactions is the existence of charge correlations due e.g. to resonances. We mentioned in the preceding section the importance of this effect in the QS interpretation of multiplicity distributions. In this section we shall briefly outline an approach to this problem due to Shih [13] which could serve as an explanation why despite the existence of strong charge correlations, the multiplicity distributions of charged particles are nevertheless similar to those of identical particles.
The idea consists in trying to parametrize the charged field correlations in the same way as the identical field chaotic correlations, the difference being concentrated in the strength factor multiplying the corresponding correlations. One extends thus eq. (3) by writing

$$\Gamma(y_1, y_2; --) = \Gamma(y_1, Y_2; ++) = \frac{1}{A}\Gamma(y_1, y_2; +-) = \Gamma_0 \exp(-|y_1 - y_2|/\xi). \qquad (30)$$

The chaoticity p and the correlation length ξ are assumed to be the same for charged and identical particles. This is one of the simplest possible generalizations of QS, demanding only one parameter more than in the case of identical particles, namely the specific charge-correlation strength A, and surprisingly enough it appears sufficient to describe the available intermittency data, both for charged and identical data. This is seen in Fig. 4. The fact that the parameter A turns out to be much larger than unity confirms the expectation that charge correlations are much stronger than identical particle correlations. Given the simplicity of the above ansatz, the phenomenological success of fitting both charged and identical data within the same QS formalism, with only 3 parameters (p, ξ, A) is quite remarkable and suggests that the quoted multiplicity data, at least in one dimension y, can be understood without invoking the intermittency conjecture.

4.4 Three dimensional analysis and non-stationary effects

The importance of a three-dimensional analysis of the multiplicity data was emphasized in ref. [12] and in [17] as well as in all subsequent experimental studies (cf. e.g. this volume). Furthermore the role of the stationarity assumption in rapidity in the QS approach had been realized from the very beginning [1]. The only support for this assumption lies in the presumed flatness of the rapidity distributions. As a matter of fact this flatness is restricted to the very central region. Moreover, it is only a necessary condition for stationarity as can be readily seen from eq. (3) which correlates <u>two</u> points y_1, y_2, rather than referring only to the particular case $y_1 = y_2$ which enters the rapidity distribution. A first investigation of the role of a three-dimensional analysis and of the stationarity assumption in the QS analysis of the multiplicity data has been performed in ref. [18].
One generalizes eq. (3) by the ansatz

$$< \pi^+(y_1, p_{T1})\pi(y_2, p_{T2}) > = \gamma_y(y_1, y_2)\gamma_T(\vec{p}_{T1}, \vec{p}_{T2}). \qquad (31)$$

This is still not quite general because it assumes factorization in (y, p_T). Partial support for this assumption comes from the observation that in a first approximation the single inclusive cross section factorizes.

Eq. (23) retains its form but the functions $B, \tilde{B}$ of eqs. (24),(25) are replaced by

$$B_r = B_{yr} B_{Tr}, \qquad \tilde{B}_r = \tilde{B}_{yr} \tilde{B}_{Tr} \qquad (32)$$

where B_{yr} are the expressions (24),(25) and $B_{Tr}, \tilde{B}_{Tr}$ are the corresponding integrals in p_T space. γ_T is either parametrized by a Gaussian

$$\gamma_T \sim \exp(-|\vec{p}_{T1} - \vec{p}_{T2}|^2/\xi_T^2) \qquad (33)$$

or an exponential

$$\gamma_T \sim \exp(-|\vec{p}_{T,1} - \vec{p}_{T,2}|/\xi_T). \qquad (34)$$

The differences between these two forms appear to be small in applications. A further generalization of the QS treatment performed in [18] refers to non-stationary processes in rapidity. This problem has two aspects: (i) the change of multiplicity distribution with the change of the width δy of the symmetric rapidity window, (ii) the change of $P(n)$ with the shift of the rapidity window. Property (i) is already a consequence of the simplified formalism (23). To account for (ii) two approaches were used. The first interpreted the effect (ii) in terms of two independent sources, one completely coherent and another completely chaotic. A discussion of this mechanism will be given in the next section. The other approach attempts to describe this effect again by a linear superposition of a coherent and chaotic field (eq. (2)) as described below. This second approach applies as long as the deviation from stationarity is small. One generalizes the function $\gamma(y_1 - y_2)$ which enters eq. (3) by writing instead

$$\gamma(y_1, y_2) = \sqrt{p(y_1)p(y_2)\frac{dn}{dy_1}\frac{dn}{dy_2}} \exp(-|y_1 - y_2|/\xi). \qquad (35)$$

