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Quasi-Long Range Order in a Flavored Anyon Model

C. Aragão de Carvalho* and D. Boyanovsky

Department of Physics and Astronomy

University of Pittsburgh

Pittsburgh, P.A, U.S.A

We study aspects of long range order in a model consisting of two interacting layers of anyon gases with opposite statistical charge. We start with bosons with a statistical interaction and second quantize the theory using a novel approach that uses a local and gauge invariant order parameter. We then study this model in the Bogoliubov approximation. We explicitly compute the depletion from the condensate and *gauge invariant* correlation functions. We show that the ground state is a "weak" condensate of charge-flux composites with "quasi long- range order" but finite helicity modulus at zero temperature. The condensate becomes statistically neutral in the thermodynamic limit. It is pointed out that the "quasi long range order" at zero temperature has its origins in the logarithmic infrared singularities arising from the *two dimensional* Coulomb repulsion induced by the long range statistical interaction. The quasiparticle spectrum consists of one massless mode, and a plasmon mode with a weak gap. The system is quasi-ordered at any non-zero temperature.

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Introduction and Motivation:

Since its original conception[1–4], fractional statistics gained much attention both in condensed matter physics and field theory. In two-space, one time dimensional field theories, it was shown at the semiclassical level that the solitons of the nonlinear sigma models with Hopf term have fractional spin[2, 3]. Halperin[5] observed that the quasiparticles of the Laughlin states[6] for the fractional Hall effect obey fractional statistics, namely, they are *anyons*[7]. Effective theories describing excitations with fractional statistics, were first proposed within the context of the Fractional Quantum Hall Effect (FQHE) by Girvin[8], Girvin and MacDonald[9], Girvin, MacDonald and Platzman[10], Zhang, Hansson and Kivelson[11], Read[12], and Lee and Zhang[13]. This description incorporates a "statistical" gauge field whose dynamics is determined by a Chern-Simons (CS) term in the Lagrangian[14]. The (CS) term, attaches magnetic fluxes to charged particles and fractional statistics is a result of the Aharonov-Bohm phase[15].

In an important development, Laughlin[16] and Fetter, Hanna and Laughlin[17] developed a mean field theory for the anyon gas and showed that fluctuations in an RPA-like approximation give a Goldstone pole in the current-current correlation function implying the Meissner effect. Chen et. al.[18] argued that this Goldstone pole is exact. This anyon gas has also been studied in mean field approximation by Hosotani et. al[19]. More recently, the RPA and Hartree treatments have been improved incorporating the conservation laws for anyons[20, 21].

The motivation of this paper is to study aspects of off-diagonal long range order in anyon theories, by proposing a model of two layers of anyon gases of opposite statistical charge, that also interact via two-body potentials. We start with bosons and introduce the statistical interaction by coupling the bosons to a Chern-Simons "statistical" gauge field. We first introduce a consistent gauge invariant method of second quantization that reveals a local gauge invariant order parameter associated with charge-flux composites.

The reason for this step is that a local gauge symmetry *cannot be broken* (see below) and off-diagonal long range order must be studied with gauge invariant operators. A proper order parameter must be gauge invariant *and* transform non-trivially under the *global* symmetry in question, in this case global phase transformations. A gauge non-invariant operator will have arbitrary expectation values in a fixed gauge. One may instead look for long range order without invoking an order parameter by studying correlation functions, but then one must necesarily append non-local line integrals of the gauge fields to ensure gauge invariance. Our quantization method bypasses these ambiguities that are often overlooked in the literature. This we believe to be a novel and consistent treatment.

Secondly, we study this model of two interacting anyon gases (two flavors) in the Bogoliubov approximation. We show, in this approximation, that the ground state is a condensate of charge-flux composites, that becomes statistically neutral in the thermodynamic limit.

When the two-body potentials are short ranged we find both a plasmon and an acoustic phonon mode. In the case of *three dimensional* Coulomb interactions, we find a plasmon and a massless mode with a singular dispersion relation.

In both cases we find that the system has "quasi-long range order" at zero temperature, caused by a *two dimensional Coulomb repulsion* induced by the long range statistical interaction. This induced *two dimensional* Coulomb interaction introduces logarithmic divergences. Even zero temperature correlation functions for the acoustic branch (in case of short-ranged two body potentials) fall-off algebraically at long distances, signaling "quasi-long range order".

Despite this "quasi long range order" the helicity modulus (superfluid density) is finite. The system is disordered at non-zero temperature, in agreement with rigorous inequalities, which we confirm explicitly in the present model.

