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CLASSICAL AND QUANTUM STRINGS NEAR SPACE-TIME SINGULARITIES: GRAVITATIONAL PLANE WAVES WITH ARBITRARY POLARIZATION

H.J. DE VEGA
LPTHE*, Paris

M. RAMON MEDRANO
Departamento de Fisica Teorica, Madrid**

and

N. SANCHEZ
Observatoire de Paris, Section de Meudon***

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* Laboratoire Associé au CNRS UA280
Postal Address: LPTHE, Université Paris VI, Tour 16, 1er étage, 4, Place Jussieu, F-75252, PARIS Cedex 05, **FRANCE**.

** Postal address: Facultad de Ciencias Fisicas, Universidad Complutense, Ciudad Universitaria, E-28040, MADRID, **SPAIN**.

***DEMIRM, Laboratoire Associé au CNRS UA336, Observatoire de Meudon et Ecole Normale Supérieure Paris.
Postal Address: DEMIRM, Observatoire de Paris, Section de Meudon, 92195 MEUDON Principal Cedex, **FRANCE**.

ABSTRACT

We study the classical and quantum dynamics of strings near spacetime singularities. We deal with singular gravitational plane waves of arbitrary polarization and arbitrary profile functions $W_1(U)$ and $W_2(U)$, whose behavior for $U \to 0$ is $W_1(U) = \alpha_1 / U |^{\beta_1}$, $W_2(U) = \alpha_2 / U |^{\beta_2}$ (U being a null variable). In these spacetimes, the string dynamics is exactly (and explicitly) solvable even at the spacetime singularities.

The string behavior depends crucially on whether both parameters β_1 and β_2 are smaller or larger than two. When both $\beta_1 < 2$ and $\beta_2 < 2$, the string passes smoothly through the spacetime singularity. When $\beta_1 \geq 2$ and/or $\beta_2 \geq 2$, the string does not cross the singularity $U = 0$ but goes off to infinity in a given direction α depending on the polarization of the gravitational wave : (i) if the singularity of W_1 dominates over that of W_2, then $\alpha = 0$ and we recover the string in the constant polarization case, (ii) if the singularity of W_2 dominates (with $\beta_2 \geq 2$), then $\alpha = \mathrm{sign}[\alpha_2]\pi/4$, and (iii) when $\beta_1 = \beta_2 \geq 2$, then $\mathrm{tg}(2\alpha) = \alpha_2/\alpha_1$. Perpendicular to the escape direction, the string approaches the singularity with fixed frequency and vanishing amplitude. The string time evolution is fully determined by the spacetime geometry, whereas the overall σ-dependence is fixed by the initial string state. The proper length at fixed $\tau \to 0$ $(U \to 0)$ strechts infinitely.

For $\beta_1 < 2$ and $\beta_2 < 2$, the string passes smoothly through the spacetime singularity and reaches the $U > 0$ region. In this case, outgoing operators make sense and we find the explicit transformation relating in and out operators. For the quantum string states, this implies spin polarization rotations and particle transmutations. The expectation values of the outgoing mass and mode number operators are computed and the excitation of the string modes after the crossing of the spacetime singularity is analysed.

I - INTRODUCTION AND RESULTS

The study of string dynamics in curved space-times, reveals new insights with respect to string propagation in flat space-time[1-9]. Particularly interesting is the string behaviour in strong gravitational fields and in the vicinity of space-time singularities. Gravitational shock-waves and gravitational plane waves are physically relevant and mathematically tractable space-times for studying the string dynamics. Singular gravitational waves are specially attractive since the string equations in these space-times turn out to be exactly (and explicitly) solvable even at the space-time singularities.

In this paper, we consider strings propagating in gravitational plane-wave space-times described by the metric

$$ds^2 = \left[W_1(U)(X^2-Y^2) + 2W_2(U)\,XY\right]dU^2 - dU\,dV + (dX^j)^2 \tag{1.1}$$

Here, U, V are null coordinates

$$U = X^0 - X^{D-1} \quad , \quad V = X^0 + X^{D-1}$$

and $X \equiv X^1$, $Y \equiv X^2$, X^3, X^4,..., X^{D-2} are spatial coordinates. $W_1(U)$ and $W_2(U)$ are arbitrary functions describing the profile of the gravitational wave. These space-times are exact solutions of the vacuum Einstein equations. The case when $W_2(U) = 0$ describes waves of constant polarization and was studied in ref.[8]. The case with both $W_1 \neq 0$ and $W_2 \neq 0$ describes waves with arbitrary polarization.

If $W_1(U)$ and/or $W_2(U)$ are singular functions, space-time singularities will be present. The singularities will be located on the null plane U = constant. Here, we will consider profile functions with power-type singularities,

$$W_1(U) \underset{U\to 0}{=} \frac{\alpha_1}{|U|^{\beta_1}} \quad , \quad W_2(U) \underset{U\to 0}{=} \frac{\alpha_2}{|U|^{\beta_2}} \tag{1.2}$$

In this class of space-times, one can choose the light-cone gauge $U = 2\alpha' p^U \tau$. After Fourier expansion in the world sheet coordinate σ, the Fourier components $X_n(\tau)$ and $Y_n(\tau)$ satisfy a pair of coupled one-dimensional Schrödinger-type equations with τ playing the role of the spatial coordinate, and

$$W(\tau) = \left(2\alpha' p^U\right)^2 \begin{pmatrix} W_1 & W_2 \\ W_2 & -W_1 \end{pmatrix}\Bigg]_{U=2\alpha' p^U(\tau)} ,$$

as an effective potential. (Here p_U stands for the U-component of the string momentum). The transverse coordinates X^j (j = 3,..., D–2) propagate freely. We study the string propagation when it approaches the singularity at U = 0, from say, U < 0. We find that the string behaviour depends crucially on whether both power parameters β_1 and β_2 are smaller or bigger than two. When $\beta_1 < 2$ and $\beta_2 < 2$, the solutions $X(\sigma,\tau)$ and $Y(\tau,\sigma)$ are regular and the string passes smoothly through the gravitational singularity at the null plane U = 0. When $\beta_1 \geqslant 2$ and/or $\beta_2 \geqslant 2$, the string does not cross the singularity plane U=0, but it goes off to infinity in the (X,Y) plane. We find here that the string scapes to infinity in a given direction α with respect to the X-axis, which depends on the polarization of the gravitational wave. We obtain: (i) for $\beta_1 > \beta_2$ (and $\beta_2 \geq 2$), then the angle α=0 and the string goes off to infinity in the X-direction. In this case, the singularity of W_1 dominates over that of W_2 and we recover the situation of ref.[8]. (ii) for $\beta_2 > \beta_1$ (and $\beta_2 \geqslant 2$), then $\alpha = (\pi/4)\,\mathrm{sgn}\,\alpha_2$, and (iii) for $\beta_1 = \beta_2 \geqslant 2$, then

$$\mathrm{tg}\,\alpha = \frac{\alpha_2}{\alpha_1 + \sqrt{\alpha_1^2 + \alpha_2^2}} \quad , \quad \text{i.e.} \quad \mathrm{tg}\,2\alpha = \frac{\alpha_2}{\alpha_1} \; .$$

