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QCD Coupling Constant from Anomalous Magnetic Moments of Quarks in Null Pontryagin Index Model of Baryons

Syurei IWAO

Physics Institute, Sue 13-52-44, Kanazawa 920-13

Abstract

Assuming the constancy of the QCD coupling constant near zero momentum transfer, it is shown that its value estimated from the constituent quark masses and anomalous magnetic moments is consistent with that obtained from the renormalization group equation for beta function. The inputs are determined from the mass formula of Shuryak and Rosner in the static SU_6 model with additional color degree for the low-mass baryons with null Pontryagin index and available data.

1. Introduction

We know that the magnetic moments of leptons is one of the most prominent source to test QED.[1] Recently, Mattingly and Stevenson (MS)[2] have suggested that the QCD coupling constant should be freezing to the fixed value a* ($a = \alpha_s(s)/\pi$), in the certain range of low s, say s=0 to (a few hundred MeV)^2. This idea seems to be natural from the gauge theory point of view in conjunction with QED. Kataev[3] has reported on their most recent QCD result of the cross section ratio R up to a^3 for hadrons versus muon-pair production in e^+e^- annihilation. Shuryak and Rosner (SR)[4] have reported on the baryon mass formula for low-mass states by assuming solely spin-zero attractive q-q' exchange beloging to the instanton and anti-instanton ball, denoted as IB and $\overline{IB}$, respectively. Here IB and $\overline{IB}$ represent that one of the quarks of the baryon is associated with the instanton and the remaining two quarks with the anti-instanton and vice versa. Notice also that the single quark associated with two instantons and one anti-instanton may also be counted as the same IB and so on. In addition to this a pair of instanton and anti-instanton may also be produced in the hadronic matter without associating with any of the quark in the hadron etc. We only have the constraint $N_I - N_{\overline{I}} = 0$ for a given hadron, where N_I and $N_{\overline{I}}$ are the total number of instantons and anti-instantons, respectively.

We have shown[5] that the SR mass formula may naturally be extended to high mass region in a straightforward manner, including meson states, by making use of solely the spin, flavor and instanton structure without referring to the details of the associated symmetry group. We can show that the similar idea can be accommodated with the static SU_6 model[6),7] by taking a color degree of freedom into account in addition without modifying the SR mass formula. The magnetic moments and the effective masses of quarks estimated roughly by the Particle Data Group[8] from the observed baryon magnetic moments indicate that those masses are

well below the corresponding ones in the SR mass formula. The observations made in above two paragraphs suggest us to calculate a* from the anomalous magnetic moments of quarks by choosing the quark masses of the SR mass formula as real ones.

The purpose of this paper is to estimate a* at zero momentum transfer by reanalyzing the available experimental information.[8] We expect that the QCD radiative correction to the magnetic moments of the quarks should be the same for the light u, d and s quarks. However, the mass as well as the magnetic moment for the individual quark will teaches us the allowed (mean) value or range of a*, owing to the uncertainties involved both theory and experiment. For this reason we perform the least squares fit to totally 15 octet and decuplet baryons by taking u-d mass difference into account, and 9 magnetic moments (including $\Sigma^0\Lambda$ transition one) for semi-stable baryons by 5 and 3 parameters, respectively. One can use, e.g., the SU(3) electromagnetic-mass sum rule as a subsidiary condition in order to find a better fit to the observed masses, whereas such a consideration may not be necessary for our present purpose.

The radiative correction to the magnetic moment of the quarks may approximately be given by taking the square root of R by omitting color factor 3 and quark charge one and expanding it to a^3 order. There remains the question that this method really takes effectively into account the full radiative effect arising from three bound quarks. Let us assume an approximate validity of this approach. The numerical coefficient of a^3 is large negative compared to those for a and a^2, so that there does not exits a reasonable a^* if we include up to a^3. Up to a^2 we find a*= 0.345 and 0.360 as an average for the number of flavors 3 and 5, respectively. They are about 31 and 37 % higher than the one found by MS from the renormalization-group-equation analysis of beta function.

Sections 2 and 3 are devoted to the mass formula and magnetic moments of baryons, respectively, and to the derivation of the corresponding parameters for the quarks. In Section 4 the fixed value a* assumed is estimated by combining the theoretical Kataev value with the corresponding ones determined by previous two sections. We shall discuss our results as well as a relevant problems in the final section.

2. Baryon Mass Formula

The expression of the SR mass formula does not change from the original one, so our task is to show that it holds in SU_6 with additional color degree for quarks.