The dependence of the chaoticity on rapidity reflects the fact mentioned above that multiplicity distributions change their shape when shifting the rapidity window. In particular it is known that in the center the distribution is broader than at large y. This is also reflected by the forward-backward correlation. As to the rapidity distribution dn/dy it is taken from experiment, considering the deviations from the plateau. After comparing this generalized QS formalism with data, one finds [18] that the generalized QS formalism leads to a "renormalization" of the parameter p and ξ. In particular an increase of p and a slight decrease of ξ is obtained, as compared with the one dimensional, stationary formalism. Agreement with the data for identical particles of the Na22 collaboration is satisfactory as can be seen from Fig. 5.

4.5 Two-component approach to multiplicity distribution

As mentioned in the previous section, an alternative interpretation of the change of $P(n)$ with the position of the rapidity window was given [19] in terms of two independent sources, one concentrated in the center and totally chaotic described by a negative binomial distribution P_{NB} and another concentrated at larger rapidities and completely coherent, described by a Poisson distribution P_P:

$$P(n) = \sum_{n_a, n_b} P_P(n_a) P_{NB}(n_b) \delta(n - n_a - n_b). \tag{36}$$

With this conjecture not only the multiplicity distribution in shifted rapidity windows could be explained but also the forward-backward asymmetry [20].
The remarkable outcome of the comparison of the consequences of this ansatz with data at various energies was that the mean multiplicity n_a emerged as a slow (logarithmic) function of s suggesting a bremsstrahlung-like mechanism while n_b appeared as a much stronger function ($s^{1/4}$) consistent with the expectations of the Landau hydrodynamical model. This consequence illustrates quite clearly the double-prong strategy outlined in the introduction for a description of fluctuations via QS on the one hand, and mean values via a hydrodynamical model on the other. Given the possible important implications of this result a more microscopic derivation of the two component model sketched above appeared desirable. Such a derivation was given in ref. [21] in terms of a stochastic branching equation:

$$\begin{aligned} \frac{\delta}{\delta t} P(n_a, n_b; t) &= \mu_0 [P(n_a - 1, n_b; t) - P(n_a, n_b; t)] \\ &+ \lambda_0 [P(n_a, n_b - 1; t) - P(n_a, n_b; t)] \\ &+ \lambda [(n_b - 1) P(n_a, n_b - 1; t) - n_b P(n_a, n_b; t)] \end{aligned} \tag{37}$$

where $\mu_o, \lambda_0, \lambda$ are production rates in the interval dt (cf. Fig. 6).

5 Conclusions

Energy dependence of multiplicity distributions in the quantum statistical approach

As stated in the introduction one of the main goals of the strategy outlined in this review is to determine from experiment the "laws" which govern multiparticle dynamics, i.e. the energy dependence of the main physical observables. The choice of these observables is dictated by very general considerations based on QS. In particular in our case these observables for identical particles are the mean multiplicity $< n >$, the chaoticity p and the correlation length L (ξ). The implementation of such a program is unfortunately hampered at present for hadronic reactions by the absence of data for identical particles at higher energies. The only available data for both charged and identical particles is due to the Na22 collaboration at $\sqrt{s} = 22$ GeV. On the other hand we have seen that (i) at this (rather low) energy both charged and identical data can be described by the QS formalism, the difference between charged and identical data being taken care of by the specific charge correlation parameter A (cf. eq. (32)); (ii) the charged data can be described at all energies available (up to $\sqrt{s} = 900$GeV) with the same QS formalism. This encouraged us to study the energy dependence of multiparticle production as reflected in the multiplicity distributions by the QS formalism and the results of this study will be presented below. Given the necessarily preliminary character of such a study, due to the absence of identical particle data, we have neglected, in a first approximation, the

