Fermilab Library
0 1160 0019196 9

PAR-LPTHE-92-41

PAR-LPTHE 92-41

BRS Invariance, Supersymmetry and the Shift of the bare Parameter in Chern-Simons Theory*

G. Giavarini,[a)] C. P. Martin[b)] and F. Ruiz Ruiz[c)]

a) L.P.T.H.E., Université Paris VI-VII, 2 Place Jussieu, 75251 Paris Cedex 05, France.

b) Department of Mathematics and Statistics, University of Guelph, Guelph, Ontario N1G 2W1, Canada.

c) Niels Bohr Institute, University of Copenhagen, Blegdamsvej 17, DK-2100 Copenhagen O, Denmark.

The Chern-Simons (CS) action on $\mathbb{R}^3$ for an $SU(N)$ gauge connection A^a_μ in the fundamental representation is given by

$$S_{CS} = -\frac{ik}{4\pi}\int d^3x\ \epsilon^{\mu\rho\nu}\left(\frac{1}{2}\ A^a_\mu\partial_\rho A^a_\nu + \frac{1}{3!}\ f^{abc}A^a_\mu A^b_\rho A^c_\nu\right)\ . \tag{1}$$

In this note we shall briefly report on some aspects of perturbative quantization of CS theory in the Landau gauge and on the related problem of radiative corrections to the classical parameter k. The action in eq. (1) is invariant under infinitesimal gauge transformations for any value of k. In order to obtain a quantum theory single valued under large gauge transformations with non-zero winding number, k must take on integer values only. However, large gauge transformations lie beyond the perturbative regime and therefore k does not necessarily need to be an integer in a perturbative analysis. Consequently, there is no reason a priory why k should undergo the quantization process without getting non-integer corrections. The Landau gauge fixing term is

$$S_{GF} = \int d^3x\ \left[(J^{a\mu} - \partial^\mu\bar{c}^a)D^{ab}_\mu c^b - b^a\partial A^a - \frac{1}{2}\ f^{abc}H^a c^b c^c\right]\ . \tag{2}$$

As customary, b^a denotes the Lagrange multiplier imposing the gauge condition $\partial A^a = 0$, c^a and $\bar{c}^a$ are Faddeev-Popov ghosts, $D^{ac}_\mu = \partial_\mu\delta^{ac} + f^{abc}A^b_\mu$ is the covariant derivative and J^a_μ and H^a are auxiliary fields introduced for later convenience. Apart from the usual BRS symmetry, the gauge-fixed action is invariant under the following transformations [1]

$$\begin{aligned} &s_\nu A^a_\mu = i\epsilon_{\nu\mu\rho}\,\partial^\rho c^a\,, && s_\nu c^a = 0\,, && s_\nu J^a_\mu = 0\,,\\ &s_\nu b^a = -\frac{k}{4\pi}(D_\nu c)^a\,, && s_\nu\bar{c}^a = \frac{k}{4\pi}A^a_\nu\,, && s_\nu H^a = -\frac{ik}{4\pi}\epsilon_{\nu\mu\rho}\,\partial^\mu J^{a\rho}\,, \end{aligned} \tag{3}$$

whose generators s_ν form, together with the BRS generator, a $\mathcal{N} = 1$ abelian superalgebra. In this gauge, the theory is known to be perturbatively finite [2,3], that is, the beta function and the anomalous dimensions of the fields vanish to all orders in perturbation theory. The proof in Ref. [3] makes use of the Ward identities (WIs) associated to the supersymmetry above. Perturbative finiteness of Green functions however does not imply that the corresponding Feynman diagrams should also be finite. Actually, since the theory is by power counting only renormalizable, the Feynman diagrams contributing to a given Green function are in general UV divergent and a regularization method is required. Now, UV finiteness of the theory ensures the cancellation of the divergences when summing over all Feynman diagrams contributing to the Green functions at a given perturbative order. The values of the Green functions so obtained depend on the regulator used which, being a purely technical device, should not entail any physical consequences. By now several regularization schemes have been proposed for CS theory. They have explicitly confirmed finiteness at first perturbative order and in some instances to second order. These regularization schemes can be divided into two groups. On the one hand there are manifestly BRS invariant regularizations, such as η-function regularization [4], higher covariant derivatives (HCD) and Pauli-Villars fields [5,6], large mass limit of dimensionally regularized topologically massive Yang-Mills (TMYM) theory [7] and geometric regularization [8]. In all these schemes the parameter k gets a one-loop integer shift $k \to k + c_v$, where $c_v = N$ is the quadratic Casimir operator in the adjoint representation of $SU(N)$. This shift is the key to understanding the correspondence between 3-dimensional CS theory and current algebra in 2 dimensions [4]. On the other hand, there are methods that explicitly break BRS invariance at the regularized level, like the one in [9]. In this case no radiative correction to k is found up to two loops. It is important to realize that the BRS invariant regularizations in Refs. [4-8] happen to break the supersymmetry (3), while the one in Ref. [9] preserves it. We are not aware of any regularization method invariant under both BRS and supersymmetry transformations. Moreover, it has been explicitly shown [10,11] that, for the regularizations in Refs. [7] and [5], the supersymmetric WIs remain broken in the limit where the regulator is removed and that the gauge invariant part of the breaking is responsible for the shift of k. Since the symmetry (3) is not anomalous [3], the natural question to ask is if the absence of any shift is the price to pay to recover the supersymmetry at the quantum level. In the following we shall concisely illustrate how a parametrization can in fact be naturally singled out in CS theory from gauge invariance arguments and show that supersymmetry can be reconciliated with the shift of k [10]. First of all, we must make clear what the meaning of renormalization is for a finite theory. Let us symbolically denote by Λ the regulator (or set of regulators) needed to regularize CS theory at any perturbative order and by $\Lambda \to \infty$ the limit in which the regulator is removed. Since finiteness ensures that this limit exists, the bare effective action Γ and the bare E-point Green functions are defined as

