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Another calculation of the Higgs mass and the top mass from the principles of H. B. Nielsen: $m_H \cong 163$ GeV for $M_t \cong 190$ GeV

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Abstract

We calculate the Higgs mass and the top mass starting from the principle that there are two, essentially degenerate minima in the Higgs effective potential: the second is at about the Planck energy scale $M_P = 1.2 \times 10^{19}$ GeV. Thus the parameter of the quartic self-coupling λ_H vanishes, as does β_{λ_H} at M_P. The new element is the addition of a quantum interaction term which couples the square of the Higgs field to the square of a pseudoscalar field, in the domain of the energy scale between about 10^{14} GeV and M_P. We modify β_{λ_H} at one loop. The pseudoscalar field which is introduced may be the field which is responsible for a spontaneous breakdown of discrete symmetry - for CP noninvariance at an energy scale of (10^{15} - 10^{16}) GeV. The result is then a closer value for $m_H \cong 163$ GeV for the top pole-mass $M_t \cong 190$ GeV; both values are now close to the electroweak scale parameter $\langle \phi_H \rangle / \sqrt{2} = 175$ GeV. In terms of dimensionless running coupling parameters, which determine the masses near to the electroweak scale, we get $\sqrt{\lambda_H} \cong 0.66$ and $g_t/\sqrt{2} \cong 0.72$, values that are close to each other and close to unity.

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1 Introduction

The purpose of this paper is to present the results of another calculation of the Higgs mass and the top mass. The calculation follows the principles recently put forward by Frogatt and Nielsen [1] in their calculation of these quantities. The essential hypothesis as applied to the standard-model Higgs field, is that there is a minimum in the effective potential at about the Planck energy scale $M_P = 1.2 \times 10^{19}$ GeV. At this energy scale the running parameter for the quartic self-coupling

$\lambda_H(\mu \cong \phi_H = M_P) = 0$ and also $\beta_{\lambda_H} = 0$. The latter condition determines the running top mass parameter $m_t(\mu \cong \phi_H = M_P)$. Using the standard model β functions, evaluated for two loops [2,3], extrapolation down to the electroweak scale gives the results [1], $m_H \cong 135$ GeV and $M_t \cong 173$ GeV (for the top pole-mass). This value of m_H is essentially the same as the lower bound set forth by Sher [3] We have modified this calculation in one way: we introduce a quartic, quantum coupling of the square of the Higgs field ϕ_H^2 to the square of a hypothetical, pseudoscalar field b^2. The coupling is effective in the domain of energy scale from M_P down to a about 10^{14} GeV; below this scale, decoupling occurs. With this coupling we modify β_{λ_H} at one loop. The new result is $m_H \cong 163$ GeV for $M_t \cong 190$ GeV. In terms of the dimensionless running coupling parameters which determine the masses near to the electroweak scale $\langle\phi_H\rangle \cong 247$ GeV, we have $\sqrt{\lambda_H} = m_H/\langle\phi_H\rangle \cong 0.66$ and $g_t/\sqrt{2} = m_t/\langle\phi_H\rangle \cong 0.72$. Thus the Higgs mass and the running top mass $m_t \cong 178$ GeV are close to each other and both are close to the electroweak scale parameter $\langle\phi_H\rangle/\sqrt{2} \cong 175$ GeV. The two numbers for the coupling parameters are nearly the same and are close to unity. We consider that this possibility has some intuitive and aesthetic appeal; therefore we report the calculation here. As to the nature of the pseudoscalar field b, the possibility has been considered [4] that this is the field which is responsible for a spontaneous breakdown of CP invariance [5] at a scale of $F_b = (10^{15} - 10^{16})$ GeV. This scale is set by an attempt [4] to describe the empirical baryon asymmetry in the present universe in terms of the squared ratio of an energy scale F_b to the Planck scale. The role of the b field is explained in detail in ref. [5] and in ref. [4].

2 Calculation and Results

The starting point of our considerations is a modification of the standard-model renormalization group improved, effective Higgs potential, through addition of a quantum coupling to a pseudoscalar field b, with a running coupling parameter λ_b. We add to the Lagrangian terms $\mathcal{L}_b$, where

$$-\mathcal{L}_b = \frac{\lambda_b}{4}\phi_H^2 b^2 + \frac{\tilde{\lambda}_b}{2} b^4 \tag{1}$$

Following Nielsen [1] we require that the quantum corrected effective potential has a minimum at about the Planck scale, with an energy density which is essentially the same as that of the present vacuum at the electroweak scale. Since the quadratic terms of the potential are negligible at the Planck scale, the Higgs part of the effective potential is given by

$$V = \frac{\lambda_H(\phi_H)}{8}\phi_H^4 \tag{2}$$

The running coupling parameter $\lambda_H(\mu \cong \phi_H)$ is determined by the renormalization group equation (RGE)

$$\frac{d\lambda_H}{d\tilde{t}} = \beta_{\lambda_H} \tag{3}$$

with $\tilde{t} = \ln(\phi_H)$. Degeneracy with the present electroweak vacuum requires the potential to be nearly zero at the Planck scale, thus $\lambda_H(M_P)$ is very nearly zero.