corrections for non-stationarity and three-dimentional kinematics. This also implies that the correlations are defined in rapidity space (eq.(3)) and not in configuration space. [3]
Another aspect of the energy dependence is related to the fact that most multiplicity data refer to the central rapidity region. It is known that in this region the inelasticity K is also energy dependent. This means that the energy available for particle production

$$w = K\sqrt{s} \tag{38}$$

which determines the observables $< n >, p, \xi$ is a rather involved function of $\sqrt{s}$. This fact was taken into account in the analysis of the data.
The results of this analysis [22] are contained in the Figures (7-9) where the three characteristic quantities are represented as a function of s. In these figures predictions for Tevatron and SSC energies are also made. Besides the previously known increase of the mean multiplicity $< n >$ with s, the remarkable feature of these results is that both the rapidity correlation length ξ and the chaoticity p increase quite substantially with s. The absolute values of p as given in Fig. 7 are, of course, not realistic because the p_T dependence was not considered (cf. section 4.4). This omission amounts to the fact that the p values of Fig. 7 are strongly underestimated. They have to be "renormalized" by a factor of order 2. Nevertheless we believe that the overall s dependence is correct and reflects the broadening with s of the multiplicity distribution (deviation from KNO scaling). It appears that at low ($\sqrt{s} = 22$GeV) energies, at least the chaoticity $p << 1$, i.e. there is an appreciable amount of coherence. This conclusion, which follows from the analysis of multiplicity data is consistent with the results of Bose-Einstein correlations obtained with the same Na22 data, although the interpretation of BEC still suffers from many ambiguities.One of the main tasks for future theoretical and experimental work is the elucidation of the role of coherence in Bose-Einstein correlations and a more detailed joint analysis of multiplicity and BEC data.
Since the chaoticity is an order parameter limited, by definition from above by 1, Fig. 7 gives an indication at what energy the distribution will become completely chaotic and beyond which the present trend will have to change. (The energy at which p reaches its maximum corresponds to a phase transition.)

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Figure Captions

1. Second order BEC as a function of $y_g = |y_1 - y_2|$ for a purely chaotic distribution ($p = 1$). The successive dashed curves represent the analytical result of the unbiased correlation function as given by QS [5]. In Fig. 1a the data points represent the

[3] The relation between the correlation length in momentum (rapidity) space and configuration space is the subject of present investigations. Preliminary results are presented in I.V. Andreev and R.M. Weiner, Multiparticle Dynamics 1990, eds. R. Baier, D. Wegener, World Scientific, p. 519.

results of a Monte Carlo simulation without any bias on n. The other parts of Fig. 1 represent the results of the MC simulation with various fixed n_{min} (from ref. [8]).

2. Normalized factorial moments ϕ_q of order q in finite (pseudo) rapidity windows of width δy centered around $y_{CMS} = 0$, plotted against δy or $\delta\eta$ (pseudorapidity) for : (a) $p\bar{p}$ collisions at $\sqrt{s} = 540$GeV,(η) and (b) pp collisions at $\sqrt{s} = 22$ GeV(y). The order q is indicated next to the curves. These have been computed using eq. (23) with: (a) $p = 0.44$ and $\xi = 4.0$ and (b) $p = 0.32$ and $\xi = 1.0$ (from ref. [11]).

3. Averages of normalized factorial moments ϕ_q of order q in finite rapidity windows of width δy shifted across a wide rapidity range in each event, plotted against δy or $\delta\eta$ for: (a) pp collisions at $\sqrt{s} = 22$GeV(y), and (b) $^{32}S + Au$ collisions at 200 A GeV (η). The order q is indicated next to the curves. These have been computed using eq. (24) with : (a) $p = 0.20$ and $\xi = 0.9$ and (b) $p = 0.015$ and $\xi = 0.2$ (from ref. [11]).