The model

We begin our description of the model by considering two bosonic particle fields

$(\psi_1\,;\psi_2)$ statistically charged with opposite statistical charge. The statistical interaction is introduced by coupling[8, 15, 18] these fields to a Chern-Simons gauge field[14]. It is more convenient to write the Lagrangian in terms of the doublet $\Psi = (\psi_1, \psi_2)$ as

$$\mathcal{L} = \Psi^\dagger \left[i\frac{\partial}{\partial t} - q\tau_3 A_0 \right] \Psi - \frac{1}{2m}\Psi^\dagger \left[-i\vec{\nabla} - q\tau_3 \vec{A} \right]^2 \Psi - V(\Psi^\dagger, \Psi) + \frac{\theta}{2}\partial_\mu A_\nu A_\lambda \tag{1}$$

With τ_3 the diagonal Pauli matrix and the potential $V(\Psi^\dagger, \Psi)$ is gauge invariant and will be specified later. For only one species, the above Lagrangian density coincides with the one proposed by Girvin[8], Girvin and MacDonald[9], Girvin, MacDonald and Platzman[10], and studied as an effective field theory for the FQHE by Zhang, Hansson and Kivelson[11], Read[12] and Lee and Zhang[13]. In this approach, the superfluid properties of the bosonic theory were identified with the properties of the fractional Hall state, and argued to describe qualitatively the long-wavelength physics.

However as mentioned in the introduction we are motivated to study the above model with two flavors in its own right, with particular attention to a detailed study of long range order. It is clear that we are *not* studying a Hall system in which parity and time reversal invariance are explicitly broken by the external magnetic field, and in which the statistical interaction is a result of the external magnetic field.

Rather, we motivate this model as a system of two layers of anyon gases with the two species corresponding to the layers. We will assume that there is no hopping between the layers and the number of anyons for each layer is conserved, this situation may be relaxed.

We depart from previous treatments of the one flavor case by the above mentioned authors, in that we introduce a novel gauge invariant second quantization scheme. This new scheme allows the unambiguos identification of local and gauge invariant order parameters which are necesary, as mentioned in the introduction, to study aspects of long range order. We then proceed to study the effects of fluctuations around the mean-field.

The gauge- invariant second quantization procedure that we now describe, begins by identifying the Hamiltonian. It is given by

$$H = \int d^2x \{ \frac{1}{2m} \psi_1^\dagger \left[-i\nabla - q\vec{A} \right]^2 \psi_1 + \frac{1}{2m} \psi_2^\dagger \left[-i\nabla + q\vec{A} \right]^2 \psi_2 + V + A_0 \mathcal{G} \} \quad (2)$$

$$\mathcal{G}(x) = \theta B(x) + q(\psi_1^\dagger \psi_1 - \psi_2^\dagger \psi_2) \quad (3)$$

$$B(x) = (\partial_1 A_2 - \partial_2 A_1) \quad (4)$$

Where we have written Gauss' law (3), in terms of the original fields ($\psi_{1,2}$) to show clearly the contribution of opposite statistical charges. Since we are imposing that the total number of particles of each species is conserved, the above Hamiltonian has a *global* $U(1) \times U(1)$ symmetry, under which the fields transform as

$$\begin{aligned} \psi_1(x) &\rightarrow \psi_1(x) e^{i\theta_1} \\ \psi_2(x) &\rightarrow \psi_2(x) e^{i\theta_2} \end{aligned} \quad (5)$$

with $\theta_{1,2}$ arbitrary.

The most general two-body potential that is compatible with this symmetry is:

$$V = \int d^2x d^2y \sum_{\alpha,\beta} \psi_\alpha^\dagger(x) \psi_\beta^\dagger(y) \mathcal{V}_{\alpha\beta}(x-y) \psi_\alpha(x) \psi_\beta(y) \quad (6)$$

Of the two $U(1)$ symmetries, only a diagonal combination is coupled to the gauge field corresponding to the local gauge transformation

$$\begin{aligned} \psi_1(x) &\rightarrow \psi_1(x) e^{i\theta(x)} \\ \psi_2(x) &\rightarrow \psi_2(x) e^{-i\theta(x)} \\ \vec{A}(x) &\rightarrow \vec{A}(x) + \frac{1}{q} \vec{\nabla} \theta(x) \end{aligned} \quad (7)$$

We will work in a sector in which the total number of particles of both species are equal. This sector is then statistically neutral and there is *zero* mean statistical magnetic field.

