A PARADOX IN THE ASYMPTOTIC SCALING OF DYNAMICALLY GENERATED FERMION MASSES

R. D. BALL & J. TIGG

Theoretical Physics
1 Keble Road
Oxford
OX1 3NP, U.K.

ABSTRACT

Paradoxically, the asymptotic behaviour of dynamically generated fermion masses in asymptotically free vectorlike gauge theories determined using the Schwinger–Dyson equation differs by a logarithm from that found with the operator product expansion. We show that this inconsistency can be resolved only if the scaling of the fermion condensate is asymptotically nonperturbative.

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Consider a nonabelian asymptotically free gauge theory, with gauge group G, and n_f fermions each in a vectorlike representation r_f of G, of dimension n_c. QCD is the best known example; $G = SU(3)$, $r_f = \mathbf{3}$, $n_c = 3$. Then the Schwinger–Dyson equation for the fermion self energy $\Sigma(p)$ takes the form (in Euclidean space)

$$\Sigma(p) = m_0 + g^2 \int \frac{d^4q}{(2\pi)^4} D^{ab}_{\mu\nu}(p-q)\,\gamma_\mu \lambda^a S(q) \Gamma^b_\nu(p,q), \tag{1}$$

where $D^{ab}_{\mu\nu}(p-q)$ is the full gauge field propagator, m_0 the bare fermion mass, $S(q) \equiv (i\not{q} + \Sigma(q))^{-1}$ the full fermion propagator, and $\Gamma^b_\nu(p,q)$ the gauge-fermion vertex function.

Now since the theory is asymptotically free, the form of the fermion self energy may be precisely determined in the deep Euclidean limit. In fact the Schwinger–Dyson equation (1) admits solutions with two distinct types of asymptotic behaviour[1,2]:

(i) 'Irregular' solutions;

$$\Sigma_+(p^2) \underset{p^2\to\infty}{\sim} m_R(\ln p^2/M^2)^{-d}, \tag{2}$$

where the renormalisation group invariants m_R and M are interpreted as the renormalised 'current' fermion mass, and the typical mass scale of the gauge field interactions, respectively. The number d is twice the asymptotic ratio of the anomalous dimension $\frac{1}{2}\gamma_m(g^2) \equiv -\partial \ln m_0(\kappa)/\partial \ln \kappa$ and the beta function $\beta(g^2) \equiv \partial g^2(\kappa)/\partial \ln \kappa$ (where $\kappa \equiv \mu^2/M^2$, μ^2 being the renormalisation scale); perturbatively, as $\kappa \to \infty$,

$$\begin{aligned} \beta(g^2) &= -\beta^0 g^2 + \cdots, & \beta^0 &= \tfrac{1}{48\pi^2}\big(11C_2(G) - 4n_f T(r_f)\big), \\ \tfrac{1}{2}\gamma_m(g^2) &= \gamma^0 g^2 + \cdots, & \gamma^0 &= \tfrac{3}{16\pi^2} C_2(r_f), \end{aligned} \qquad d \equiv \frac{\gamma^0}{\beta^0}. \tag{3}$$

Typically d is a small number; for QCD, with $2 \le n_f \le 6$ flavors, $\frac{12}{29} \le d \le \frac{4}{7}$. We assume (with little loss of generality) that $d < 1$ in what follows.

The irregular solution (2) is perturbative, in the sense that it sums the leading log corrections to the bare mass m_0. Since it is hard (it only falls off slowly with p^2), the loop integral in eq.(1) requires renormalisation (by a single subtraction), giving the running mass $m_0(\kappa)$. Clearly the irregular solution only arises when $m_0(\kappa)$ is nonzero; the $U(n_f) \times U(n_f)$ chiral symmetry is then broken explicitly, and the chiral limit is precisely the limit $m_R \to 0$.

(ii) 'Regular' solutions;

$$\Sigma_-(p^2) \underset{p^2\to\infty}{\sim} \frac{4m_D^3}{p^2}(\ln p^2/M^2)^{\delta}, \tag{4}$$

where the renormalisation group invariant m_D is interpreted as a dynamically generated 'constituent' fermion mass; $\Sigma_-(4m_D^2) \simeq m_D$. It is expected to be the same order of magnitude as M, and we shall show in §2 that it is closely related to the fermion condensate. δ will be discussed shortly.