If $\alpha_1 > 0$, ($\alpha_1 < 0$), the string escape directions are within the cone $|\alpha| < \pi/4$, ($|\alpha - \pi/2| < \pi/4$) as depicted in figure 1.
In addition to escaping to infinity, the string oscillates in the (X,Y) plane, perpendicularly to the escape direction, and with vanishing amplitude for $U \to 0$.

We find that the time coordinate $X^0(\sigma,\tau)$ near the U = 0 singularity behaves as:

$$X^0(\sigma,\tau) = \begin{cases} B^2(\sigma)\ (-\tau)^{-\nu} & \text{for } \beta = 2 \\ B^2(\sigma)\exp\left[\dfrac{2\sqrt{\bar{\alpha}}}{\frac{\beta}{2}-1}\left(-2\alpha' p\tau\right)^{1-\beta/2}\right] & \text{for } \beta > 2 \end{cases} \qquad (1.3)$$

where $v = \sqrt{1+4\bar{\alpha}}$ and

$$\bar{\alpha} = \begin{cases} \alpha_1 & \text{for } \beta_1 > \beta_2 \\ \alpha_2 & \text{for } \beta_2 > \beta_1 \\ \sqrt{\alpha_1^2 + \alpha_2^2} & \text{for } \beta_1 = \beta_2 \end{cases}$$

We find that, as a function of the time coordinate $X^0(\sigma,\tau)$, the transverse string coordinate $\rho = \sqrt{X^2 + Y^2}$, approaches asymptotically (for $X^0 \to \infty$), the U = 0 singularity as

$$\rho(\sigma, X^0) \underset{X^0 \to \infty}{\sim} \begin{cases} B(\sigma)^{1/\nu}\left(\sqrt{X^0}\right)^{1-1/\nu} & \text{for } \beta = 2 \\ \sqrt{X^0}\left[\ln\left(\dfrac{X^0}{C_2\, B^2(\sigma)}\right)\right]^{-\frac{\beta}{2(\beta-2)}} & \text{for } \beta > 2 \end{cases} \qquad (1.4)$$

We find that when the string approaches the U = 0 singularity, its proper length at a fixed $\tau \to 0^-$, stretchs infinitely as

$$\sqrt{-\tau}^{\,1-\nu}, \quad \beta = 2$$

$$\sqrt{-\tau}^{\,\beta/2}\exp\left[\frac{\sqrt{\bar{\alpha}}}{\frac{\beta}{2}-1}\left(-2\alpha' p^0\tau\right)^{1-\beta/2}\right], \quad \beta > 2$$

We find that the string behaviour near the singularity expresses naturally in terms of the variable

$$\hat{U} = \begin{cases} \ln(-U) & \beta = 2 \\ \dfrac{(-U)^{1-\beta/2}}{1-\beta/2} & \beta > 2 \end{cases}$$

For instance, the oscillatory modes in the (X,Y) plane are not harmonic in U but in $\hat{U}$. $\hat{U}$ is like the cosmic time for strings in cosmological backgrounds (in terms of which the string oscillates), whereas U is like the conformal time.

We would like to remark that the string evolution near the space-time singularity is a **collective motion** governed by the nature of the gravitational field. The state of the string fixes the overall σ-dependent coefficient $B(\sigma)$ [see eqs.(1.3)-(1.4)], whereas the τ-dependence is fully determined by the space-time geometry. In other words, the τ-dependence is the same for all modes n. In some directions, the string collective propagation turns to be an infinite motion (the escape direction), whereas in other directions, the motion is oscillatory, but with a fixed (n-independent) frequency. In fact, these features are not restricted to singular gravitational waves, but **are generic** to strings in strong gravitational fields.

In section III, we study the quantum propagation of the string for the case $\beta_1 < 2$ and $\beta_2 < 2$. In this case, the string **crosses** the singularity and reaches the region U > 0. Therefore, outgoing scattering states and outgoing operators can be defined in the region U > 0. We explicitly find the transformation relating the ingoing and outgoing string mode operators. For the particles described by the quantum string states, this relation implies two types of effects: (i) rotation of spin polarization in the (X,Y) plane, and (ii) transmutation between different particles. We compute the expectation values of the outgoing mass ($M^2_>$) operator and

of the mode-number operator $N_>$, in the ingoing ground state $|O_<>$. We find for $M^2_>$ and $N_>$ different VEV's than for $M^2_<$ and $N_<$. This difference is due to the excitation of the string modes after crossing the space-time singularity. In other words, the string state is not an eigenstate of $M^2_>$, but an infinity superposition of one-particle states with different masses. This is a consequence of the particle transmutation which allows particle masses different from the initial one $(m_0{}^2)$.

II - THE STRING DYNAMICS IN SINGULAR GRAVITATIONAL WAVES WITH ARBITRARY POLARIZATION. THE STRING ESCAPE ANGLE.

Let us consider the exact D-dimensional gravitational plane wave space-time

$$ds^2 = F(U,X,Y)\,dU^2 - dU\,dV + (dX^j)^2 \tag{2.1}$$

where U and V are null coordinates

$$U = X^0 - X^{D-1} \quad , \quad V = X^0 + X^{D+1} .$$

$X \equiv X^1$, $Y \equiv X^2$, X^3, X^4,.... X^{D-2} are spatial coordinates.

In order to satisfy the vacuum Einstein equations, F(U,X,Y) in eq.(2.1) must obey

$$\left(\frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2} \right) F(U,X,Y) = 0 \tag{2.2}$$

A gravitational wave with arbitrary polarization is described by

$$F(U,X,Y) = W_1(U)\,(X^2 - Y^2) + 2\,W_2(U)\,XY \tag{2.3}$$

where $W_1(U)$ and $W_2(U)$ are arbitrary functions describing the profile of the wave.

The case where $W_2 = 0$ corresponds to gravitational waves with constant polarization and was considered in ref.[8].