Setting $\lambda_H(M_P) = 0$, the minimum condition $dV/d\phi_H = 0$ gives the constraint $\beta_{\lambda_H} = 0$. From this, the value for the top Yukawa coupling $g_t(\mu \cong \phi_H = M_P)$ entering into the β-function can be extracted. Extrapolating to the electroweak scale with the renormalization group yields the couplings at the electroweak scale and thus the Higgs mass and the top mass.

The additional interactions in eq. (1) modify the β-function of λ_H above the mass of the b field. At one loop, we have

$$\beta_{\lambda_H} = \beta_{SM} + \frac{1}{16\pi^2}\lambda_b^2 \tag{4}$$

We use the standard model RGE for the gauge and Yukawa couplings g_k [2,3]

$$\frac{dg_k}{d\tilde{t}} = \beta_{g_k} \tag{5}$$

As in ref. [1] we use the two-loop expressions [2, 3] for all of the standard-model β-functions β_{SM} and β_{g_k}. In addition, we have the RGE for λ_b and $\tilde{\lambda}_b$. At one loop, the β-functions are

$$\begin{aligned} \beta_{\lambda_b} &= \frac{1}{16\pi^2}\lambda_b\left(3\lambda_H + 4\lambda_b + 12\tilde{\lambda}_b\right) \\ \beta_{\tilde{\lambda}_b} &= \frac{1}{16\pi^2}\left(\frac{1}{4}\lambda_b^2 + 36\tilde{\lambda}_b^2\right) \end{aligned} \tag{6}$$

The field b decouples below a scale which is taken here as $\approx 10^{15}$ GeV (near to the hypothetical b mass [4]), and λ_b can then be set equal to zero. Far below the b mass, any contribution of the b field can be absorbed by a finite redefinition of the quartic coupling λ_H, which then evolves according to the β-functions of the standard model. It is always possible to choose a decoupling point, which is naturally near to the b mass, so that any threshold corrections vanish. The running λ_H then becomes a continuous function of the scale $\mu \cong \phi_H$. The extra positive λ_b^2 term in the β-function for λ_H in eq. (4) will produce a larger initial value for $g_t(\mu = M_P)$, since the latter parameter enters with a negative sign in β_{λ_H}. This will in turn produce a larger top and Higgs mass at the electroweak scale. The renormalization group development of the coupled eqs. (3-6) is evaluated numerically. Fig. 1 shows the prediction for the Higgs pole-mass, for the top pole-mass as a function of the initial value $\lambda_b(\mu = M_P)$. We have taken $\tilde{\lambda}_b(M_P) = \lambda_b(M_P)/2$; the results have very little sensitivity to $\tilde{\lambda}_b$ in our range of λ_b. The pole-masses are given by [3]

$$\left.\begin{aligned} M_t &= \frac{g_t(m_t)\langle\phi_H\rangle}{\sqrt{2}}\left(1 + \tfrac{4}{3\pi}\alpha_s + 10.95\alpha_s^2\right) \\ m_H &= \sqrt{\lambda_H(m_t)}\langle\phi_H\rangle \end{aligned}\right\} \quad m_t = m_t(\mu = M_t \cong \langle\varphi_H\rangle) \tag{7}$$

The choice $\lambda(m_t)$ in eq. (7) minimizes one-loop corrections from virtual top loops to the tree-level formula $m_H = \sqrt{\lambda_H}\langle\phi_H\rangle$. We use $\alpha_s = 0.117$ in eq. (7). Figs. 2 and 3 show the evolution of the running coupling parameter $\lambda_H(\mu \cong \phi_H)$ and the running top mass $m_t(\mu \cong \phi_H)$ as functions of $\ln\phi_H$ for different initial values of $\lambda_b(\mu \cong M_P)$. Fig. 4 shows the running of $\lambda_b(\mu)$ and also of $\tilde{\lambda}_b$. Note the opposite direction of evolution from $\lambda_H(\mu)$ in fig. 2.