4. Factorial moments $f_{q,ch}$ and $f_{q,-}(q = 2-4)$ as a function of the rapidity window δy ($|y| < \delta y/2$). (from ref. [13]). Dashed curves: $C = 2, p = 0.17, \xi = 0.70$. Solid curves: $C = 3.5, p = 0.11, \xi = 0.65$. Data points are experimental values obtained by the Na22 collaboration.

5. Comparison of experimental data on negative particles from Na22 experiment with values derived from Eqs. (31) through (35). Data points of the normalized factorial moments $f_q(q = 2,3,4)$ are derived from the experimental negative binomial fits of Na22 data [16] plotted against the rapidity window Y on a log-log plot, with $|y| < Y$ (from ref. [18]).

6. Typical evolution processes where two types of particles (partons) a and b are produced by a virtual source (s). The production vertices are characterized by couplings μ_0 and λ_0 respectively. The bottom process corresponds to the evolution (branching) of a single particle (partons) b. The corresponding production vertex is characterized by a coupling λ.

7. s-dependence of p. Errors are shown for values extrapolated to 2 TeV and 40 TeV from ref. [22]).

8. s-dependence of the correlation length ξ. Errors are shown for values extrapolated to 2 TeV and 40 TeV (from ref. [22]).

9. s-dependence of $< n >_{ch}$. Experimental data shown as diamonds; extrapolations are shown as asterisks; extrapolations according to the quadratic expression in lns are shown in squares (from ref. [22]).

10. s-dependence of normalized initial moments C_q of the charged multiplicity distribution for $q = 2,...,5$. Symbols are the same as in Fig. 9 (from ref. [22]).

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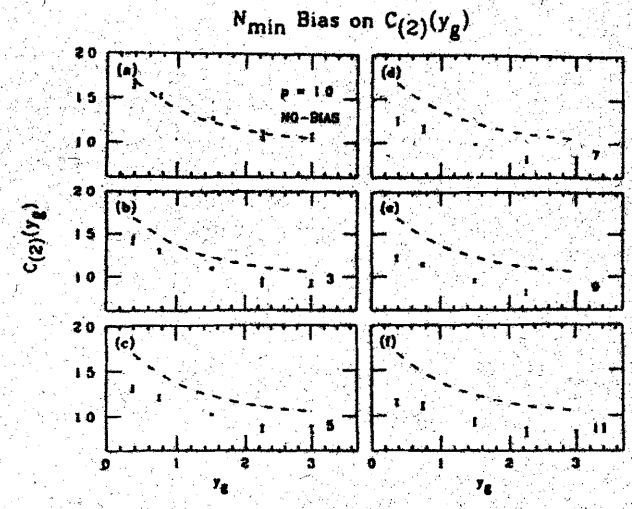


Fig. 1

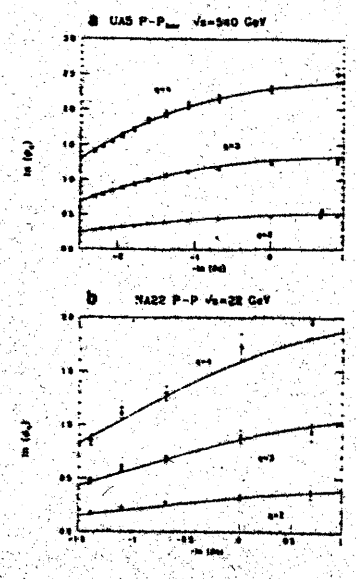
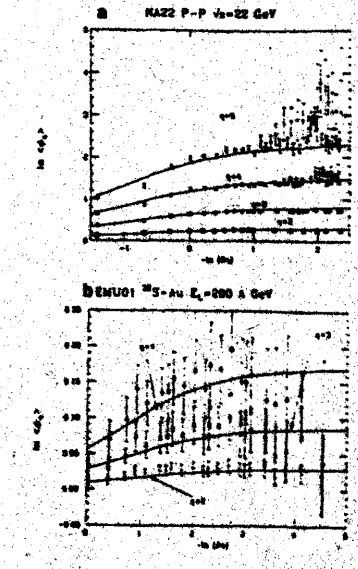