Let us consider strings propagating in the above space-time. In curved space-time, the string equations of motion are given by

$$2\,\partial_a \left(G_{\lambda\nu}\, \partial^a X^\nu \right) = \partial_\lambda G_{\mu\nu} \left(\partial_a X^\mu \partial^a X^\nu \right)$$

The equations of motion for the string coordinates $X^A(\sigma,\tau)$ take the form

$$(\partial_\tau^2 - \partial_\sigma^2)\, U(\sigma,\tau) = 0 \tag{2.4}$$

$$(\partial_\tau^2 - \partial_\sigma^2)\, V + \frac{\partial F}{\partial U} \left[(\partial_\sigma U)^2 - (\partial_\tau U)^2 \right] + 2\,\frac{\partial F}{\partial X} \cdot \left(\partial_\sigma X\, \partial_\sigma U - \partial_\tau X\, \partial_\tau U \right) + 2\,\frac{\partial F}{\partial Y} \left(\partial_\sigma Y\, \partial_\sigma U - \partial_\tau Y\, \partial_\tau U \right) = 0 \tag{2.5}$$

$$(\partial_\tau^2 - \partial_\sigma^2)\, X^j = 0 \quad , \quad 3 \leq j \leq D-2 \tag{2.6}$$

$$(\partial_\tau^2 - \partial_\sigma^2)\, X^k + \frac{1}{2}\,\frac{\partial F}{\partial X^k} \left[(\partial_\sigma U)^2 - (\partial_\tau U)^2 \right] = 0 \quad , \quad k = 1,2 \tag{2.7}$$

In this space-time, the string constraints are

$$T_{\pm\pm}(\sigma,\tau) = -\partial_\pm U\, \partial_\pm V + F(U)\,(\partial_\pm U)^2 + \sum_{j=1}^{D-2} (\partial_\pm X^j)^2 \simeq 0 \tag{2.8}$$

where $\quad x_{\pm} = (\sigma \pm \tau) \quad , \quad \partial_{\pm} = \frac{1}{2}(\partial_\sigma \pm \partial_\tau)$

Since U obeys the d'Alembert equation, we can choose the light-cone gauge

$$U = 2\alpha' p^U \tau \tag{2.9}$$

Therefore, with F given by eq.(2.3), equations (2.5)-(2.7) become

$$V'' - \ddot{V} + (2\alpha' p^U)^2 \left[(\partial_U W_1)(X^2 - Y^2) + 2(\partial_U W_2) X Y \right] + \\ + 4(2\alpha' p^U) \left[W_1 (X\dot{X} - Y\dot{Y}) + W_2 (\dot{X} Y + \dot{Y} X) \right] = 0 \tag{2.10a}$$

$$X'' - \ddot{X} + (2\alpha' p^U)^2 \left[W_1 X - W_2 Y \right] = 0 \tag{2.10b}$$

$$Y'' - \ddot{Y} + (2\alpha' p^U)^2 \left[W_2 X - W_1 Y \right] = 0 \tag{2.10c}$$

and

$$(\alpha' p^U)^2 \left[W_1(U)(X^2 - Y^2) + 2 W_2(U) X Y \right] \mp \\ \mp (\alpha' p^U) \partial_{\pm} V + (\partial_{\pm} X)^2 + (\partial_{\pm} Y)^2 + \sum_{j=3}^{D-2} (\partial_{\pm} X^j)^2 = 0 \tag{2.11}$$

Interesting profile functions $W_1(U)$ and $W_2(U)$ to be considered here are those which are nonzero only on a finite interval $-T < U < T$, and which have a singular behaviour for $U \to 0$ say, like

$$W_1(U) \underset{U \to 0}{=} \frac{\alpha_1}{|U|^{\beta_1}} \quad , \quad W_2(U) \underset{U \to 0}{=} \frac{\alpha_2}{|U|^{\beta_2}} \tag{2.12}$$

Let us analyse now the solutions of the string equations (2.10) and (2.11) for a closed string. The transverse coordinates obey the free equation (2.6), with the solution

$$X^i = q^i + 2\alpha' p^i \tau + \frac{i}{2}\sqrt{2\alpha'} \sum_{n \neq 0} \frac{1}{n} e^{in\sigma} \left(\alpha_n^i e^{-2in\tau} - \tilde{\alpha}_{-n}^i e^{2in\tau} \right) \\ (i = 3, \ldots, D-2) \tag{2.13}$$

For the X and Y components it is convenient to Fourier expand as

$$X(\sigma, \tau) = \sum_{n=-\infty}^{+\infty} e^{2in\sigma} X_n(\tau) \\ Y(\sigma, \tau) = \sum_{n=-\infty}^{+\infty} e^{2in\sigma} Y_n(\tau) \tag{2.14}$$

Then, eqs.(2.10) for X and Y yield

$$\ddot{X}_n + 4n^2 X_n - (2\alpha' p^U)^2 (W_1 X_n + W_2 Y_n) = 0$$
$$\ddot{Y}_n + 4n^2 Y_n - (2\alpha' p^U)^2 (W_2 X_n - W_1 Y_n) = 0 \qquad (2.15)$$

Formally, these are two coupled one-dimensional Schrödinger-like equations with τ playing the role of a spatial coordinate. It is convenient to write them in the matrix form

$$\left[\frac{d^2}{d\tau^2} + 4n^2 - W(\tau) \right] \begin{pmatrix} X_n \\ Y_n \end{pmatrix} = 0 \qquad (2.16a)$$

with

$$W(\tau) = (2\alpha' p^U)^2 \begin{pmatrix} W_1 & W_2 \\ W_2 & -W_1 \end{pmatrix} \qquad (2.16b)$$

playing the role of the effective potential. We study now the interaction of the string with the gravitational wave. For $\alpha' p\, \tau < -T$, $W(\tau) = 0$ and therefore X, Y are given by the usual flat-space expansions

$$X(\sigma,\tau) = q^X_< + 2\alpha' p^X_< \tau + \frac{i}{2}\sqrt{2\alpha'} \sum_{n\neq 0} \frac{e^{2in\sigma}}{n} \left(\alpha^X_{n<} e^{-2ni\tau} - \tilde{\alpha}^X_{-n<} e^{2in\tau} \right)$$
$$Y(\sigma,\tau) = q^Y_< + 2\alpha' p^Y_< \tau + \frac{i}{2}\sqrt{2\alpha'} \sum_{n\neq 0} \frac{e^{2in\sigma}}{n} \left(\alpha^Y_{n<} e^{-2ni\tau} - \tilde{\alpha}^Y_{-n<} e^{2in\tau} \right) \qquad (2.17)$$

These solutions define the initial conditions of the string propagation for $\tau \geqslant \tau_0$, $\tau_0 \equiv T/2\alpha' p^U$.