The mass operator of SR formula may be written as

$$M = \sum_i M_{q(i)} P_{q(i)} + O_{(ij)} P^0_{ij} , \qquad (2.1)$$

where $P_{q(i)}$ is the projection operator to pick out the i-th quark q(i), $M_{q(i)}$ the corresponding mass operator, P^0_{ij} the exchange operator of i-th and j-th quark in spin 0 state and $O_{(ij)}$ the strength of the corresponding exchange. The structure of the mass operator teaches us clearly that the main ingredient of the problem involved is to show the validity of the mass formula for the baryons with spin and parity $1/2^+$ states. In order to answer the question one has to look at the way of construction of SU_6 wave function.

The quarks in flavor SU_3 in spin 1/2 states appear either 8_s or 8_a: symmetric and antisymmetric arrangement for the first two quarks in Young tableux by convention. The first two quarks beloging to 8_a is multiplied by spin zero wave function and then the third quark with either spin up or down, depending upon the 3rd component of total spin of the baryon, is multiplied to it. By an appropriate interchange

of flavor and spin labels[7] one can find spin 1/2 16-plet members of the completely symmetric 56-plets of SU_6 wave functions. Before the construction of above final step one may label further the quarks with color indices, a, b and c say, for the first, second and third one, respectively, with the antisymmetric color Levi-Civita tensor as a multiplicative factor. One can consider that the exchange operator P^0_{ij} operates either to the 1st and 2nd 8_a qurks in SU_3 for simplicity or to the corresponding full labeled quarks for completeness. Here the specification of IB and $\overline{IB}$ should, of course, be made suitably. The net effect of P^0_{ij} brings the negative sign to the exchange term.

Parameterizing the expectation values of the spin-zero u-d and u-s (or d-s) exchange term by $-3\alpha/2$ and $-3\beta/2$, respectively, and those for the constituent quark masses by U, D and S for u, d and s quark, one may find easily the baryon masses expressed by their symbols for short in terms of these five parameters. Since Δ state has a large decay width, we shall express it by an average of four states by a suitable combination of U and D. Totally 15 baryon masses are given by

$$p = 2U + D - \frac{3}{2}\alpha \ , \tag{2.2a}$$

$$n = U + 2D - \frac{3}{2}\alpha \ , \tag{2.2b}$$

$$\Lambda = U + D + S - \alpha - \frac{1}{2}\beta \ , \tag{2.2c}$$

$$\Sigma^+ = 2U + S - \frac{3}{2}\beta \ , \tag{2.2d}$$

$$\Sigma^0 = U + D + S - \frac{3}{2}\beta \ , \tag{2.2e}$$

$$\Sigma^- = 2D + S - \frac{3}{2}\beta \ , \tag{2.2f}$$

$$\Xi^0 = U + 2S - \frac{3}{2}\beta \ , \tag{2.2g}$$

$$\Xi^- = D + 2S - \frac{3}{2}\beta \ , \tag{2.2h}$$

$$\Delta = \frac{3}{2}(U+D) \ , \tag{2.2i}$$

$$\Sigma^{*+} = 2U + S \ , \tag{2.2j}$$

$$\Sigma^{*0} = U + D + S \ , \tag{2.2k}$$

$$\Sigma^{*-} = 2D + S \ , \tag{2.2\ell}$$

$$\Xi^{*0} = U + 2S \ , \tag{2.2m}$$

$$\Xi^{*-} = D + 2S \ , \tag{2.2n}$$

$$\Omega^{-} = 3S \ . \tag{2.2o}$$

The numerical results are tabulated in Table I. The general feature

Table I.

of the parameters confirms the consistency of our previous fit,[5] so that it does not affects the conclusion obtained there.

3. Magnetic Moments of Baryons

The magnetic moment operator for a baryon may be defined as

$$\vec{\mu} = \sum_i \frac{eQ_i}{2m_i}(1 + \delta_i)\vec{\sigma}_i = \sum_i \mu_i \vec{\sigma}_i \ , \tag{3.1}$$

where eQ_i, m_i, μ_i and δ_i are the charge, constituent quark mass, magnetic moment and its anomalous part of the quark i, respectively, σ_i the Pauli matrices. The expectation value of the operator (3.1) for SU_6 wave functions[6)-8)] is given by

$$\mu = \sum_i a_i \mu_i \ , \tag{3.2}$$

where the numerical coefficients a_i satisfy

$$\sum_i a_i = 2J \ , \tag{3.3}$$

except for the transient moment. Here J is the eigenvalue for the total spin. For the sake of completeness we shall list the magnetic moments of 9 baryons. They are