The quantization of the theory is achieved by imposing the following equal time commutation relations

$$\left[\psi_\alpha(x), \psi^\dagger_\beta(y)\right] = \delta_{\alpha\beta}\delta^2(x-y) \;\; ; \;\; \alpha,\ \beta = 1,2 \tag{8}$$

$$[A_i(x), A_j(y)] = \frac{i}{\theta}\epsilon_{ij}\delta^2(x-y) \tag{9}$$

$$[\psi_\alpha(x), A_i(y)] = [\psi^\dagger_\alpha(x), A_i(y)] = 0 \tag{10}$$

All other commutators vanish.

The Euler-Lagrange equations are obtained as Heisenberg equations of motion from the Hamiltonian (2) with the above commutation relations. Gauss' law $\mathcal{G}(x) = 0$, however, does not arise as a Heisenberg equation of motion and must be imposed as a constraint on the physical states. Gauss' law operator, $\mathcal{G}(x)$, is the generator of time independent gauge transformations, and the constraint on the physical states ensures gauge invariance of these states. This is the statement that the *local* gauge symmetry is never broken.

By writing $\vec{A}(x)$ in terms of single valued fields as

$$A_i(x) = \partial_i\eta(x) + \epsilon_{ij}\partial_j\chi(x) = \left(\vec{A}_L + \vec{A}_T\right)_i . \tag{11}$$

One has $-\nabla^2\chi(x) = B(x)$, and it is straightforward to show

$$[B(x), \eta(y)] = \frac{i}{\theta}\delta^2(x-y)\ ;\ \ [\eta(x), \eta(y)] = 0\ ;\ \ [B(x), B(y)] = 0. \tag{12}$$

In contrast to other treatments, we emphasize on studying the *physical spectrum and gauge invariant operators.* We are interested in studying broken symmetries and want to identify a *local and gauge invariant* order parameter. The local gauge symmetry cannot be broken because Gauss' law operator, the generator of gauge transformations, must annihilate the states as a constraint. The fields $\psi_\alpha(x)$, ; $\alpha = 1,2$ are not invariant under local gauge transformations, thus their expectation value in the ground state must vanish by gauge invariance. One may fix a gauge, for example Coulomb gauge, however in a fixed gauge this operator may acquire a non-vanishing expectation value, but this is a gauge artifact. Alternatively one may decide not to look at the expectation value of $\psi(x)$ as an order parameter, but compute correlation functions. To properly ensure a gauge invariant result one must then append the line

integral of the gauge field, a highly non-local object. The main point of this discussion is to emphasize that only gauge invariant operators can create physical excitations and can reveal off-diagonal long range order.

These we consider to be important points that had not been addressed in previous treatments, and our consistent quantization without gauge fixing is one of the main differences between this work and previous attempts. The gauge invariant operators $\mathcal{O}$ must satisfy

$$[\mathcal{G}(x), \mathcal{O}(y)] = 0 \tag{13}$$

From equations (8- 10, 12) it follows directly that

$$\Phi_1(x) = \psi_1(x)e^{-iq\eta(x)} \; ; \;\; \Phi_1^\dagger(x) = \psi_1^\dagger(x)e^{iq\eta(x)} \tag{14}$$

$$\Phi_2(x) = \psi_2(x)e^{iq\eta(x)} \; ; \;\; \Phi_2^\dagger(x) = \psi_2^\dagger(x)e^{-iq\eta(x)} \tag{15}$$

$$\tag{16}$$

are gauge invariant operators. It is clear from the commutation relations (8,12) that these operators carry charge ∓ 1 respectively, and since

$$e^{iq\eta(y)}B(x)e^{-iq\eta(y)} = B(x) + \frac{q}{\theta}\delta^2(x-y),$$

these operators create an infinitely thin solenoid, i.e. a vortex in the gauge field, with flux $\mp q/\theta$ at the position of the charge. These are *gauge invariant operators that create charges and flux tube composites* (charge-vortex) as required by Gauss' law.