In the language of the Schrödinger-like equations we have a (matrix) potential barrier (or a potential well) in $-T < U < T$. The string solutions eq.(2.17) correspond in the Schrödinger-like language to the incident and reflected waves from the left side

$$X_{n<}(\tau) = \frac{i}{n}\sqrt{\frac{\alpha'}{2}} \left(\alpha^X_{n<} e^{-2in\tau} - \tilde{\alpha}^X_{-n<} e^{2in\tau} \right)$$
$$Y_{n<}(\tau) = \frac{i}{n}\sqrt{\frac{\alpha'}{2}} \left(\alpha^Y_{n<} e^{-2in\tau} - \tilde{\alpha}^Y_{-n<} e^{2in\tau} \right) \qquad (2.18a) \qquad \text{for } \tau < -\tau_0$$

and

$$X_{0<}(\tau) = q^X_< + 2\alpha' p^X_< \tau$$
$$Y_{0<}(\tau) = q^Y_< + 2\alpha' p^Y_< \tau \qquad (2.18b)$$

We consider the propagation of the string when it approaches the singularity at $U = 0 = \tau$ from $\tau < 0$. When W_1 is more singular at $U = 0$ than W_2, i.e. $\beta_1 > \beta_2$ in eq.(2.12), the string behaviour is determined by $W_1(U)$ and this is the situation we have analized in ref.[8]. Let us consider here the case when both singularities are of the same type; i.e. $\beta_1 = \beta_2 = \beta$. This case is actually generic since the case when W_2 (W_1) is more singular than W_1 (W_2) can be obtained by setting $\alpha_1 = 0$ ($\alpha_2 = 0$) in the $\beta_1 = \beta_2$ solution. Eq.(2.15) can be approximated near $\tau = 0^-$ as

$$\ddot{X}_n - \frac{(2\alpha' p^U)^{2-\beta}}{|\tau|^\beta} (\alpha_1 X_n + \alpha_2 Y_n) = 0$$
$$\ddot{Y}_n + \frac{(2\alpha' p^U)^{2-\beta}}{|\tau|^\beta} (\alpha_1 Y_n - \alpha_2 X_n) = 0 \qquad (2.19)$$

The behaviour of the solutions $X_n(\tau)$ and $Y_n(\tau)$ for $\tau \to 0$ depend crucially on the value range β. Namely,

i) $\beta > 2$

ii) $\beta = 2$

iii) $\beta < 2$

For simplicity, we start our analysis with the case $\beta = 2$ where we find the solution

$$X_n(\tau) \underset{\tau \to 0^-}{=} C|\tau|^{\lambda} \quad , \quad Y_n(\tau) \underset{\tau \to 0^-}{=} D|\tau|^{\lambda} \tag{2.20}$$

Here, λ fulfils

$$\lambda^2(\lambda-1)^2 - \alpha_1^2 - \alpha_2^2 = 0$$

which has four solutions

$$\lambda_{1,2} = \frac{1 \pm \sqrt{1-4\sqrt{\alpha_1^2+\alpha_2^2}}}{2} \quad , \quad \lambda_{3,4} = \frac{1 \pm \sqrt{1+4\sqrt{\alpha_1^2+\alpha_2^2}}}{2} \tag{2.21}$$

Notice that for any real value λ_1 and λ_2, we have $\lambda_4 < 0$, $\lambda_3 > 0$, $\mathrm{Re}\,\lambda_1 > 0$, $\mathrm{Re}\,\lambda_2 < 0$.

The solution associated with λ_4 diverges at $\tau = 0$

$$\begin{pmatrix} X_n(\tau) \\ Y_n(\tau) \end{pmatrix}_4 \underset{\tau \to 0^-}{\sim} (-\tau)^{\lambda_4} \begin{pmatrix} \sqrt{\alpha_1^2+\alpha_2^2} + \alpha_1 \\ \alpha_2 \end{pmatrix} \tag{2.22}$$

The solutions associated with $\lambda_{1,2}$ and λ_3 vanish for $\tau \to 0^-$:

$$\begin{pmatrix} X_n(\tau) \\ Y_n(\tau) \end{pmatrix}_3 \underset{\tau \to 0^-}{\sim} (-\tau)^{\lambda_3} \begin{pmatrix} \sqrt{\alpha_1^2+\alpha_2^2} + \alpha_1 \\ \alpha_2 \end{pmatrix} \tag{2.23}$$

$$\begin{pmatrix} X_n(\tau) \\ Y_n(\tau) \end{pmatrix}_{1,2} \underset{\tau \to 0^-}{\sim} (-\tau)^{\lambda_{1,2}} \begin{pmatrix} -\sqrt{\alpha_1^2+\alpha_2^2} + \alpha_1 \\ \alpha_3 \end{pmatrix} \tag{2.24}$$

For $\sqrt{\alpha_1^2 + \alpha_2^2} > 1/4$, the solutions $X_{n,(1,2)}$ and $Y_{n,(1,2)}$ approach the singularity oscillating with decreasing amplitudes.

As it is clear, for generic initial conditions, the string behaviour near $\tau \to 0^-$ is dominated by the $(X_n(\tau), Y_n(\tau))_4$ solutions. The fact that $(X_n(\tau), Y_n(\tau))_4$ diverges when $\tau \to 0^-$, means that the string goes to infinity as it approaches the singularity plane. From eq.(2.24), we see that the string goes to infinity in a direction forming an angle α with the X-axis in the X, Y plane. The string escape angle α is given by

$$\mathrm{tg}\,\alpha = \frac{\alpha_2}{\alpha_1 + \sqrt{\alpha_1^2+\alpha_2^2}} \quad , \quad \text{ie} \quad \mathrm{tg}\,2\alpha = \frac{\alpha_2}{\alpha_1} \tag{2.25}$$

(see fig.1).

When $\alpha_2 = 0$, then $\alpha = 0$ and we recover the result of ref.[8] in which the string escapes to infinity in the X-direction. For $\alpha_1 = 0$, then $\mathrm{tg}\,\alpha = \mathrm{sign}(\alpha_2) = \pm 1$ and the string goes to infinity with an angle $(\pi/4)$ sign α_2. If $\alpha_1 > 0$, and arbitrary α_2, the allowed directions are within the cone $|\alpha| < \pi/4$, as depicted in fig.1 If $\alpha_1 < 0$ the string escape angle is within the cone $|\alpha - \pi/2| < \pi/4$. In summary, for arbitrary values of α_1, α_2, the string escape angle can take any value between 0 and 2π.

