Probing the Fixed Point Structure of the Quantum Hall System

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Abstract

Recent scaling experiments have thrown the universality hypothesis for the quantum Hall effect into some confusion. While they appear to confirm a novel form of "super-universality" of the delocalization exponent relating transitions between different Landau levels, contrary to expectation, the experiments have not found universal values for the dissipative conductivities at the delocalization fixed points. We discuss this behaviour in the context of two phase and renormalization group flow diagrams that have been suggested to describe the quantum Hall system. We show that the experimental results are well described by the renormalization group structure predicted if the quantum Hall system is invariant under an infinite discrete parameter space symmetry.

It is the purpose of this letter to argue that recent scaling experiments on the quantum Hall system performed by Koch et al.[1] lend additional support to the conjectured parameter space geometry recently proposed in Ref.[2]. We first summarize the current status of scaling experiments on the quantum Hall system and discuss these results in the context of two renormalization group (RG) flow diagrams that have been proposed to describe the system.

Two physical length scales may be identified in the experimental configuration used in Ref.[1]. These are the inelastic scattering length L and the characteristic size W of the sample (for simplicity the Hall-bar geometry is frozen and we consider only the variation of an overall scale W). The experimental observations in the metallic regime in large samples have a straight-forward interpretation [3] if the resistivities depend on a single variable $|\kappa| = (L/\xi)^{1/\nu}$, where ξ is the localization length. The condition for metallic behaviour is that the mean free path between inelastic collisions should not exceed the localization length of the levels near the Fermi energy. In an infinite sample, one expects the scaling behaviour of the halfwidth $\Delta\rho \equiv \rho_{xy} - \rho^*_{xy}$ of the dissipative resistivity, which recovers between the plateaus, to be given by $\Delta\rho \sim \Delta B \sim \xi^{1/\nu} \sim \kappa \sim L^{-1/\nu}$. When the sample is sufficiently cold L is believed to scale with T ("Thouless scaling" [4]), conventionally expressed as $L \sim T^{-p/2}$ $(p > 0)$, so that we alternatively have: $\Delta\rho \sim T^{\mu}$, with $\mu = p/2\nu$. For finite size samples at sufficiently low temperature the inelastic scattering length will be larger than the sample width W. In this case only electron states with localization length larger than W contribute to a finite resistivity and $\Delta\rho \sim W^{-1/\nu}$. The central theoretical and experimental issue is which exponents, if any, are universal in the usual sense, i.e. sample independent, and if there are separate exponents controlling the transition between different pairs of phases (plateaus) of the Hall system [3].

In AlGaAs-GaAs heterojunctions the *crossover* between the two scaling behaviours happens at about $T_x \simeq 10^2$mK [1]. At temperatures above T_x the scaling at delocalization is driven by L, below T_x it is driven by W. Koch et al.[1] report that at $T = 25$mK and W varying between 10μm and 64μm their AlGaAs-GaAs samples appear to be solidly into the domain of W-scaling, with ΔB independent of T. By exploiting this they were able to extract the values of the scaling

exponents defined above. Their results support the scaling hypotheses described above, but both μ and p vary considerably (by up to a factor of two) between high and low mobility samples. Remarkably, the ratio $p/\mu = 2\nu$ remains unchanged, and the delocalization exponent ν appears to be $\nu = 2.3 \pm 0.1$ for *any* delocalization transition in *any* sample[1]. This is consistent with numerical simulation results [5] giving $\nu = 2.34 \pm 0.04$ for the lowest Landau level, as well as a semi-classical argument [6] showing that the effect of tunnelling on the percolation picture of the quantum Hall effect [7] is to shift the value of the classical exponent by 1, giving $\nu = 7/3$.