Notice, however, that since $[\eta(x), \psi(y)] = 0$, these operators obey the equal time commutation relations of the underlying charged fields

$$\left[\Phi_\alpha(x), \Phi_\beta^\dagger(y)\right]_\pm = \delta_{\alpha,\beta}\delta^2(x-y). \tag{17}$$

In the gauge invariant sector, the Hamiltonian may be written solely in terms of these operators. This is readily done by using the decomposition (11), writing the original fields $\psi_{1,2}$; $\psi_{1,2}^\dagger$ in terms of the operators $\Phi_{1,2}$; $\Phi_{1,2}^\dagger$ in (15) and normal ordering the Hamiltonian to avoid self divergences. One finds

$$H = \int d^2x \{ \frac{1}{2m} \Phi_1^\dagger(\vec{x}) \left[-i\vec{\nabla} - q\vec{A}_T \right]^2 \Phi_1(\vec{x}) + \frac{1}{2m} \Phi_2^\dagger(\vec{x}) \left[-i\vec{\nabla} + q\vec{A}_T \right]^2 \Phi_2(\vec{x}) \}$$
$$+ \int d^2x d^2y \sum_{\alpha,\beta} \Phi_\alpha^\dagger(x) \Phi_\beta^\dagger(y) \mathcal{V}_{\alpha\beta}(x-y) \Phi_\alpha(x) \Phi\beta(y) \tag{18}$$
$$A_{T,i} = \frac{q}{\theta} \epsilon_{ij} \partial_j \int d^2y G(\vec{x} - \vec{y}) (\Phi_1^\dagger(\vec{y}) \Phi_1(\vec{y}) - \Phi_2^\dagger(\vec{y}) \Phi_2(\vec{y})) \tag{19}$$

with $G(\vec{x} - \vec{y}) = 1/2\pi \ln(|x-y|/R)$ and R and infrared cutoff.

The Hamiltonian still maintains the *global* $U(1) \times U(1)$ symmetry corresponding to conservation of particle-flux composites of each species, which in terms of the gauge invariant operators becomes

$$\Phi_1(x) \rightarrow \Phi_1(x) e^{i\theta_1}$$
$$\Phi_2(x) \rightarrow \Phi_2(x) e^{i\theta_2} \tag{20}$$

Thus the gauge invariant charge-flux composite operators $\Phi_{1,2}$, $\Phi_{1,2}^\dagger$ are ideal candidates for an order parameter, they are invariant under the *local* gauge transformation but transform non-trivially under the *global* phase transformation. They can obtain a non- vanishing vacuum expectation value, and their correlation functions are automatically gauge invariant without the addition of line integrals.

The identification of these local gauge invariant order parameters is the main motivation and bonus of our rather un- conventional second quantization procedure.

Bogoliubov Approximation

We now turn to the analysis of the model defined above within the Bogoliubov approximation. We will work grand-canonically and introduce two different chemical potentials for the independently conserved particle-flux numbers of each species. Thus we introduce the operator

$$K = H - \mu_1 \int d^2x \Phi_1^\dagger \Phi_1 - \mu_2 \int d^2x \Phi_2^\dagger \Phi_2 \tag{21}$$

We work in the statistically neutral sector and assume that $< N_1 >=< N_2 >= N$ is macroscopically large and that the zero momentum condensate has N_{01}, N_{02} particles respectively. We expand the particle-flux operators in a volume Ω as

$$\Phi_1(x) = \sqrt{n_{01}} + \frac{1}{\sqrt{\Omega}} \sum_k b_k e^{-ikx} \tag{22}$$

$$\Phi_2(x) = \sqrt{n_{02}} + \frac{1}{\sqrt{\Omega}} \sum_k d_k e^{-ikx} \tag{23}$$

where $n_{0i} = N_{0i}/\Omega$. The Bogoliubov approximation consists in assuming $N_{01}, N_{02} \gg 1$ and keeping the quadratic terms in the b_k, d_k in the Hamiltonian. It is straightforward to see that, in this approximation, the two-body interactions arising from (19) in (18) *do not contribute* because of the transversality of $\vec{A}_T$. The three body interaction from (19) and the two-body potential terms are the only relevant contributions to the Hamiltonian in this approximation. We would like to stress that the zero momentum mode is a condensate of *charge-flux composites.*