Let us now consider the case $\beta > 2$. We find

$$\begin{pmatrix} X_n(\tau) \\ Y_n(\tau) \end{pmatrix}_{\tau \to 0^-} = (-\tau)^{\beta/4} \exp\left[K(-\tau)^{1-\beta/2}\right] \begin{pmatrix} A \\ B \end{pmatrix} \tag{2.26}$$

where K can take the four following values:

$$K_{1,2} = \mp \frac{i}{(\beta/2 - 1)} (2\alpha' p^0)^{1-\beta/2} (\alpha_1^2 + \alpha_2^2)^{1/4} \tag{2.27}$$

$$K_{3,4} = \mp \frac{1}{(\beta/2 - 1)} (2\alpha' p^0)^{1-\beta/2} (\alpha_1^2 + \alpha_2^2)^{1/4} \tag{2.28}$$

And A, B are constants such that

$$\left(\frac{A}{B}\right)_{1,2} = \frac{\sqrt{\alpha_1^2 + \alpha_2^2} + \alpha_1}{\alpha_2}, \tag{2.29}$$

for the solution eqs.(2.25), (2.26), and

$$\left(\frac{A}{B}\right)_{3,4} = \frac{\alpha_1 - \sqrt{\alpha_1^2 + \alpha_2^2}}{\alpha_2}, \tag{2.30}$$

for the solution eqs.(2.26), (2.28).

As for the $\beta = 2$ case, we have a divergent solution for $\tau \to 0^-$ associated to K_4 :

$$\begin{pmatrix} X_n(\tau) \\ Y_n(\tau) \end{pmatrix}_4 \simeq (-\tau)^{\beta/4} \exp\left\{2 \frac{(\alpha_1^2 + \alpha_2^2)^{1/4}}{\beta - 2} (2\alpha' p^0)^{1-\beta/2} (-\tau)^{1-\beta/2}\right\} \begin{pmatrix} \alpha_1 + \sqrt{\alpha_1^2 + \alpha_2^2} \\ \alpha_2 \end{pmatrix} \tag{2.31}$$

The other three solutions vanish for $\tau \to 0^-$:

$$\begin{pmatrix} X_n(\tau) \\ Y_n(\tau) \end{pmatrix}_3 \simeq (-\tau)^{\beta/4} \exp\left\{-2 \frac{(\alpha_1^2 + \alpha_2^2)^{1/4}}{\beta - 2} (2\alpha' p^0)^{1-\beta/2} (-\tau)^{1-\beta/2}\right\} \begin{pmatrix} \alpha_1 + \sqrt{\alpha_1^2 + \alpha_2^2} \\ \alpha_2 \end{pmatrix} \tag{2.32}$$

$$\begin{pmatrix} X_n(\tau) \\ Y_n(\tau) \end{pmatrix}_{1,2} \simeq (-\tau)^{\beta/4} \exp\left\{\mp i\, 2 \frac{(\alpha_1^2 + \alpha_2^2)^{1/4}}{\beta - 2} (2\alpha' p^0)^{1-\beta/2} (-\tau)^{1-\beta/2}\right\} \begin{pmatrix} \alpha_1 - \sqrt{\alpha_1^2 + \alpha_2^2} \\ \alpha_2 \end{pmatrix} \tag{2.33}$$

The solutions (1,2) oscillate for $\tau \to 0^-$ with decreasing amplitude.

It can be noticed that the asymptotic behaviours for $\tau \to 0^-$ are not uniform in β. As one can sees from eqs.(2.21)-(2.24) and (2.31)-(2.33), the limits $\tau \to 0^-$ and $\beta \to 2$ do not commute.

As in the $\beta = 2$ case, and for generic initial conditions, the string behaviour for $\tau \to 0^-$ is dominated by the $(X_n(\tau), Y_n(\tau))_4$ solutions. The string goes to infinity in the same way as for the $\beta = 2$ case and with the same escape angle given by eq.(2.25).

Finally, let us discuss now the situation $\beta < 2$. In this case, the solution for $\tau \to 0^-$ behaves as

$$X_n(\tau) = A_1 + A_2\tau + O\left(|\tau|^{2-\beta}\right)$$
$$Y_n(\tau) = B_1 + B_2\tau + O\left(|\tau|^{2-\beta}\right) \qquad (2.34) \qquad \beta \neq 1$$

In the special case $\beta = 1$ one should add a term $0\,(\tau \ln|\tau|)$.
For $\beta < 2$, the string coordinates X, Y are always regular indicating that the string propagates smoothly through the gravitational-wave singularity $U = 0$. (Nevertheless, the velocities X and Y diverge at $\tau = 0$ when $1 \leqslant \beta < 2$).

Let us discuss now more extensively the string behaviour near the singularity $\tau \to 0^-$ for $\beta \geqslant 2$.

For generic initial conditions from eqs.(2.22) and (2.31), we see that the string behaves as

$$\begin{pmatrix} X(\sigma,\tau) \\ Y(\sigma,\tau) \end{pmatrix}_{\beta=2} \underset{\tau\to 0^-}{=} A(\sigma)\left(\sqrt{-\tau}\right)^{1-\sqrt{1+4\bar{\alpha}}} \begin{pmatrix} \alpha_1+\bar{\alpha} \\ \alpha_2 \end{pmatrix}, \qquad (2.35)$$

$$\begin{pmatrix} X(\sigma,\tau) \\ Y(\sigma,\tau) \end{pmatrix}_{\beta>2} \underset{\tau\to 0^-}{=} B(\sigma)(-\tau)^{\beta/4} \exp\left\{ 2\frac{\bar{\alpha}^{1/2}}{\beta-2}(2\alpha' pU)^{1-\beta/2}(-\tau)^{1-\beta/2} \right\} \begin{pmatrix} \alpha_1+\bar{\alpha} \\ \alpha_2 \end{pmatrix} \qquad (2.36)$$

where $\bar{\alpha} = \sqrt{\alpha_1^2 + \alpha_2^2}$, the functions $A(\sigma)$, $B(\sigma)$ depend on the initial conditions. The above solutions imply that the string does not cross the $U = 0$ singularity plane. The string goes off to infinity in the (X,Y) plane, grazing the singularity plane $U = 0$ (therefore never crossing it). Here, the string escapes to infinity with an angle α that depends upon the strenghts of the profile singularities, α_1 and α_2, but not on β. At the same time, the presence of the oscillatory modes (2.24) and (2.33) imply that the string oscillates in the X,Y plane perpendicularly to the escape direction, with an

amplitude vanishing for $\tau \to 0^-$. The non-oscillatory modes (2.23) and (2.32) are negligible since they are in the same direction as the divergent solutions (2.22) and (2.31).