From fig. 1, the value $\lambda_b(M_P) = 0$ reproduces the prediction of Frogatt and Nielsen [1] for m_H and M_t. On the other hand, a possible empirical [6] value for

$M_t \cong 190$ GeV gives $m_H \cong 163$ GeV and the parameter $\sqrt{\lambda_b(M_P)}$ has a value $\cong 0.8$. This number is close to the values $\sqrt{\lambda_H(m_t)} = 0.66$ and $g_t/\sqrt{2} \cong 0.72$; these dimensionless coupling parameters are all close to unity. Clearly $g_t \cong 1$, whereas $\sqrt{3\lambda_H} \cong 1.14$ (this corresponds to the sometimes used convention of placing a coefficient 1/4! instead of 1/8 in eq. (2)). Note that the running mass is now $m_t \cong 178$ GeV. Both m_H and m_t are now near to $\langle\phi_H\rangle/\sqrt{2} = 175$ GeV. For $\sqrt{\lambda_b} \cong 0.70$, one gets from fig. 1, $M_t \cong 185$ GeV and $m_H \cong 153$ GeV. For $\sqrt{\lambda_b} \cong 1$, one gets $M_t \cong 200$ GeV and $m_H \cong 180$ GeV. The uncertainty in the value of the QCD parameter $\alpha_s(M_t) = 0.117 \pm 0.006$ leads to an uncertainty with a magnitude of about 4 GeV for the top mass and 3 GeV for the Higgs mass. Also one keeps in mind that the Planck mass minimum is an order of magnitude estimate [1]. Varying the minimum between 10^{18} and 10^{20} GeV produces an uncertainty with a magnitude of about 3 GeV for the top mass and 5 GeV for the Higgs mass prediction. The decoupling point produces an uncertainty which is less than 1 GeV when decoupling is varied between 10^{14} to 10^{17} GeV.

3 Conclusion

The calculation of the Higgs and top masses given here is an example of what happens to the values calculated in the standard model using the physical principles put forth in ref. [1], when an additional quantum interaction involving ϕ_H^2 and a second spin-zero field b^2 is introduced in the region of energy scale between about 10^{14} GeV and M_P. While the physical motivation for b is its role in the spontaneous breakdown of CP invariance [4,5], the fact that m_H then approaches M_t (fig. 1) and that both masses are then close to $\langle\phi_H\rangle/\sqrt{2}$ is of interest, in particular to the experimental search for the Higgs boson.

