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Origin of Energy of the Universe

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Abstract

Assuming the standard models of cosmology and fundamental particles, and the conservation laws, the production of the large energy of the Universe is explained naturally so as to join the classical and quantum field theories.

1. INTRODUCTION

A general theory of relativity[2] has been developed a great deal by many authors either in sophisticated forms[2]-[7] or in an easily approachable manner.[8]-[11]

After the discovery of Hubble[12] the predictions by the standard model of Universe, based on a Friedmann-Lemaître-Robertson-Walker (FLRW) spacetime metric, are compared with ever untiringly increasing astronomical observations. The basis of the expanding picture of the Universe is strengthened further by the discovery of a background radiation by Penzais and Wilson[13] as a remnant of an early epoch of the Universe. We agree commonly now that the Universe has started from an extremely limited spacetime portion, or ideally a point. The new discovery of temperature fluctuation by COBE[14] will give us an important clue to the structure of the Universe, but it may not be relevant directly to our main study (see, later discussion).

We have now the well-tested Glashow-Salam-Weinberg (GSW) electroweak theory[15] and well-recognized quantum chromodynamics (QCD).[16,17] Here the spontaneous breaking of symmetry (SBS) known as a Higgs mechanism is an important ingredient. The P, PC and baryon number non-conservation in baryogenesis[18] are the important subjects in cosmology.

We are not going to challenge whole the problems and confine ourselves to the production of energy of the Universe as a result of transition from a false vacuum to a true vaccum. There will be various possibilities on the choice of the former in terms of the fundamental particles, since we do not have a way to realize such an epoch. The conservation law will admit us to identify in principle

the total classical energy to the appropriately adjusted net zero-point energy in the standard model. On arriving at this picture, the author is greatly indebted to the critical study made by Dirac[19] on quantum field theory. We shall discuss shortly our scenario in the following.

We know that the zero-point momentum and angular momentum do not contribute at all after the normal ordering of creation and annihilation operators in the quantum field theory, while the zero point energy remains as a positive and negative infinity for boson and fermion field, respectively. We were taught that one can choose arbitrarily the origin of energy at his will, so that these infinities can be discarded with no care in the fundamental processes, except that the effect of the vacuum filled with the associated particle-antiparticle pairs is automatically taken care of in the relativistic field theory. Notice that such an infinity will appear even with the discontinuous spacetime formulation of the theory, where the corresponding manifold in the energy-momentum space consists of granules of countable (enumerable) infinity. We believe that the zero point energy should be taken care of seriously in the intermediate step of transition from the original Universe to the classical One.

Our scenario is as follows. The big bang is caused from the uneven statistical weight between the full bosonic and fermionic zero-point energies. We suppose that the initial state of the Universe is null (nothing) vacuum which carries no quantum number. We shall choose this as a constraint for the sake of clarity. To fill up the Dirac sea an excessive energy is released.

The conservation of energy, momentum and angular momentum in the general relativity is certified by Hawking and Ellis during the lengthy period of the development of the Universe.

A reason why we rely on its development of the Universe on the fundamental particle theory is pointed out in Section 2, where the estimate of the total energy in the classical theory is given from the matter dominated era. Section 3 is devoted to the discussion of the net zero point energy in the standard $SU_C(3)\times SU_F(2)\times U_Y(1)$ theory. A possible choice of the field combination which gives a null value for everything at the beginning of the Universe is also pointed out there. A way to estimate the expected value of a variable in the quantum theory of tunnelling[20] from the false to true vacuum in terms of the given parameters is shown in Section 4, by making use of the associated maximum entropy for the probability.[21] The result of our study is summarized in the last section.

2. CLASSICAL GROUND STATE ENERGY

Before entering a subject of this section it would be useful to point out a reason why to study the problem of the Universe from both classical and quantum field theory. For this purpose we shall compare the Newtonian coupling constant in an appropriate mass unit with the average value of the known electromagnetic, weak and strong coupling constants at a certain energy and assume that they are proportional to each of their occurrence probabilities (inverse lifetime) of relevant events. One finds then, by taking the units of time in seconds as

$$t\ G\ m^2 = 10^{-22}\ \bar{\alpha}\ . \tag{2.1}$$

Here G is the gravitational constant, m the mass scale, t their relevant lifetime and $\bar{\alpha}$ is defined as[22]

$$\bar{\alpha} = \frac{1}{3}(\alpha_1(m_Z)+\alpha_2(m_Z)+\alpha_3(m_Z)) = 0.05337\ , \tag{2.2}$$

where m_Z is the mass of the weak intermediate Z boson. One finds $t = 3\cdot 10^7$ and 10^{14} y for $m = m_e$ and m_p (e and p denote the electron and proton), respectively. The observed age of the Universe is approximated by $t_O = 2/3h_O\cdot 10^{10}$ y $(1/2 \leq h_O \leq 1)$, which is in between the above crude bounds. If one uses the average mass of the three-light current quarks[23] in place of m he may find a closer agreement with the observation. There remains an ambiguity in the choice of time scale 10^{-22} seconds on the right-hand-side input in Eq.(2.1). We want to show the reasonableness of the direction of the study rather than a detailed numerical agreement.