The conditions

$$< G|b_k|G >= 0 \ ; \ < G|d_k|G >= 0 \tag{24}$$

(where $|G >$ represents the true quasiparticle ground state) are ensured in this approximation by the relations

$$\mu_1 = 2\mathcal{V}_{11}(0)n_{01} + 2\mathcal{V}_{12}(0)n_{02}$$

$$\mu_2 = 2\mathcal{V}_{22}(0)n_{02} + 2\mathcal{V}_{12}(0)n_{01}$$

with $\mathcal{V}_{\alpha\beta}(k)$ the Fourier transform of the two-body potentials. Up to constant terms we find

$$\begin{aligned} K = \sum_{k\neq 0} \{ & (b_k^\dagger b_k + d_k^\dagger d_k)\varepsilon(k) + (\frac{q^2}{\theta})^2 \frac{(n_{01}+n_{02})}{2mk^2} \times \\ & \left[\sqrt{n_{01}}(b_k + b_{-k}^\dagger) - \sqrt{n_{02}}(d_k + d_{-k}^\dagger)\right] \left[\sqrt{n_{01}}(b_{-k} + b_k^\dagger) - \sqrt{n_{02}}(d_{-k} + d_k^\dagger)\right] + \\ & \mathcal{V}_{11}(k)n_{10}(b_k^\dagger + b_{-k})(b_k + b_{-k}^\dagger) + \mathcal{V}_{22}(k)n_{20}(d_k^\dagger + d_{-k})(d_k + d_{-k}^\dagger) + \\ & 2\mathcal{V}_{12}(k)\sqrt{n_{01}n_{02}}(b_k^\dagger + b_{-k})(d_{-k}^\dagger + d_k)\} \end{aligned} \tag{25}$$

with $\varepsilon(k) = k^2/2m$.

It is evident in the above Hamiltonian, the instantaneous two- dimensional Coulomb repulsion induced by the statistical interaction. Its origin may be traced to

(19). The stability of the system is ensured by the statistical charge neutrality between the flavors. In the effective description of the FQHE in reference([11, 13]), the external magnetic field has precisely the value that neutralizes the statistical charge. Remember that the Chern-Simons Gauss' law (3) associates statistical charges with magnetic fields. In reference ([22]) a neutralizing background charge (equivalent to a magnetic field) was introduced to stabilize the system.

The above Hamiltonian may be diagonalized by a canonical transformation to quasiparticle operators. Since the steps are rather cumbersome and not very illuminating, we just give the main ingredients. It becomes convenient for later purposes to introduce the following intermediate variables:

$$\varepsilon_{11}(k) = \varepsilon(k) + \frac{4n_{01}n_{02}}{(n_{01}+n_{02})}\left[\mathcal{V}_{11}(k) + \mathcal{V}_{22}(k) + 2\mathcal{V}_{12}(k)\right] \tag{26}$$

$$\varepsilon_{22}(k) = \varepsilon(k) + 2(\frac{q^2}{\theta})^2\frac{(n_{01}+n_{02})^2}{mk^2} + \frac{4}{(n_{01}+n_{02})}\left[\mathcal{V}_{11}(k)n_{01}^2 + \mathcal{V}_{22}(k)n_{02}^2 - 2\mathcal{V}_{12}(k)n_{01}n_{02}\right] \tag{27}$$

$$\varepsilon_{12}(k) = \frac{4\sqrt{n_{01}n_{02}}}{(n_{01}+n_{02})}\left[\mathcal{V}_{11}(k)n_{01} - \mathcal{V}_{22}(k)n_{02} + \mathcal{V}_{12}(k)(n_{01}-n_{02})\right] \tag{28}$$

$$\Delta = \sqrt{(\varepsilon_{11}(k) - \varepsilon_{22}(k))^2 + 4\varepsilon_{12}^2(k)} \tag{29}$$

$$\tan(2\beta) = \frac{2\varepsilon_{12}(k)}{(\varepsilon_{11}(k) - \varepsilon_{22}(k))} \tag{30}$$

$$E_1(k) = \frac{\varepsilon_{11}(k) + \varepsilon_{22}(k)}{2} + \frac{\Delta}{2} \tag{31}$$

$$E_2(k) = \frac{\varepsilon_{11}(k) + \varepsilon_{22}(k)}{2} - \frac{\Delta}{2} \tag{32}$$

$$\cos(\alpha) = \frac{\sqrt{n_{01}}}{\sqrt{n_{01}+n_{02}}} \tag{33}$$

The Coulomb repulsion induced by the statistical Chern-Simons interaction only enters in $\varepsilon_{22}(k)$.