Fig 2



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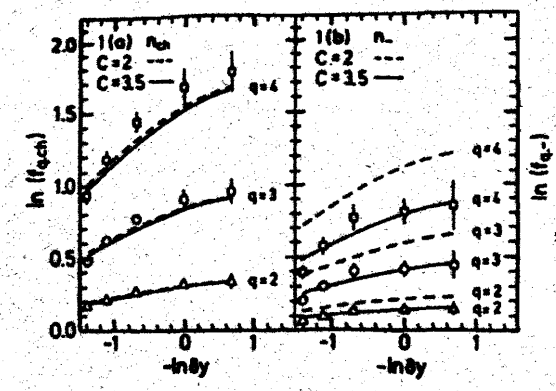

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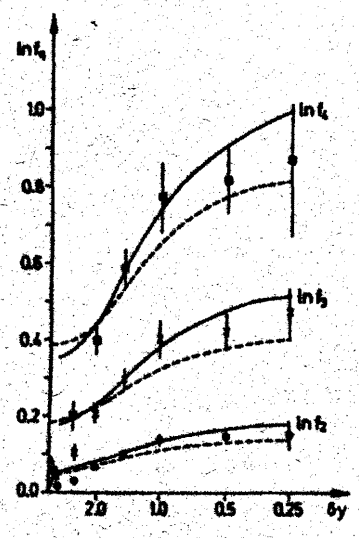

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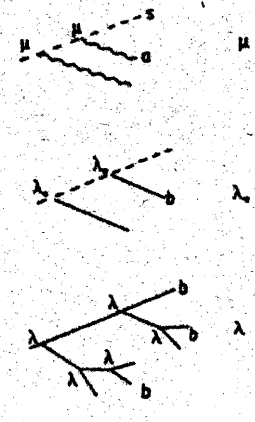

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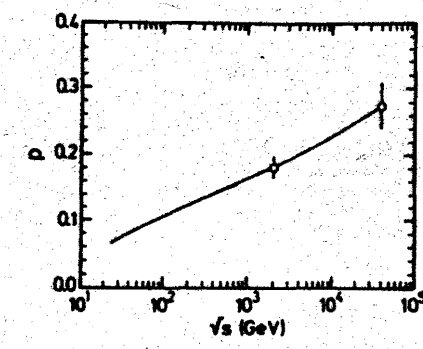


Fig. 7

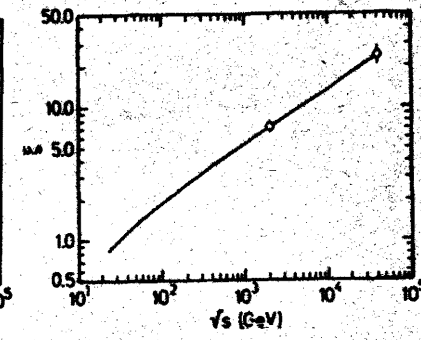
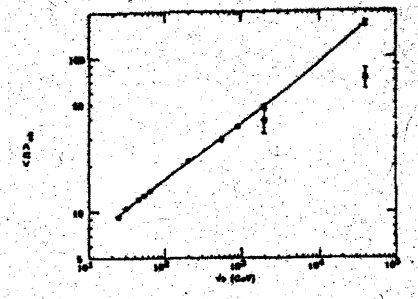


Fig. 8



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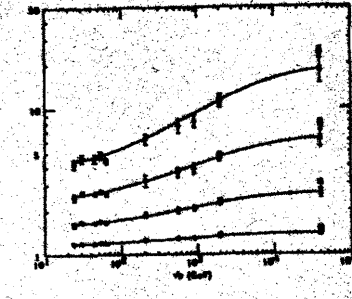

Fig. 10