$$\mu_p = \frac{1}{3}(4\mu_u - \mu_d) , \tag{3.4a}$$

$$\mu_n = \frac{1}{3}(4\mu_d - \mu_u) , \tag{3.4b}$$

$$\mu_\Lambda = \mu_s , \tag{3.4c}$$

$$\mu_{\Sigma^+} = \frac{1}{3}(4\mu_u - \mu_s) , \tag{3.4d}$$

$$\mu_{\Sigma^-} = \frac{1}{3}(4\mu_d - \mu_s) , \tag{3.4e}$$

$$\mu_{\Xi^0} = \frac{1}{3}(4\mu_s - \mu_u) , \tag{3.4f}$$

$$\mu_{\Xi^-} = \frac{1}{3}(4\mu_s - \mu_d) , \tag{3.4g}$$

$$\mu_{\Omega^-} = 3\mu_s , \tag{3.4h}$$

$$\mu_{\Sigma^0\Lambda} = \frac{1}{\sqrt{3}}(\mu_d - \mu_u) . \tag{3.4i}$$

The numerical results are given in Table II. Note that the magnetic moment of s quark is about 15 % larger in absolute value than the one given by the Particle Data Group,[8] while those for u and d are in agreement with their values as they should be. Combining the results in Tables I and II we find the anomalous magnetic moments of u, d and s quark in units of n.m. as

$$\delta_u = 0.33058, \quad \delta_d = 0.40103 \quad \text{and} \quad \delta_s = 0.34936 . \tag{3.5}$$

4. QCD Coupling Constant at Zero Momentum Transfer

The cross section ratio R for hadrons and muon-pair production in e^+e^- annihilation is given by

$$R = R^{(0)}(1 + a + c_2a^2 + c_3a^3 + \dots), \tag{4.1}$$

where $R^{(0)}=3 \sum_i Q_i^2$ and $a = \alpha_s(s)/\pi$. The numerical coefficient c_2 and the renormalization scheme dependent one c_3 may be obtained

numerically from Kataev expression[3] for number of flavors f=3;

$$c_2 = 1.639820 \ , \qquad c_3 = -9.041821 \ , \tag{4.2a}$$

and for f=5;

$$c_2 = 1.409230 \ , \qquad c_3 = -9.249935 \ , \tag{4.2b}$$

Taking the square root of $R/R^{(0)}$ and expanding it in power series of a one finds

$$\delta_i = \frac{1}{2}a + \frac{1}{2}(c_2 - \frac{1}{4})a^2 + \frac{1}{2}(c_3 - \frac{1}{2}c_2 + \frac{1}{8})a^3 + \ldots \ . \tag{4.3}$$

Notice that a factor 1/2 in the first term arises from two causes: The one comes from the cross term between uncorrected and one loop diagram to the photon vertex as a part in Eq.(4.1), which is expected from one gluon exchange (diagram) effect to the anomalous magnetic moment of the quark by taking the trace of the product of two SU_3 color generators into account in Eq.(4.3) in contradistinction to that for QED.[9),10)] The other is due to the absolute square of one gluon emission effect in Eq.(4.1), which may be reduced to the gluon emitted from the quark of interest is absorbed by the other bound quark. The higher order term will be caused by the similar but complicated effects. One noticeable point is that in Eq.(4.1) and then in Eq.(4.3) no fractional power dependence of the coupling $a = \alpha_s(s)/\pi$ appears.

As pointed out in the introduction the anomalous magnetic moments of quarks should be independent from a kind of flavors. However, the indirect information from their composite system tells us that there is some discrepancy in δ_i (i = u, d and s). The theoretical coefficient

of the third term on the right-hand side of Eq.(4.3) is large negative relative to the other ones in the first and second term, so that an unsatisfactory solution for a* is gained if we include that term. In the numerical work we shall omit it and determine a* including term up to a^2. The f dependent effect for a* is determined for each δ_i separately. We list them in the corresponding order for u, d and s.