The spatial string coordinates $X^i(\sigma,\tau)$ $[3 \leqslant i \leqslant D-2]$ behave freely [eq.(2.13)].

The longitudinal coordinate $V(\sigma,\tau)$ follows from the constraints eqs.(2.11) and the solutions for $X(\sigma,\tau)$, $Y(\sigma,\tau)$ and $X^j(\sigma,\tau)$ $[3 \leqslant j \leqslant D-2]$. We see that for $\tau \to 0^-$, $V(\sigma,\tau)$ diverges as the square of the singular solutions (2.35)-(2.36).

It can be noticed that the oscillatory modes in the (X,Y) plane, eqs.(2.24) and (2.33), are not harmonic in τ but in

$$\sqrt{\bar{\alpha}}\,\log(-\tau) \qquad \text{for} \quad \beta = 2$$

and

$$\sqrt{\bar{\alpha}}\,\frac{(-\tau)^{1-\beta/2}}{1-\beta/2} \qquad \text{for } \beta > 2 \qquad (2.37)$$

In fact, the space-time metric

$$dS^2 = \frac{(dU)^2}{|U|^\beta}\left[\alpha_1(X^2-Y^2) + 2\alpha_2 XY\right] - dU\,dV + (dX^j)^2 \qquad (2.38)$$

suggests to introduce the null variable

$$d\hat{U} = dU / U^{\beta/2}$$

i.e.

$$\hat{U} = \begin{cases} \ln U & \beta = 2 \\ \dfrac{U^{1-\beta/2}}{1-\beta/2} & \beta > 2 \text{ and } \beta < 2 \end{cases} \qquad (2.39)$$

That is, the string behaviour expresses naturally in terms of the above variable. $\hat{U}$ is like the cosmic time in cosmological space-times.

Moreover, it is interesting to compare the string behavior for $\tau \to 0$ in the inflationary backgrounds for which we have[7]

$$dS^2 = dX^{0\,2} - R^2(X^0)\,(dX^i)^2$$

with $R(\tau) = -(H\tau)^{-\beta/2}$,

$$X^0(\sigma,\tau)\underset{\tau\to 0}{=}\begin{cases} \text{const.} & \beta < 2 \text{ (superinflationary)} \\ -H^{-1}\ln[H\tau L(\sigma)] & \beta = 2 \text{ (de Sitter, inflationary)} \\ -\dfrac{(\tau L(\sigma))^{1-\beta/2}}{1-\beta/2} & \beta > 2 \text{ (power type, inflationary)} \end{cases}$$

$$X^i(\sigma,\tau)\underset{\tau\to 0}{=}\begin{cases} A^i(\sigma) + D^i(\sigma)\dfrac{\tau^2}{2} + \tau^{1+\beta}F^i(\sigma) & \beta \neq 2 \\ A^i(\sigma) + \tilde{D}^i(\sigma)\dfrac{\tau^2}{2} + \dfrac{\tau^2}{2}\ln\tau\,\tilde{F}^i(\sigma) & \beta = 2 \end{cases}$$

The $A^i(\sigma)$ are arbitrary functions of σ , $D^i(\sigma)$, $D^i(\sigma)$, $F^i(\sigma)$, and $F^i(\sigma)$ are fixed by the constraints. Here X^0 is the cosmic time, while $\eta \approx \tau$ is the conformal time. For the singular plane waves we have

$$\hat{U}(\sigma,\tau)\underset{\tau\to 0}{=}\begin{cases} \text{const.} & \beta < 2 \\ \ln(-U) & \beta = 2 \\ \dfrac{(-U)^{1-\beta/2}}{1-\beta/2} & \beta > 2 \end{cases}$$

We see that the behavior of the string time coordinate is in both cases

similar and completely determined by the spacetime geometry. (That is, it is independent of the string initial state).

In the inflationary cases, all transverse spatial coordinates are **non-oscillating** and **regular** for $\tau \to 0$ ("frozen"), while for singular plane waves, in addition, one of the transverse coordinates is non-oscillatory and singular for $\tau \to 0$.

The above behaviors for X^0 and U are characteristic of strings in strong gravitational fields. Notice that the inflationary backgrounds are non-singular, whereas the plane waves [eqs.(1.1)-(1.2)] are singular spacetimes. This is connected to the fact that the string coordinates X, Y exhibit divergences in these plane waves spacetimes.

Let us consider the spatial length element of the string, i.e. the lenght at fixed $U = 2\alpha' p^u \tau$, between (σ,τ) and $(\sigma+d\sigma,\tau)$,

$$dS^2 = dX^2 + dY^2 + \sum_{j=3}^{D-2}(dX^j)^2 \qquad (2.40)$$

For $\tau \to 0^-$ eqs.(2.35) and (2.36) yield

$$dS^2_{\tau\to 0^-} = \left[(\alpha_1+\bar{\alpha})^2 + \alpha_2^2\right] B'(\sigma)^2 d\sigma^2 \begin{cases} (-\tau)^{1-\sqrt{1+4\bar{\alpha}}} & \beta = 2 \\ (-\tau)^{\beta/2}\exp\left[\dfrac{2\sqrt{\bar{\alpha}}}{\beta/2-1}(-2\alpha' p^u \tau)^{1-\beta/2}\right] & \beta > 2 \end{cases} \qquad (2.41)$$

That is, the proper length between (σ_0,τ) and (σ_1,τ) is given by

$$\Delta S = \left[B(\sigma_1) - B(\sigma_0)\right]\sqrt{(\alpha_1+\bar{\alpha})^2+\alpha_2^2}\begin{cases} \sqrt{-\tau}^{\,1-\sqrt{1+4\bar{\alpha}}} & \beta = 2 \\ (-\tau)^{\beta/4}\exp\left[\dfrac{\sqrt{\bar{\alpha}}}{\beta/2-1}(-2\alpha' p^u \tau)^{1-\beta/2}\right] & \beta > 2 \end{cases} \qquad (2.42)$$

We see that $\Delta s \to \infty$ for $\tau \to 0^-$, that is, the string stretches infinitely when it approaches the singularity plane.