To get some idea let us assume that the Universe is approximated by a sphere with a uniform density of radius r. The potential energy stored in it is

$$V = -\frac{3}{5}\frac{GM^2}{r}\ , \tag{2.3}$$

where M is the total mass of the Universe. Here r is in general a function of time. For a closed Universe at a slow expansion epoch the mean kinetic energy is given approximately by $\bar{T} = -\bar{V}/2$ by a virial theorem. For this choice the ground state takes a half of

the negative enrgy specified by Eq.(2.1). To get 1 the ratio of the observed and critical matter density, it is expected to have 5 to 10 times more missing mass than the estimate made from the nearby bright mass within an event horizon. The content of them classified by cold, warm and hot matter, whose names depend upon each of their kinetic energies, is not yet clarified. The total energy of the Universe will be specified completely by including the missing mass. The question arises then how this energy is supplied from its null initial value? This is a main theme of the next section.

3. ZERO POINT ENERGY

We are interested in the net zero point energy arising from the standard model of fundamental particles. The building blocks are the anomaly free sets of quarks and leptons in three generations, gluons, photon, weak intermediate bosons, graviton and two-coplex-isodoublet Higgs scalars. The neutrino will be counted as a single (left-handed) or double (both left- and right-handed) helicity states. For our purpose it is only necessary to consider the free field Hamiltonians. In order to evaluate the formally infinite constants we can consider the objects even before the SBS, so that all the particles may be treated massless ones. We shall choose this convention in the comparison of the relative statistical weight for zero point energies. Dropping the operator parts and summing over the two spin degrees of freedom the zero point energy for an electron is given by

$$E_{0e} = -2 \int_{-\infty}^{\infty} d^3p \; E_p \; \delta^3(\vec{p}-\vec{p}) \; . \qquad (3.1)$$

Here E_p is the electron energy with momentum $\vec{p}$. The corresponding expression for the photon becomes

$$E_{0ph} = \int_{-\infty}^{\infty} d^3k \; \omega_k \; \delta^3(\vec{k}-\vec{k}) \; , \qquad (3.2)$$

summing over the polarization degrees. Here ω_k is the photon energy with momentum $\vec{k}$. The similar applies to the graviton.

There arise factors 8, 3 and 2 as the statistical weights for the expressions analogous to Eq.(3.2) from gluons, massless weak-intermediate bosons and isodoublet Higgs scalars, respectively. The net statistical weight for bosons becomes 15. Similarly the total statistical weight for fermions in the analogous form of Eq.(3.1) becomes 24+3/2 and 27, respectively, if the neutrino is only in left-handed and both-handed object. The importance is the relative weight 15 versus 51 (or 54) for the bosonic versus fermionic zero-point energies. Even if we consider solely the first generation of fermions the negative sign of the net zero-point energy does not change.

If there does not exist additional fundamental particles than those we know at presnt, the transition from the initial null to the net negative energy occurs from the difference between the statistical weights for bosonic and fermionic zero-point energies explosively, so as to fill up the vacant Dirac holes left unoccupied completely in the initial state. Here there occurs the change to

the new ground state which corresponds energetically to the classical one.

If the world is supersymmetric (SUSY)[(24)] we expect that the SUSY partners should appear so as to cancel comletely the net bosonic and fermionic zero-point energies in the initial Universe. In this scenario there is no difference with null (none) from the beginning. One can also say that the SUSY vaccum is completely filled by the corresponding fundamental particles and no bosonic zero point energy. The unsuccessful search of the superpartners[(25)] may be consistent with this observation.

4. COMMENT ON QUANTUM TUNNELLING

The subject of this section is not directly relevant to the main idea of this paper, however it is joined closely with the successive development of the Universe after the first transition discussed in the previous section.

We shall summarize at first the theoretical work of Vilenkin[(20)] suitable for our purpose. An action of a simple model is given by

$$S = \int \left(\frac{R}{16\pi G} - \rho_V\right) \sqrt{-g} \; d^4x \; , \qquad (4.1)$$

where R is the scalar curvature, ρ_V the false-vacuum energy density and g is the determinant of the metric tensor.