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Figures

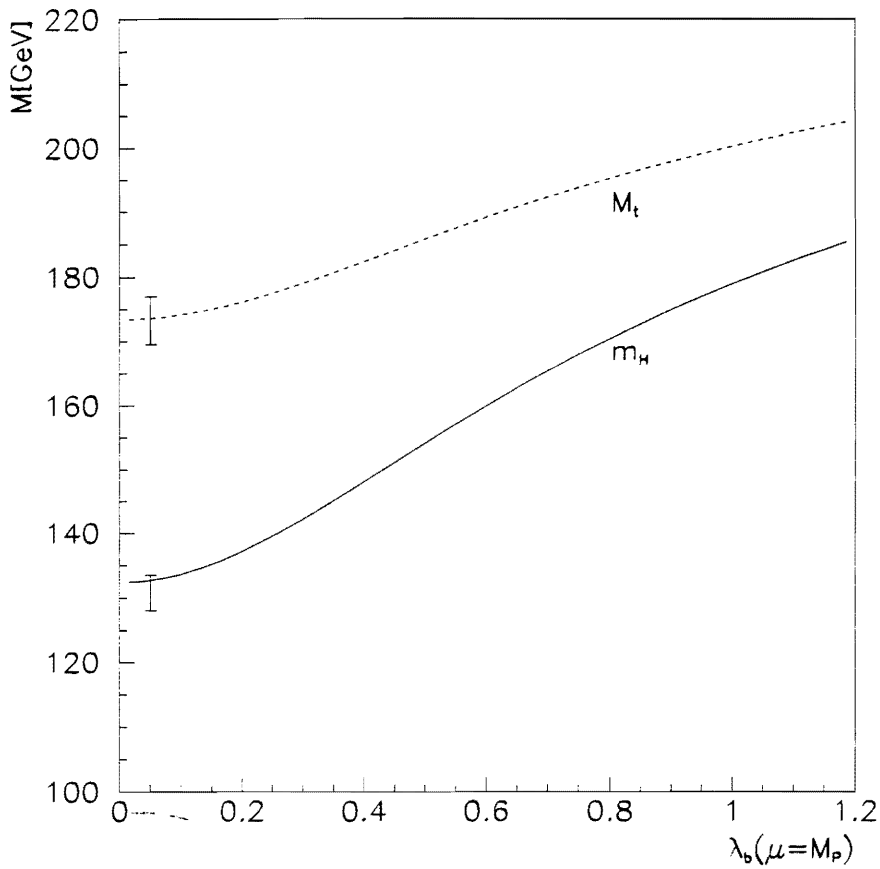

Figure 1: Predictions for the Higgs pole-mass m_H (solid curve) and top pole-mass M_t (dashed curved) as a function of $\lambda_b(\mu = M_P)$, the additional quartic coupling parameter, which is introduced in the text.

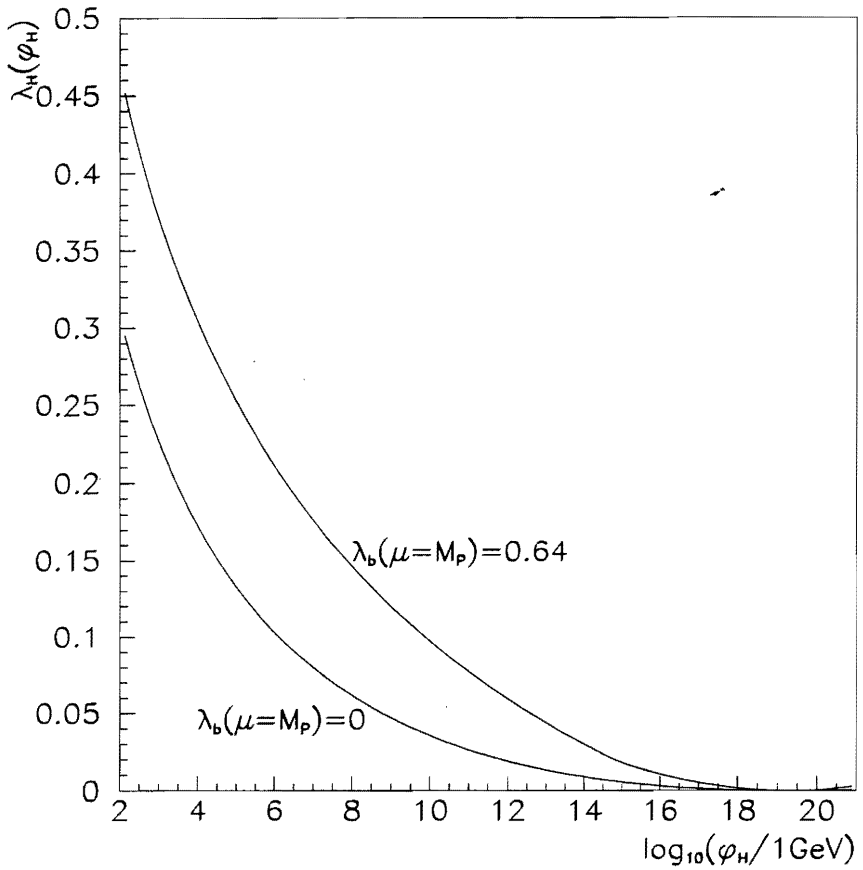

Figure 2: Renormalization group evolution of the Higgs quartic self-coupling parameter $\lambda_H(\mu = \phi_H)$, for different values of the additional quartic coupling parameter which is introduced in the text.