Koch et al.[1] also report the maximum diagonal conductivity $\sigma_{xx}^{\max}(B_c)$ measured at critical magnetic field B_c, which in the scaling limit should contain information about the location of the delocalization fixed points in the parameter space spanned by $(\sigma_{xy}, \sigma_{xx})$. With all other parameters fixed the observed value of $\sigma_{xx}^{\max}(B_c)$ varied by less than 20% for the Hall-bar widths W considered and, given the universality observed in the delocalization exponent, it is reasonable to assume that the value of $\sigma_{xx}^{\max}(B_c)$ in the largest sample ($W = 64\mu$m) approximates the scaling limit, i.e. the value at the delocalization fixed point. However, they did find that $\sigma_{xx}^{\max}(B_c)$ varied considerably for transitions between different Landau levels and from sample to sample.

If this is interpreted as indicating that the *location* of the delocalization fixed points are not universal, then this would seem to contradict the universality hypothesis which the measurement of the exponent ν seems to verify. It is the purpose of this letter to suggest another interpretation of these data, and argue that they provide valuable information about the scaling diagram going beyond the observed universality of critical exponents.

To focus the comparison of the theoretical expectations with the experimental measurements we consider two RG scaling diagrams that have been suggested to describe the properties of the quantum Hall system. The diagram shown in Fig. 1a was proposed [8] on purely phenomenological grounds in order to explain

[1] An apparent discrepancy with the earliest experiments in MOSFETs [3] can be attributed to an erroneous assumption about the value of p. The recent experiments by Koch et al.[1] show that p is non-universal and sensitive to the magnetic field, so that the result for metals cannot be taken over to the Hall system.

how integer and fractional phases may be accessed by varying the initial conditions (corresponding to different initial choices of σ_{xx}). In proposing it there was no suggestion that the critical exponents associated with different fixed points involving integer and fractional levels or fractional levels and fractional levels should be the same, although the expectation was that transitions shifted by integers would be described by the same diagram. Thus the observation that the critical exponents are the same requires further explanation.

In Ref.[2] we suggested that a unified description of the integer and fractional phases of the quantum Hall system is possible if the scaling diagram of transport coefficients (conductivities) is invariant under an infinite discrete group of transformations. We proposed that this could be achieved by making the partition function invariant under a modular symmetry ($\mathrm{SL}(2,\mathbb{Z})$) acting on the parameter space $(\sigma_{xy}, \sigma_{xx})$ of the effective quantum field theory, in such a way that it maps all critical points into each other. A symmetry of this type will organize the parameter space into equivalence classes which are the different phases of the system. The modular group leads to an infinitely nested "tree" of fractional phases, with the remarkable property that only fractional values of σ_{xy} are available in the strong coupling limit ($\sigma_{xx} \to 0$), thus accounting both for the observed quantization of Hall plateaus, as well as the hierarchy of states believed to be present in this system. Furthermore, this symmetry also forces the scaling equations to have the same form close to all fixed points, thus explaining why the delocalization exponent is "super-universal". The RG flow diagram consistent with this symmetry is shown in Fig. 1b. The most significant difference between this and Fig. 1a is the appearance of even-denominator phases. As discussed in Ref.[2], at high magnetic field strengths the low-lying Landau levels will not be able to populate these regions meaning that the system will not have flowed to the self-dual plane in which the phase diagram is that given in Fig. 1b. However the critical points associated with the transitions between odd and even denominator phases will still influence the behaviour of the system in their vicinity.

Prior to the publication of Ref.[1] all scaling data on the position of delocalization fixed points in the integer case [9], and mobility fixed points in the fractional case [10], agreed with the RG flow shown in Fig. 1b within experimental accu-

racy [2]. In both these experiments the location of the fixed points were extracted from temperature driven scaling experiments which produce a rather immediate picture of the RG flow. The recent experiments by Koch et al.[1] are not quite so straight-forward to translate into the RG language, since, as mentioned above, they appear to show that the location of the RG fixed points, and consequently the geometry of the RG flow, is not universal in this system. This may however be an illusion – an artefact of the way these experiments are done, as we now explain.