In terms of quasiparticle operators, the Hamiltonian, within this approximation becomes (up to a constant term)

$$K = \sum_k \left[\sqrt{\varepsilon(k)E_1(k)}\gamma_1^\dagger(k)\gamma_1(k) + \sqrt{\varepsilon(k)E_2(k)}\gamma_2^\dagger(k)\gamma_2(k)\right] \tag{34}$$

The bare particle and quasiparticle operators are related by

$$b_k^\dagger = \frac{\sin(\alpha+\beta)}{2}(\gamma_1^\dagger(k)F_1^+(k) + \gamma_1(-k)F_1^-(k)) + \frac{\cos(\alpha+\beta)}{2}(\gamma_2^\dagger(k)F_2^+(k) + \gamma_2(-k)F_2^-(k)) \tag{35}$$

$$d_k^\dagger = \frac{\cos(\alpha+\beta)}{2}(\gamma_1^\dagger(k)F_1^+(k) + \gamma_1(-k)F_1^-(k)) - \frac{\sin(\alpha+\beta)}{2}(\gamma_2^\dagger(k)F_2^+(k) + \gamma_2(-k)F_2^-(k)) \tag{36}$$

with

$$F_{1,2}^{\pm} = \left(\frac{\varepsilon(k)}{E_{1,2}(k)}\right)^{\frac{1}{4}} \pm \left(\frac{E_{1,2}(k)}{\varepsilon(k)}\right)^{\frac{1}{4}} \tag{37}$$

We now study the spectrum of collective excitations. To do this, we must decide what are the interactions $\mathcal{V}_{\alpha,\beta}(k)$. We consider to particular cases:

i) "short-range" interactions with $\mathcal{V}_{\alpha,\beta}(k \to 0) = \mathcal{V}_{\alpha,\beta}(0) =$ finite constant.

ii) *Three dimensional* Coulomb interaction with potentials in (6) $\mathcal{V}_{\alpha,\beta}(\vec{x}-\vec{y}) \propto |\vec{x}-\vec{y}|^{-1}$ with $\vec{x}$ three dimensional vectors. Since we are assuming a situation in which there are two parallel layers of anyon gases, separated by a distance d, this situation corresponds to

$$\mathcal{V}_{11}(k) = \frac{v_{11}}{k} \;;\; \mathcal{V}_{22}(k) = \frac{v_{22}}{k} \;;\; \mathcal{V}_{12}(k) = \frac{v_{12}e^{-kd}}{k} \tag{38}$$

with $v_{\alpha,\beta}$ being constant coefficients. In both cases, however, it becomes clear that the strongest infrared singularity is arising from the *two-dimensional* Coulomb interaction induced by the Chern-Simons term as is evident in $\varepsilon_{22}(k)$. For the case i) of "short-range" interactions, we find in the long-wavelength ($k \to 0$) limit

$$\sqrt{\varepsilon(k)E_1(k)} \to \omega_{pl} = \frac{q^2(n_{01}+n_{02})}{m\theta} \tag{39}$$

$$\sqrt{\varepsilon(k)E_2(k)} \to ck \tag{40}$$

Then we see that one branch corresponds to a "plasmon" mode and the other to an acoustic phonon mode. The physical interpretation of these two modes is very

simple: by retracing the canonical transformations we see that the plasma mode corresponds to the combination of flavors that couples with the two- dimensional (logarithmic) interaction induced by the statistical interaction. It corresponds to an out-of-phase oscillation of the anyon gases and a statistical charge density wave, and ω_{pl} is the plasma frequency of the charged bose- gas[23]. The other branch corresponds to an in-phase oscillation, a "sound wave". The condensate breaks the original $U(1) \times U(1)$ global symmetry, however a particular combination has a *local gauge invariant coupling* and thus interacts with long-range forces. The low energy mode that would correspond to a Goldstone boson of the global broken symmetry in the case of short range forces, dissapears giving rise to a gap in the spectrum. This is the equivalent of the Anderson-Higgs mechanism. The orthogonal combination, however, interacts with short-range forces, the acoustic phonon mode is the resulting Goldstone mode arising from the other U(1) global symmetry that is broken by the condensate.

In the effective Chern-Simons field theory for the FQHE, the plasma frequency ω_{pl} is identified with the inter- Landau level cyclotron mode[13, 25]. As we will se shortly, however, this plasma frequency, the gap in the plasmon spectrum, is *very weak* in the thermodynamic limit. The same infrared singularity that opens up the gap in the spectrum, is responsible for its vanishing in the thermodynamic limit in two dimensions (see discussion below).

For the case ii), when the interactions correspond to the *three-dimensional* Coulomb interaction, we find in the long- wavelength ($k \to 0$) limit

$$\sqrt{\varepsilon(k)E_1(k)} \to \omega_{pl} + C_1 k \tag{41}$$

$$\sqrt{\varepsilon(k)E_2(k)} \to C_2\sqrt{k} \tag{42}$$

with C_1 ; C_2 being constants. The linearly dispersing mode (41) (for low momentum) in the case of *three dimensional* Coulomb forces, agrees with the results of Kallin and Halperin[24] and Lopez and Fradkin[25]. The singularity in the quasiparticle density of states given by (42) is a peculiarity of the small two-dimensional phase space.