The action for a grand unified model with a Higgs field ϕ and the effective potential $V(\phi)$ is

$$S = \int \left(\frac{R}{16\pi G} + \frac{1}{2}(\partial_\mu \phi)^2 - V(\phi)\right) \sqrt{-g} \quad d^4x \tag{4.2}$$

with

$$V(\phi) = \rho_v - \frac{1}{2}\mu^2\phi^2 \ . \tag{4.2a}$$

The action for Starobinsky model is defined as

$$S = \frac{1}{16\pi G} \int \left(R + \frac{R^2}{6M^2} + \frac{R^2}{R_0} \ln \frac{R}{R_0}\right) \sqrt{-g} \quad d^4x \ . \tag{4.3}$$

Here M is a free parameter, $R_0 = 12\ H_O^2$ and the scale factor of FLRW metric in the Starobinsky model is known to have an inflationary solution

$$a(t) = \frac{1}{H_O} \cosh H_O t \quad . \tag{4.3a}$$

In case of Eqs. (4.2) and (4.3) the WKB approximation was found through the corresponding Wheeler-De Witt equation, by keeping only the solvable modes.

The results of physical interest are the tunnelling probabilities for the nucleation. In case of Eq.(4.1) the probability p itself is given, while for Eqs.(4.2) and (4.3) the probability distribution P is given as the function of the variable of the model. One finds

$$p = \exp\left(-\frac{3}{8G^2\rho_v}\right) \tag{4.4}$$

for Eq.(4.1). The normalized probability distribution may be given

in general by

$$P(x) = \sqrt{\frac{a}{2\pi}} \quad \exp(-\tfrac{1}{2}ax^2),$$

where x is the variable and a an arbitrary constant. One finds

$$a = \frac{8\pi^2}{3} \frac{\mu^2\phi^2}{H_v^4}, \quad x = \phi, \quad H_v^2 = \frac{8\pi G\rho_v}{3}, \qquad \text{for model Eq.(4.2)} \tag{4.5}$$

and

$$a = \frac{8\pi}{GH_O^2}, \qquad x = x, \qquad \text{for model Eq.(4.3)} \tag{4.6}$$

If one defines

$$W(\sqrt{a}\ x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\sqrt{a}\ x} \exp(-\tfrac{1}{2}ax^2)\ dx\ , \tag{4.7}$$

he finds

$$W(\sqrt{a}\ x) + W(-\sqrt{a}\ x) = 1 \tag{4.8a}$$

and

$$W(\sqrt{a}\ x) - W(-\sqrt{a}\ x) = p(\sqrt{a}\ x). \tag{4.8b}$$

Here $p(\sqrt{a}\ x)$ is the corresponding tunnelling probability.

The entropy $S(p)$ associated with the probability p may be defined by[21]

$$S(p) = -p \ln p - (1-p) \ln(1-p). \tag{4.9}$$

The maximum value of it is given by $p = 1/2$, which corresponds to $W(\sqrt{a}\ x) = 3/4$. One finds then from Eq.(4.4)

$$\frac{3}{8G^2\rho_v} = \ln 2 \ , \qquad \text{for model Eq.(4.1).} \qquad (4.10)$$

Making use of the numerical table for the normal probability function[26] one obtaines

$$2\pi\sqrt{\frac{2}{3}}\,\frac{\mu\phi_0}{H_v^2} = 0.6745 \ , \qquad \text{for model Eq.(4.2)} \qquad (4.11)$$

and

$$2\sqrt{\frac{2\pi}{GH_O^2}}\,x_0 = 0.6745 \ , \qquad \text{for model Eq.(4.3).} \qquad (4.12)$$

Here ϕ_0 and x_0 are the corresponding expectation values in each model at the maximum entropy for the tunnelling.

5. DISCUSSION

A theory which explains an origin of eenrgy of the Universe is developed on the basis of quantum field theory and the standard models. A machinery of the scenario is a role of so far disregarded zero-point energy in the realm of the early history of the Universe.

We assume that the big bang of the Universe is fired by the uneven statistical weights between the bosonic and fermionic zero-point energies in the standard model of fundamental particles.

We have taken two possibilities: (1) there is no fundamental particles than we know at present, and (2) there are superpartners so as to make null the full zero-point energy in the initial Universe. In the first choice the null Universe to finite lower energy transition occurs with the formally infinite energy difference between the bosonic and fermionic zero-point energies. In the second case the meaning of the null in the initial Universe is more involved in its content.

We think our model does not disturb any of the existing models on the Universe, so its idea will be deepened further by the help of the existing and future information from both astronomical and fundamental particle studies.

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