Assuming that Koch et al.[1] have accessed some of the delocalization fixed points, in the absence of the full flow diagrams it is not clear precisely *which* fixed points have been accessed. It is quite possible that the different values of $\sigma_{xx}^{\max}(B_c)$ observed in the different experiments correspond to different delocalization fixed points. This possibility arises in hierarchically organized scaling diagrams with tree-like phase diagrams, like the ones shown in Fig. 1 where the abundant supply of critical points may easily lead to mis-identifications. In Fig. 1a, for example, the behaviour of transitions between levels 0 and 1 may be governed by the upper delocalization fixed point *or* by the lower delocalization fixed point(s) (through unresolved fractional levels). Since the topology and geometry of Fig. 1a can be chosen at will, because it is not constrained by any symmetry, this diagram has little predictive power. However, we can bound the positions of the delocalization fixed points since the position of the mobility fixed point controlling the appearance of the 1/3 phase is fixed by experiment [10] to lie at $\sigma_{xx}^{\mathrm{MFP}} = 0.15 \pm 0.03$. Thus the upper (lower) delocalization fixed point must lie above (below) 0.15.

The values of $\sigma_{xx}^{\max}(B_c)$ observed in Ref.[1], where only transitions between integer phases (plateaus) were considered, were approximately 0.1, 0.2 and 0.4 in units of e^2/h (we consider only the fully spin-polarized states here). No error is quoted in Ref.[1], but since $\sigma_{xx}^{\max}(B_c)$ is said to vary by less than 20% between Hall-bars, we may presumably take the error to be less than this. Previous flow experiments [9, 10], located the integer delocalization fixed point at $\sigma_{xx}^{\max} = 0.5$. This is consistent with the latter of the values found by Koch et al.[1], and is in precise agreement with the prediction ($\sigma_{xx} = 1/2$) of the self-dual scaling diagram [2], as well as the σ-model of localization in magnetic fields [11]. In

the context of Fig. 1a this value should obviously be identified with the upper delocalization fixed point. However, given the bounds derived on the positions of the delocalization fixed points in the previous paragraph, there is now no possibility of identifying the lower fixed point with the second observed value $\sigma_{xx}^{\max} = 0.2$, so the RG flow diagram of Fig. 1a and experiment are in conflict.

This conflict may disappear if the RG flow is instead given by Fig. 1b. In this case the *prediction* of the position of the mobility fixed points is $\sqrt{3}/14 = 0.124$ in excellent agreement with the experimental value quoted above. However due to the additional even-denominator phases there are new delocalization fixed points which lie *above* this mobility fixed point. These fixed points, called $\sigma_{1,2}$, $\bar{\sigma}_{2,1}$ etc. in Ref.[2], all have $\sigma_{xx} = 1/5$. The system in the odd-denominator phases may approach these fixed points arbitrarily closely and the RG flow will be strongly influenced by them. This would explain why Koch et al.[1] observe $\sigma_{xx}^{\max}(B_c)$ values around 0.2 in some of their experiments. Finally there is a class of delocalization fixed points in the self-dual diagram, with $\sigma_{xx} = 1/10$, which controls transitions between integer and $\mathbb{Z}/3$ fractional phases. These may have been mistaken for integer transitions if the fractional plateaus were not well developed. In short it is possible that all scaling data to date are consistent with Fig. 1b and with the symmetry conjectured in Ref.[2] to be responsible for the phase and RG flow diagram of the quantum Hall system.

To summarize, we have considered the implications the recent measurements of the quantum Hall system have for the RG flow properties of the system. At first sight they seem to be in conflict with the expectation, motivated by the observation of a universal delocalization exponent, that the system is close to a universal critical point. We have argued that the experimental measurements may be probing several distinct fixed points. Remarkably, this possibility is in quantitative agreement with the RG flow diagram obtained from a modular invariant partition function previously suggested as an explanation of the observed universality of critical exponents. In this approach the modular invariance relates the various fixed points and explains why the critical exponents at each should be the same.

It is clear that the experiments probing the mobility and delocalization fixed

point structure are very important in elucidating the physics of the quantum Hall system. At present the results of these experiments, while suggestive, cannot unambiguously determine the RG flow structure, but it may be hoped that more detailed scaling experiments will be able to fix this structure. Although most of the theoretical work on the quantum Hall system has concentrated on the region of the plateaus, we believe that a full understanding requires a complete description, both of the plateau region and of the transitions between plateaus, and that the recently observed universality of critical exponents strongly points to a global symmetry relating the critical points.