We now compute the depletion from the condensate for both species at zero temperature:

$$N_1' = N - N_{01} = \sum_{k\neq 0} < G|b^\dagger(k)b(k)|G > \tag{43}$$

$$N_2' = N - N_{02} = \sum_{k\neq 0} < G|d^\dagger(k)d(k)|G > \tag{44}$$

with $|G>$ the quasiparticle ground state. The Bogoliubov coefficients that relate the bare particle operators to the quasiparticle operators, have a strong infrared singularity arising from the *two-dimensional* Coulomb repulsion (originating from the statistical interaction), the leading low momentum singularities in the particle number operators are found to be

$$b^\dagger(k)b(k) \approx \{\frac{n_{01}q^2}{2\theta k^2}(2\mathcal{N}_1(k)+1) + \frac{n_{01}A}{4(n_{01}+n_{02})}\frac{(2\mathcal{N}_2(k)+1)}{k^\eta}\} \tag{45}$$

$$d^\dagger(k)d(k) \approx \{\frac{n_{02}q^2}{2\theta k^2}(2\mathcal{N}_1(k)+1) + \frac{n_{02}B}{4(n_{01}+n_{02})}\frac{(2\mathcal{N}_2(k)+1)}{k^\eta}\} \tag{46}$$

with A,B constants. $\eta = 1$ for short-range interactions, $\eta = 3/2$ for three-dimensional Coulomb interactions, and $\mathcal{N}_{1,2}(k)$ are the quasiparticle number operators.

In the thermodynamic limit we find at zero temperature

$$N_{01} \approx N_{02} \approx \frac{8\pi\theta}{q^2}\frac{N}{\ln N} \tag{47}$$

Thus we find that in the thermodynamic limit the *condensate fraction* vanishes weakly, and that the condensate becomes neutral. This results in the conclusion that the gap in the plasmon spectrum ω_{pl} vanishes weakly in the thermodynamic limit. For one flavor case, this result was previously obtained by one of us[22]. The occupation number of the bare particles (charge-flux composites) is strongly peaked near $k \approx 0$ but the logarithmic divergence arises from the statistically induced *two dimensional Coulomb interaction* and the small two dimensional phase space.

We also compute the fluctuation in the number of particles outside of the condensate

$$\delta_{1,2} = \left[< G|N'^2_{1,2}|G > - < G|N'_{1,2}|G >^2\right]^{\frac{1}{2}}$$

and we find

$$\frac{\delta_{1,2}}{N'_{1,2}} \approx \frac{q}{\theta}\frac{1}{\ln N} \tag{48}$$

vanishing more slowly that for typical short-range bose systems which have $(\ln N/N)^{\frac{1}{2}}$ behavior in two dimensions.

The zero temperature correlation functions of the *gauge invariant* charge-flux operators are found to have the asymptotic behavior

$$< G \mid \Phi^\dagger_\alpha(\vec{r})\Phi_\alpha(0) \mid G > \rightarrow n_{0\alpha}\left[1 - \frac{q^2}{4\pi\theta}\ln\left|\frac{r}{R}\right|\right] \approx n_{0\alpha}\left|\frac{\vec{R}}{r}\right|^{\frac{q^2}{4\pi\theta}} \tag{49}$$

as $r \rightarrow \infty$, and R is the size of the system introduced as an infrared cutoff and $\alpha = 1, 2$. The exponentiation on the right hand side of (49) may be understood by going to the amplitude phase representation $\Phi_\alpha = \sqrt{\rho_\alpha}e^{-i\phi_\alpha}$, in which $[\rho_\alpha(\vec{x}), \phi_\beta(\vec{y})] = -i\delta_{\alpha,\beta}\delta(\vec{x}-\vec{y})$. Writing $\rho_\alpha(x) = n_{0\alpha} + \tilde{\rho_\alpha}(x)$ and retaining the quadratic terms in the hamiltonian, one finds that as $k \rightarrow 0$

$$< \phi_\alpha(k)\phi_\alpha(-k) > \rightarrow \frac{q^2}{2\theta k^2}$$

It is then clear from the results (47,48,49) that there is *almost* long range order in the ground state in this approximation despite the fact that there is a gap in the quasiparticle spectrum.