Again, this stretching of the string proper size also occurs for $\tau \to 0$ in the inflationary cosmological backgrounds[7].

Another consequence of eqs.(2.35) and (2.36) is that the string reaches infinity in a finite time τ. In particular, for σ-independent coefficients, eqs.(2.35) and (2.36) describe geodesic trajectories. The fact that for $\beta \geqslant 2$, a point particle (as well as a string) goes off to infinity in a finite τ indicates that the space-time is singular.

What we have described is the $\tau \to 0^-$ behaviour for generic initial data. In particular, there is a class of solutions where $A(\sigma) \equiv 0$ and $B(\sigma) \equiv 0$, whereas the coefficients of the regular modes are arbitrary. In this class of solutions, X and Y vanish for $\tau \to 0^-$. However, when continued to $\tau > 0$, these solutions are complex. The real valued physical solution is $X = Y = 0$ for all $\tau \geqslant 0$, it means that the string gets trapped in the singularity plane $U = 0$ at the point $X = Y = 0$, where the gravitational forces are zero.

III-QUANTUM BEHAVIOUR. PARTICLE TRANSMUTATIONS, MASS AND NUMBER OPERATORS.

When $\beta < 2$, the string crosses the gravitational wave and reaches the $U > T$ flat space-time region. In this section we study the physical properties of the string after crossing the gravitational wave, namely the string mass and mode number operators and the relation between ingoing and outgoing mode-operators.

We recall that when $\beta \geqslant 2$, the string does not reach the $U > 0$ region. There is no mass operator $M_>^2$ or string number operator $N_>$ in the outgoing region $U > T$ since there is no string there. The outgoing operators $M_>^2$ and $N_>$ make sense only for $\beta < 2$.

In order to relate ingoing and outgoing solutions, we start from eqs.(2.16), and (2.18), which are equivalent to the integral equation

$$\begin{pmatrix} X_n(\tau) \\ Y_n(\tau) \end{pmatrix} = \begin{pmatrix} X_{n<}(\tau) \\ Y_{n<}(\tau) \end{pmatrix} + \int_{-\tau_0}^{\tau_0} G(\tau-\tau')\, W(\tau') \begin{pmatrix} X_n(\tau') \\ Y_n(\tau') \end{pmatrix} d\tau' \tag{3.1}$$

here $X_{n<}(\tau)$ and $Y_{n<}(\tau)$ are given by eq.(2.18), and $G(\tau)$ obeys the equation

$$\left[\frac{d^2}{d\tau^2} + 4m^2\right] G(\tau) = \mathbb{1}\,\delta(\tau), \qquad \text{ie} \quad G(\tau) = \mathbb{1}\,\Theta(\tau)\,\frac{\sin 2m\tau}{2m} \tag{3.2}$$

where $\Theta(\tau)$ is the step function.

It is useful to introduce the Jost solutions $f_n^1(\tau)$ and $f_n^2(\tau)$. of eq.(2.16), which satisfy the initial condition

$$\begin{pmatrix} f_n^1(\tau) \\ f_n^2(\tau) \end{pmatrix} = e^{-2im\tau} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \text{for} \quad \tau < -\tau_0 \tag{3.3}$$

Then, the physical solutions X_n , Y_n are expressed in terms of the Jost solutions as

$$X_n(\tau) = \frac{i}{2n}\sqrt{2\alpha'}\left(\alpha_n^x f_n^1(\tau) - \tilde{\alpha}_{-n}^x f_n^{1*}(\tau)\right)$$

$$Y_n(\tau) = \frac{i}{2n}\sqrt{2\alpha'}\left(\alpha_n^y f_n^2(\tau) - \tilde{\alpha}_{-n}^y f_n^{2*}(\tau)\right) \tag{3.4}$$

where we used the conjugation property

$$f_n^{*i} = f_{-n}^i \quad , \quad i = 1,2 \tag{3.5}$$

For $\tau > \tau_0$, the outgoing solution ($X_{n<}(\tau)$, $Y_{n<}(\tau)$) takes the form

$$\begin{pmatrix} X_{n>}(\tau) \\ Y_{n>}(\tau) \end{pmatrix} = \begin{pmatrix} X_{n<}(\tau) \\ Y_{n<}(\tau) \end{pmatrix} + \int_{-\tau_0}^{\tau_0} \frac{\sin[2n(\tau-\tau')]}{2n} W(\tau') \begin{pmatrix} X_{n<}(\tau') \\ Y_{n<}(\tau') \end{pmatrix} d\tau' \tag{3.6}$$

Inserting eq.(2.18) and the similar expression for $X_{n>}$, $Y_{n>}$, eq.(3.6) yields the relation between the ingoing and outgoing operators

$$\alpha_{n>}^x = (1 + A_n^{11})\alpha_{n<}^x + B_n^{11}\tilde{\alpha}_{-n<}^x + A_n^{22}\alpha_{n<}^y + B_n^{22}\tilde{\alpha}_{-n}^y$$

$$\tilde{\alpha}_{n>}^x = (1 + A_n^{11})\tilde{\alpha}_{n<}^x + B_n^{11}\alpha_{-n<}^x + A_n^{22}\tilde{\alpha}_{n<}^y + B_n^{22}\alpha_{-n<}^y \tag{3.7a}$$

and

$$\alpha_n^y = (1 - A_n^{12})\alpha_{n<}^y - B_n^{12}\tilde{\alpha}_{-n<}^y + A_n^{21}\alpha_{n<}^x + B_n^{21}\tilde{\alpha}_{-n<}^x$$

$$\tilde{\alpha}_n^y = (1 - A_n^{12})\tilde{\alpha}_n^y - B_n^{12}\alpha_{-n<}^y + A_n^{21}\tilde{\alpha}_{n<}^x + B_n^{21}\alpha_{-n<}^x \tag{3.7b}$$

where

$$A_n^{ij} = -\frac{(\alpha' p^0)^2}{in}\int_{-\tau_0}^{\tau_0} e^{2in\tau'} W_i(\tau') f_n^j(\tau')\, d\tau'$$

$$B_n^{ij} = \frac{(\alpha' p^0)^2}{in}\int_{-\tau_0}^{\tau_0} e^{2in\tau'} W_i(\tau') f_n^{j*}(\tau')\, d\tau' \tag{3.8}$$

Eqs.(3.7) are Bogoliubov transformations on the oscillator mode operators. There are two types of effects here : (i) a mixing of creation and annhilation mode operators of oposite chirality and (ii) a polarization rotation in the (X,Y) plane. The effect (i) implies transmutations between different particle states[2,6]. The effect (ii) means that the spin polarization of the final particle states are different from the initial ones.