(i) for f=3;

$$a^* = 0.317, \quad 0.383, \quad 0.335 , \tag{4.4a}$$

(ii) for f=5;

$$a^* = 0.331, \quad 0.401, \quad 0.349 . \tag{4.4b}$$

We shall choose an average value of a* as a desired value temporarily. It becomes 0.345 and 0.360 for cases (i) and (ii), respectively, which are slightly higher than the value 0.263 given by MS.[2)]

5. Discussion

Assuming the existence of a fixed value of the QCD coupling constant at a certain range of low momentum transfer, the method to extract it from the anomalous magnetic moments of confined quarks is discussed. The approach is model dependent and there also exists the question on the convergence of perturbative series in the QCD at such a low momentum transfer. There still remains the question on our formula Eq.(4.3). It is desired to find a more rigorous way to extract the anomalous magnetic moments from their composite systems. We shall be content with our conjectured form for the time being. We could not find a complete agreement with the a* found from the renormalization-group-equation analysis of beta function. However, a coincidence

within 30-40 % seems to be taken as a good sign on the existence of a*.

The observed Ω_c $2719 \pm 7.0 \pm 2.5$ MeV[11] agrees with our predicted 2727 MeV[5] within an experimental error. In order to get a further test of the model it is desirable to have an experimental information on the magnetic moments for charmed baryons in addition to the observation of their unobserved partners. Unfortunately, the lifetime of them is about three order of magnitude shorter than that for the strange baryons, so it may require unsurmountable technical difficulties in addition to their abundant production. Alves et al.[12] have reported the production of $D^{\pm}$ and $D^0\bar{D}^0$ in 250 GeV π^--nucleon interactions. The production mechanism for them is assumed to be the hadro-produced by gluon-gluon fusion and quark-anti-quark annihilation. The observed D^- excess over D^+ is therefore reduced to the sea quark contribution of the nucleon. The threshold of the charmed meson and baryon production in association with π^--p interaction is 8.72 and 9.48 GeV for $D^- + \Lambda_c^+$ and $D^- + \Sigma_c^+$, respectively. We think that the part of the asymmetry is due to these mechanisms analogous to the positive K^+ excess over K^- in earlier cosmic ray study of strange particles. We hope that the production cross section for charmed baryons in the existing equipment will be examined for the feasibility of the study of the magnetic moments of charmed baryons. The counter experiment by π^+ beam will tell us the production mechanism of $D^{\pm}$ problem.

Finally we should like to make an additional rather important comment on the factor 2 difference between q-q' and q-$\bar{q}$' exchange parameters for baryons and mesons,[5] although it is not directly relevant the present work. We reduced that difference to the normalization one of spin wave functions in the previous work. It is misleading from the instanton picture of the model. Notice that above exchange for meson corresponds to the simultaneous exchange of

instanton and anti-instanton structure in IB and $\overline{IB}$, while it does not apply to the baryon. We need additional exchange of full instanton structure for the latter. This causes a factor 2 reduction in the case of baryons. These arguments should always be taken in the sense of time average of the realization of IB and $\overline{IB}$ clusters for the given hadron.

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Table I. Octet and decuplet baryon masses (MeV).

baryon	theory	experiment
p	938.27448	938.27231 ± 0.00028
n	939.56348	939.56563 ± 0.00028
Λ	1115.48	1115.63 ± 0.05
Σ^+	1191.27	1189.37 ± 0.07
Σ^0	1192.56	1192.55 ± 0.10
Σ^-	1193.85	1197.43 ± 0.06
Ξ^0	1329.93	1314.9 ± 0.6
Ξ^-	1331.22	1321.32 ± 0.13
Δ	1247	1232 ± 10
Σ^{*+}	1383.8	1382.8 ± 0.4
Σ^{*0}	1385.1	1383.7 ± 1.0
Σ^{*-}	1386.4	1387.2 ± 0.5
Ξ^{*0}	1522.47	1531.80 ± 0.32
Ξ^{*-}	1523.8	1535.0 ± 0.6
Ω^-	1661.14	1672.43 ± 0.32

Fitted parameters
U = 415.046
D = 416.335
S = 553.712
α = 205.435
β = 128.357
χ^2 = 13322

Table II. Magnetic moments of baryons (n.m.).

baryon	theory	experiment
p	2.79284700	2.79284739 ± 0.00000006
n	-1.9130480	-1.9130427 ± 0.0000005
Λ	-0.703	-0.613 ± 0.004
Σ^+	2.70	2.42 ± 0.05
Σ^-	-1.061	-1.160 ± 0.025
Ξ^0	-1.555	-1.250 ± 0.014
Ξ^-	-0.6140	-0.6507 ± 0.0025
Ω^-	-2.11	-1.94 ± 0.22
$\Sigma^0\Lambda$	-1.63	-1.61 ± 0.08

quark	Fitted parameters
u	1.851668
d	-0.971869
s	-0.703462

χ^2 =1405