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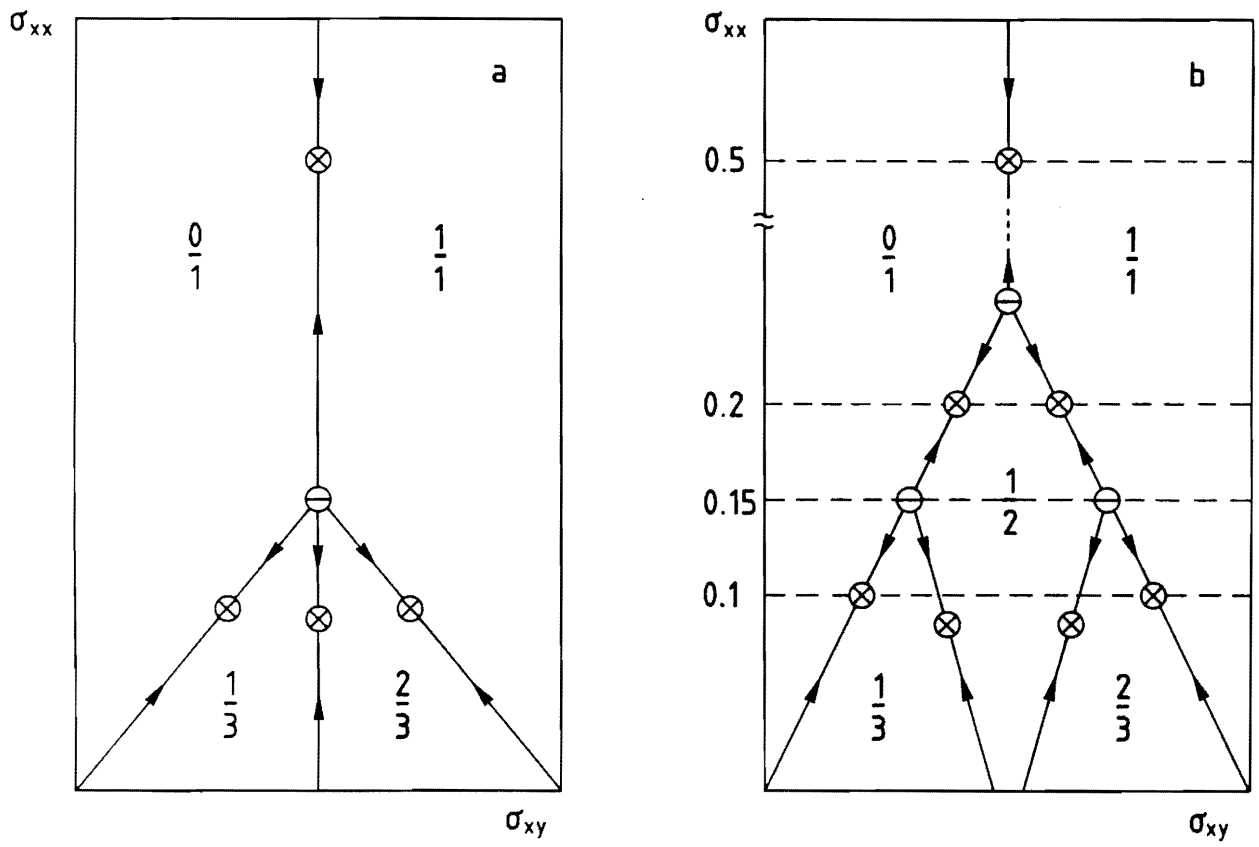
FIG. 1. Schematic comparison of (a) the scaling diagram of conductivites $(\sigma_{xy}, \sigma_{xx})$ proposed in Ref.[8] and (b) the modular invariant scaling diagram discussed in Ref.[2]. Delocalization fixed points, controlling the transition between Hall plateaus, are denoted by $\otimes$, and so-called "mobility fixed points", where new phases appear, are denoted by $\ominus$.