This is to be contrasted with the XY model, the ordinary bose gas with short range interactions and the Heisenberg antiferromagnet all of which show, in a completely analogous approximation, true long range order in the ground state. Notice that this quantum disorder at zero temperature is *not* due to Goldstone bosons. Rather the disorder and the gap in the spectrum are a consequence of the logarithmic infrared divergences in two dimensions, arising from the statistically induced Coulomb interaction. Furthermore, all these results are at *zero temperature.*

At finite temperature, not only the first term on the right hand sides of equations (45, 46) contribute to the logarithmic singularities, but also the terms proportional to $\mathcal{N}_2(k)$. For both cases $\eta = 1, 3/2$, at long wavelengths one finds

$$\frac{< \mathcal{N}_2(k) >}{k^\eta} \approx \frac{1}{k^2}$$

this happens, in either case, because the low momentum behavior of the dispersion relation and Bogoliubov coefficient conspire to produce a $1/k^2$ singularity at long wavelenghts, responsible for a logarithmic singularity in two dimensions.

This result is completely analogous to that in the two dimensional models mentioned above. All fail (weakly) to have true long range order at finite temperature because the fractional depletion $(N - N_0(T))/N$ (or an analogous quantity) diverges in the thermodynamic limit. The lack of long range order (logarithmic disorder) at finite temperature in these systems (including the present one) agrees with rigorous results which are a consequence of the f-sum rule and the Bogoliubov inequalities[26, 27].

The f-sum rule and Bogoliubov inequalities are *not* straightforwardly applicable to this case because of the presence of velocity dependent interactions. However, the validity of the f-sum rule has been recently proved rigorously for this problem by Khlebnikov[28]. We have computed the necessary double commutators $[[\rho_k, H], \rho_k]$ with the full Hamiltonian (before the Bogoliubov approximation) and found that because of the transversality of (19) only the kinetic term in the Hamiltonian contributes, thus validating the results of the Bogoliubov inequalities[22].

Furthermore, we have computed the helicity modulus[29] of this charged bose gas following the methods of Banavar and Jasnow[30] and found that it is finite at $T = 0$, in spite of the quasi-long range order, indicating a finite superfluid density.

Summary and further questions:

We studied aspects of off diagonal long range order in a model of two anyon gases with opposite statistical charges beginning with bosons with a Chern-Simons statistical interaction. This model is introduced to describe the physical situation of

two two-dimensional layers of anyon gases. This "flavored" anyon model has a global $U(1) \times U(1)$ symmetry corresponding to the conservation of each species of anyons. To study off-diagonal long range order we introduced a novel gauge invariant second quantization procedure that revealed a local and gauge invariant order parameter. This order parameter has the physical interpretation of a field describing charge-flux composites. We studied the model within the Bogoliubov approximation in the statistically neutral sector. The ground state is a condensate of these charge-flux composites. For both short ranged and *three dimensional* Coulomb two-body interactions of the anyon gases, we find a plasmon mode and an massless mode. The plasmon mode physically corresponds to an out of phase oscillation of the anyon gases. This is the *only combination* that feels the long range statistical forces. In the former case the massless mode is identified with an acoustic phonon branch, whereas the dispersion relation becomes singular at low momentum in the latter case.

We find that the statistical interaction induces a *two- dimensional* Coulomb interaction that is responsible for "quasi- long range order" and algebraic fall-off of the correlation functions at zero temperature but with a finite superfluid density. The plasma frequency in the plasmon branch becomes vanishingly small, and the condensate statistically neutral in the thermodynamic limit. The logarithmic singularities arising from the statistical interaction that are responsible in the first place for opening up this gap are ultimately responsible for its vanishing in the thermodynamic limit.

The system is disordered at any non-zero temperature in agreement with rigorous inequalities, which we confirmed explicitly in the present case.

It is certainly compelling to go beyond the Bogoliubov approximation, in which the quasiparticles interact. In particular, a relevant question is brought up by the result given by equation (49). This result seems to indicate that the charge-flux operators acquire anomalous dimension given in this approximation by the fractional statistics parameter $q^2/4\pi\theta$. It would be interesting to understand if this anomalous

dimension is renormalized by the interaction, and if so if the theory has interesting renormalization aspects. These are open questions, that we believe deserve further study.

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* Permanent Address: Departamento de Fisica, Pontificia Universidad Catolica do Rio de Janeiro, Rua Marques de Sao Vicente 225, C.P. 38071, Rio de Janeiro, Brazil.

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