$$\Gamma = \lim_{\Lambda\to\infty}\Gamma_\Lambda\ , \qquad G(p_1,\ldots,p_E) = \lim_{\Lambda\to\infty}G_\Lambda(p_1,\ldots,p_E)\ , \tag{4}$$

where Γ_Λ and G_Λ are the regularized effective action and Green functions. In a finite theory, eqs. (4) define a renormalization scheme in which renormalized quantities equal

* Talk presented by G. Giavarini at the XIXth International Colloquium on "Group Theoretical Methods in Physics", Salamanca (Spain), June 29-July 4, 1992.

the bare ones. However, in this scheme the values of the Green functions, and in general the value of the observables, could depend on the particular regularization used. This ambiguity can and must be resolved by a precise parametrization of the theory. Since local perturbative renormalization theory leaves complete freedom to choose a parametrization, we must look for arguments lying outside the perturbative framework. Here we take as guideline what we call the fundamental symmetries of the theory. We say that a symmetry is *fundamental* if the quantum theory does not make sense without it. The idea is that for a finite theory (of which CS is an example) the classical parameters constitute a natural parametrization of the theory [12], provided the fundamental symmetry are preserved at the regularized level. For this parameterization to make sense, all symmetry preserving regularization methods must yield the same value of the observables as function of the bare parameters. Now, BRS is obviously fundamental for CS theory. This is not the case for the symmetry (3), which does not leave the observables invariant and is only a symmetry of the gauge-fixed action in a particular gauge. Accordingly, a unique quantum theory should be defined by parametrizing CS theory in terms of the bare k, once BRS has been preserved at the regularized level. Note that this parametrization does not specify completely a renormalization scheme since any finite renormalization of the fields is still allowed. This freedom is however unphysical as the values of the observables are unaffected by rescalings of the fields. To show that all BRS invariant regularizations, with the above parametrization choice, give in fact the same value for the observables, we must first determine the most general BRS invariant bare effective action. The local part of the effective action can be found by solving the BRS identity under the only assumptiom that contributions with fewer than four fields are local, something that can be checked by explicit computations. At order $\hbar$ the local part of the effective action is found to be [6]

$$\Gamma_{\rm loc} = (1+\alpha)S_{CS}[A] - \int d^3x \; b^a\partial A^a + \Delta X \; . \tag{5}$$

Here $X = \int d^3x \; [\beta G^{a\mu}A^a_\mu - (1+\gamma)H^a c^a]$ and Δ is the Slavnov-Taylor operator

$$\Delta = \int d^3x \left[\frac{\delta\bar{S}}{\delta A^{\mu a}}\frac{\delta}{\delta G^a_\mu} + \frac{\delta\bar{S}}{\delta G^a_\mu}\frac{\delta}{\delta A^{\mu a}} + \frac{\delta\bar{S}}{\delta c^a}\frac{\delta}{\delta H^a} + \frac{\delta\bar{S}}{\delta H^a}\frac{\delta}{\delta c^a}\right] , \tag{6}$$

with $\bar{S} = S_{CS}+S_{GF}+\int d^3x \; b^a\partial A^a$ and the field G_μ denoting the combination $J_\mu - \partial_\mu\bar{c}$. $\Gamma_{\rm loc}$ receives two contributions: one is gauge invariant and provides a monodromy parameter $k(1+\alpha)$, the other is the non-observable cohomologically trivial term ΔX. This implies the existence of the wave function renormalization $\Phi = Z_\Phi\Phi_R$ ($\Phi = A_\mu, b, c, H, G_\mu$), with $Z_A = Z_G^{-1} = Z_b^{-1} = 1-\beta$, $Z_c = Z_H^{-1} = 1-\gamma$, such that $\Gamma_{\rm loc}$ takes the form

$$\Gamma_{\rm loc} = (1+\alpha)\, S_{CS}[A_R] + S_{GF}[\Phi_R] \; . \tag{7}$$

Thus α is the only observable one-loop correction. Let us emphasize that the coefficients α, β and γ may differ for different BRS invariant regulators. One would then conclude that

the emerging quantum theory is not unique. However, it happens that all BRS invariant regulators yield the same value for α, the only relevant coefficient. In the folowing table we collect the values (in units of c_v/k) of the one-loop parameters for the various BRS invariant regularization schemes used so far for CS theory.*