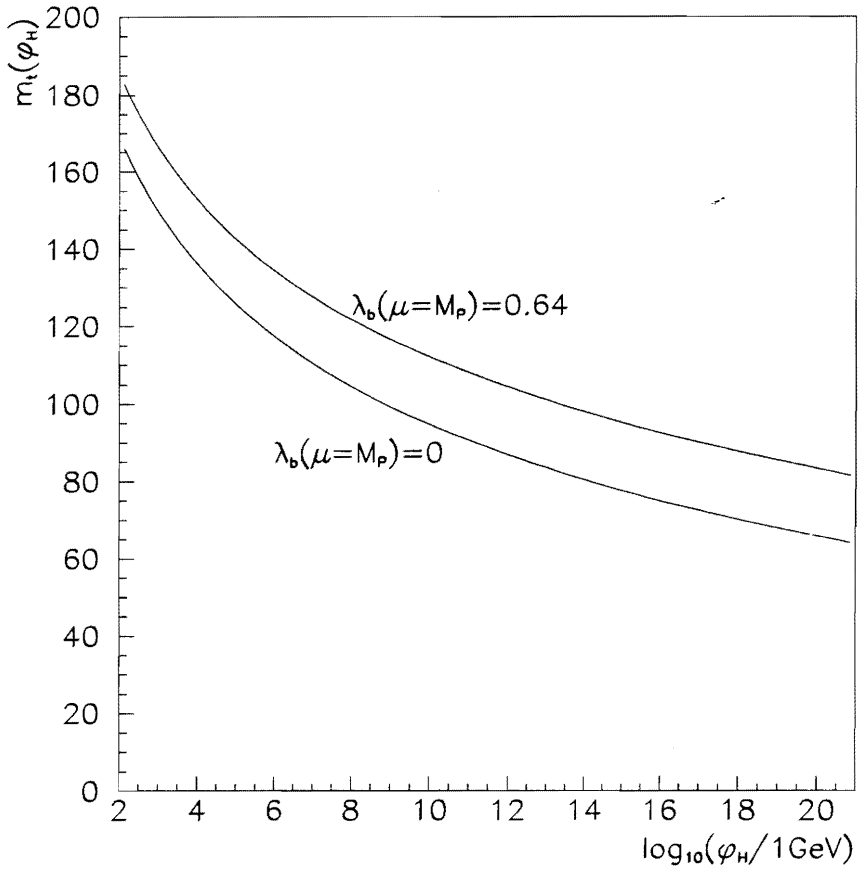

Figure 3: Renormalization group evolution of the running top mass $m_t(\tilde{t}) = g_t(\tilde{t})\langle\phi_H\rangle/\sqrt{2}$ with $\tilde{t} = \log_{10}(\phi_H/1\ GeV)$, for different values of the additional quartic coupling parameter λ_b which is introduced in the text.

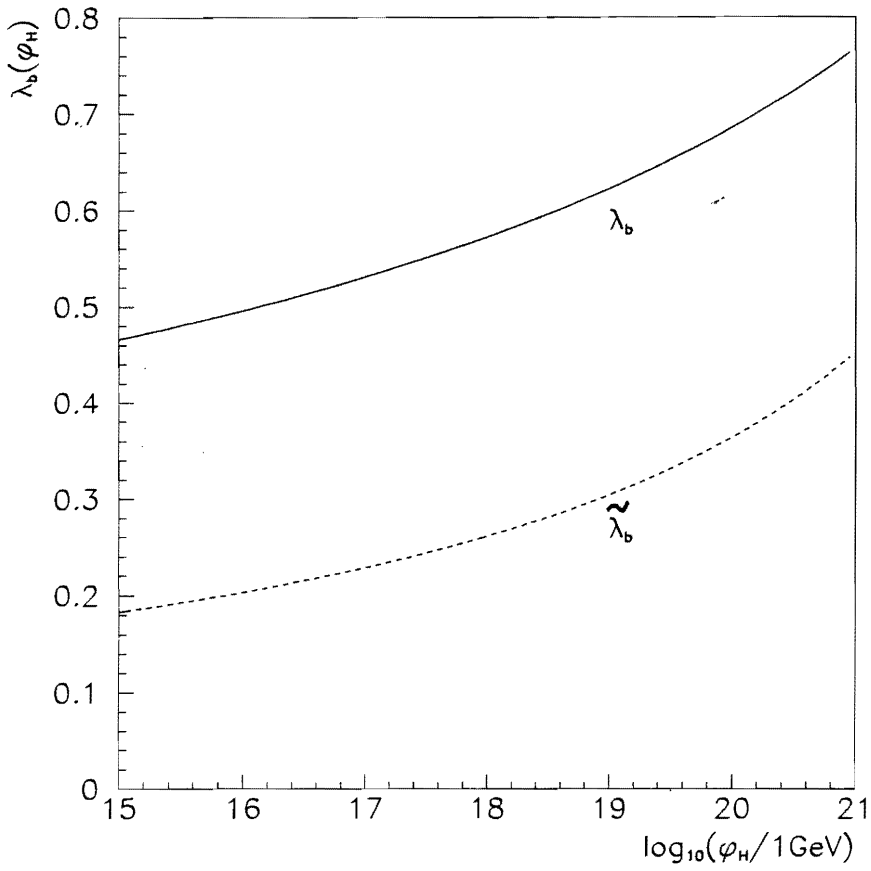

Figure 4: Renormalization group evolution of the quartic coupling parameters $\lambda_b(\mu = \phi_H)$ (solid line) and $\tilde{\lambda}_b(\mu = \phi_H)$ (dashed line), which are introduced in eq. (1) of the text. Decoupling is at $\mu \cong 10^{15}$ GeV.