Let us consider the string mass and number operators in the outgoing region

$$M_>^2 = \frac{2}{\alpha'}\sum_{n=1}^{\infty}\sum_{i=1}^{D-2}\left(\alpha_{n>}^{i\dagger}\alpha_{n>}^i + \tilde{\alpha}_{n>}^{i\dagger}\tilde{\alpha}_{n>}^i\right) + m_0^2 \tag{3.9}$$

$$N_> = \sum_{n=1}^{\infty} \sum_{i=1}^{D-2} \frac{1}{n} \left(\alpha_{n>}^{i+} \alpha_{n>}^{i} + \tilde{\alpha}_{n>}^{i+} \tilde{\alpha}_{n>}^{i} \right) \tag{3.10}$$

where m_0^2 is the tachyon mass. $\left(m_0^2 = -(D-2)/6\alpha' \right)$.

The expectation values in the ingoing ground state $|0_<\rangle$, takes the form

$$\frac{\langle 0_< | M_>^2 | 0_< \rangle}{\langle 0_< | 0_< \rangle} = \frac{4}{\alpha'} \sum_{n=1}^{\infty} n \left(|B_n^{11}|^2 + |B_n^{12}|^2 + |B_n^{22}|^2 + |B_n^{21}|^2 \right) + m_0^2 . \tag{3.11}$$

$$\frac{\langle 0_< | N_> | 0_< \rangle}{\langle 0_< | 0_< \rangle} = 2 \sum_{n=1}^{\infty} \left(|B_n^{11}|^2 + |B_n^{12}|^2 + |B_n^{21}|^2 + |B_n^{22}|^2 \right) \tag{3.12}$$

where we used eqs.(3.7), (3.9) and (3.10).

Before the collision with the gravitational wave, we have as expectation values

$$\frac{\langle 0_< | M_<^2 | 0_< \rangle}{\langle 0_< | 0_< \rangle} = + m_0^2 \qquad \frac{\langle 0_< | N_< | 0_< \rangle}{\langle 0_< | 0_< \rangle} = 0 \tag{3.13}$$

The infinite sums over modes in eqs.(3.11) and (3.12) are due to the excitation of the string modes after collisions with the gravitational wave.

In order to investigate the convergence of these series, we compute now the large n behaviour of the B_n^{ij} coefficients. This can be done by using the Born approximation, that is

$$\begin{pmatrix} f_n^1(\tau) \\ f_n^2(\tau) \end{pmatrix} = e^{-2in\tau} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \forall \tau \tag{3.14}$$

From eqs.(3.8) and (3.14) we obtain

$$B_n^{11} \underset{n\to\infty}{=} B_n^{12} = -2i \left(\frac{\alpha' p^U}{2n} \right)^{2-\beta} \alpha_1 \, \Gamma(1-\beta) \sin\left(\frac{\pi\beta}{2}\right)$$

$$B_n^{22} \underset{n\to\infty}{=} B_n^{21} = -2i \left(\frac{\alpha' p^U}{2n} \right)^{2-\beta} \alpha_2 \, \Gamma(1-\beta) \sin\left(\frac{\pi\beta}{2}\right) \tag{3.15}$$

Therefore,

$$\sum_{i,j=1}^{2} n \, |B_n^{ij}|^2 \simeq n^{2\beta-3} \quad \text{for } n \to \infty$$

Hence

$$\langle M_>^2 \rangle = \begin{cases} \text{finite} & \text{for } \beta < 1 \\ \text{divergent} & \text{for } 1 \leq \beta < 2 \end{cases} \tag{3.16}$$

and

$$\langle N_> \rangle = \begin{cases} \text{finite} & \text{for } \beta < 3/2 \\ \text{divergent} & \text{for } \frac{3}{2} \leq \beta < 2 . \end{cases}$$

For $\beta \geqslant 2$, there are no >-operators. As discussed in ref.[8], when the values of $\langle M_>^2 \rangle$ or $\langle N_> \rangle$ are divergent, such infinities are not related at all to the space-time singularity at U = 0. Such divergences are connected to the infinite transverse size (i.e. perpendicular to the propagation direction) of the gravitational wave front.

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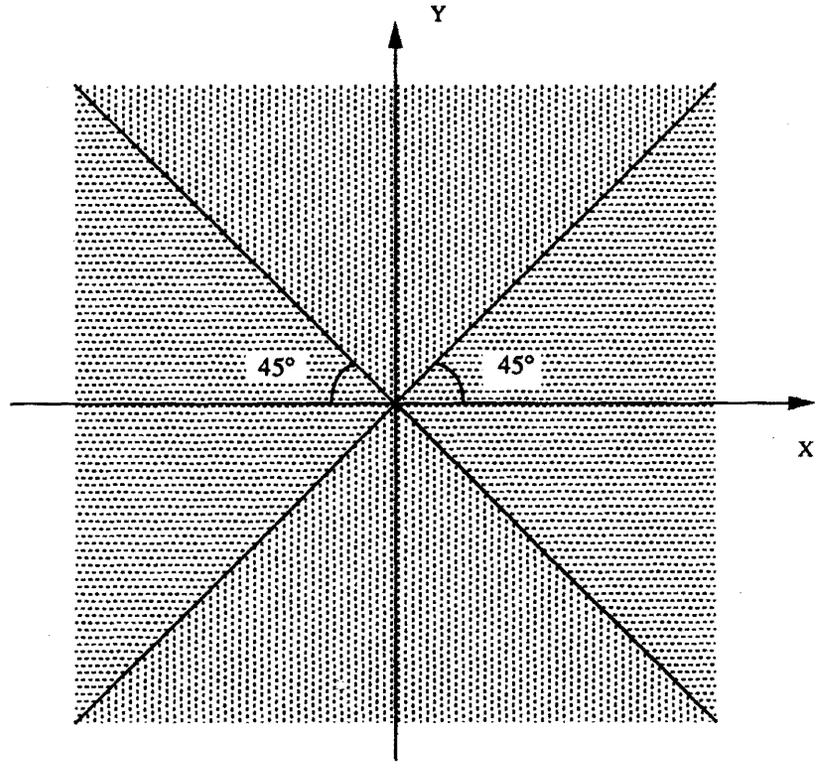

Figure 1

Escape string directions

$\alpha_1 > 0$, $\alpha_1 < 0$