Regularization Method	α	β	γ
Large m of TMYM + dimensional reg.	1	2/3	0
η-function regularization	1	0	0
HCD + Pauli-Villars	1	2/9	0
Geometric regularization	1	$4/(3\pi)\, I_n$	*

Geometric quantization makes use of ghost generations different from the standard Faddeev-Popov ones so only the pure gauge sector of Γ can be compared. In summary, different regularizations yield different Γ's but all of them predict the same observable shift $k \to k + c_v$ and therefore the same physical theory. Striking is the case of geometric regularization that defines a whole family of physically undistinguishable effective actions depending on $I_n = \int_0^\infty dp \; \frac{(1+p^2)^n}{1+p^2(1+p^2)^{2n}}$ with $n > 1$ an arbitrary integer.

Notice that neither the bare effective action (5) nor the renormalized one (7) are supersymmetric invariant. However, within our parametrization, it is easy to find another field redefinition such that the renormalized one-loop effective action becomes supersymmetric. Indeed, taking Z_Φ such that $Z_A = Z_b^{-1} = 1-\frac{1}{2}\alpha-\beta$, $Z_G = Z_{\bar{c}}$, $Z_c Z_{\bar{c}} = 1+\beta-\gamma$ and $Z_H Z_c^2 = 1-\frac{1}{2}\alpha-\gamma$, the resulting renormalized effective action can be written as:

$$\begin{aligned}\Gamma_{\rm loc} &= S_{CS}[A_R] + S_{GF}[\Phi_R] \\ &+ \frac{\alpha}{2}\int d^3x \left[\frac{ik}{4\pi}\epsilon^{\mu\rho\nu}\frac{1}{3!}f^{abc}A^a_{R\mu}A^b_{R\rho}A^c_{R\nu} - f^{abc}G^{a\mu}_R A^b_{R\mu}c^a_R + \frac{1}{2}f^{abc}H^a_R c^b_R c^c_R\right] \; .\end{aligned} \tag{8}$$

It is a matter of simple algebra to show that the latter action satisfies both the BRS identity and the supersymmetric WIs [10]. The effective actions (5), (7) and (8) give manifestly the same values for the observables as function of k since they only differ by finite rescalings of the fields. Accordingly, the supersymmetric action (8) still leads to the shifted value $k + c_v$, despite it is not as apparent as in (7). This renormalization program can be carried on at higher perturbative orders [6] under the only additional hypothesis that terms with fewer than 4 fields in Γ keep being local (that this is indeed the case has been cheked up to two loops in [7]). More precisely, as at one loop BRS invariance requires that at any order $\Gamma_{\rm loc}$ depend on three parametres, of which only one is observable as a shift of k and a finite renormalization of only the fields can be performed so that $\Gamma_{\rm loc}$ takes the supersymmetric form (8). So BRS invariance and the supersymmetry (3) alone do not

* The values given here for HCD+Pauli-Villars are those computed in [6] rather than those in [5], where strictly speaking only Pauli-Villars filelds and no HCD are used. In [5] one gets $\alpha = 1$, $\beta = \gamma = 0$.

impose any non-renormalization of k and finding no shift at two loops [7] appears as a highgly non-trivial (though expected) result.

REFERENCES

1. D. Birmingham, M. Rakowski and G. Thompson, Nucl. Phys. **B234** (1990) 367.
   F. Delduc, F. Gieres and S.P. Sorella, Phys. Lett. **B225** (1989) 367.
2. A. Blasi and R. Collina, Nucl. Phys. **B345** (1990) 472.
3. F. Delduc, C, Lucchesi, O. Piguet and S.P. Sorella, Nucl. Phys. **B346** (1990) 513.
4. E. Witten, Commun. Math. Phys. **17** (1988) 353.
   D. Birmingham, R. Kantwoski and M. Rakowski, Phys. Lett. **B251** (1990) 121.
5. L. Alvarez-Gaumé, J.M.F. Labastida and A.V. Ramallo, Nucl. Phys. **B334** (1990) 103.
6. G. Giavarini, F. Ruiz Ruiz, C.P. Martin, *Perturbative quantization of Chern-Simons theory*, preprint.
7. G. Giavarini, C.P. Martin and F. Ruiz Ruiz, Nucl. Phys. **B381** (1992) 222.
8. M. Asorey and F. Falceto, Phys. Lett. **241** (1990) 31; Int. J. Mod. Phys. **A7** No.2 (1992) 235.
9. E. Guadagnini, M. Martellini and M. Mintchev, Phys. Lett. **B227** (1989) 111.
10. G. Giavarini, C.P. Martin and F. Ruiz Ruiz, *Supersymmetry breaking in a BRS-invariant regularization of Chern-Simons theory*, preprint.
11. N. Dorey, Phys. Lett. **B246** (1990) 87.
12. A. Jaffe and A. Lesniewski, *Supersymmetric quantum fields and infinite dimensional analysis*, in *Non-perturbative quantum field theory*, G. 't Hooft, A. Jaffe, G. Mack, P.K. Mitter and R. Stora eds., Plenum Press (1988).