The regular solution (4) is nonperturbative, in the sense that it has nothing to do with summations of leading logs; it depends for its existence on appropriate

(but as yet not well understood) infrared properties of $D^{ab}_{\mu\nu}$ and Γ^b_ν. It is soft (falling off rapidly with p^2), so that the loop integral in eq.(1) requires no renormalisation; if (4) exists as a solution of (1) in the chiral limit, then the chiral symmetry will be dynamically broken to its vector subgroup. The amputated Bethe–Saltpeter amplitude of the n_f^2 (modulo $U(1)$ problems) pseudoscalar Goldstone bosons is then given by $\varphi(p^2) = \Sigma_-(p^2)/f_p$. The pseudoscalar decay constant f_p is determined by the Mandelstam normalisation condition; because φ is soft, f_p is finite. Away from the chiral limit, the Goldstone bosons will acquire small masses m_p according to the Gell-Mann–Oakes–Renner relation $m_p^2 f_p^2 = -2m_R v_R$, where v_R is a renormalisation group invariant (proportional to m_D^3). The behaviour (4) will then hold (approximately) only in the region $M^2 \ll p^2 \ll m_D^3/m_R$, the irregular behaviour (2) taking over for $p^2 \gg m_D^3/m_R$. When $m_R \sim m_D^3/M^2$, the regular behaviour disappears entirely, and the chiral symmetry is no longer broken dynamically.

The regular asymptotic behaviour (4) summarises all we really know analytically (and thus with any certainty) about dynamical fermion mass generation in asymptotically free gauge theories. There exist in the literature two essentially different arguments for it, which we shall refer to in what follows as "Lane's argument"[1] and (with less justification) "Politzer's argument"[2]. The former uses a rainbow approximation to the Schwinger–Dyson equation (1), while the latter relies on the operator product expansion. Curiously these two independent arguments result in mutually inconsistent forms for the regular behaviour (4); Lane finds $\delta = d$, while Politzer claims that $\delta = d - 1$. It is this paradox which we address in this talk.

1. Asymptotics from Schwinger–Dyson equations

1.1. Lane's argument[1]

Since in an asymptotically free theory $g^2(\kappa)$ is small at large renormalisation scales $\kappa \equiv \mu^2/M^2$, it would seem sufficient for the determination of the asymptotic behaviour of $\Sigma(p, g(\kappa))$ to use tree level expressions for the propagator $D^{ab}_{\mu\nu}$ and proper vertex Γ^b_ν in the Schwinger–Dyson equation (1). Employing Landau gauge (since it then turns out that then the fermion field is unrenormalised), the exact eqn.(1) becomes

$$\Sigma(p, g(\kappa)) = m_0 + 3C_2(r_f)g^2(\kappa)\int \frac{d^4q}{(2\pi)^4}\frac{1}{(p-q)^2}\frac{\Sigma(q, g(\kappa))}{q^2 + \Sigma(q, g(\kappa))^2}. \tag{5}$$

This approximation is known colloquially as the 'rainbow' approximation — it corresponds to the 'ladder' approximation to the Bethe–Saltpeter equation. Eqn.(5) is soon turned into a differential equation,

$$\left(p^4\left(\frac{d}{dp^2}\right)^2 + 2p^2\frac{d}{dp^2}\right)\Sigma(p, g(\kappa)) + \frac{3C_2(r_f)g^2(\kappa)}{16\pi^2}\frac{\Sigma(p, g(\kappa))}{(1 + \Sigma(p, g(\kappa))^2/p^2)} = 0, \tag{6}$$

which, when $p^2 \gg \Sigma^2$, has two solutions $\Sigma_\pm$ with irregular and regular asymptotic form respectively;

$$\Sigma_\pm(p, g(\kappa)) \sim a_\pm(\kappa)p^{-1\pm\nu(\kappa)}, \qquad \text{where} \qquad \nu(\kappa) \equiv \sqrt{1 - \tfrac{3C_2(r_f)}{4\pi^2}g^2(\kappa)}. \tag{7}$$

Taking κ to infinity at fixed p^2, then as $g^2(\kappa) \sim (\beta^0 \ln\kappa)^{-1}$,

$$\Sigma_+(p, g(\kappa)) \underset{\kappa\to\infty}{\sim} a_+\left(\frac{\ln\kappa}{\ln\kappa p}\right)^d, \qquad \Sigma_-(p, g(\kappa)) \underset{\kappa\to\infty}{\sim} \frac{a_-(\kappa)}{p^2}\left(\frac{\ln\kappa p}{\ln\kappa}\right)^d, \tag{8}$$

with d given by (3). The renormalisation group gives $\Sigma(\kappa p, g_R) = (\ln\kappa)^{-d}\Sigma(p, g(\kappa))$, so the results (8) reduce to the asymptotic forms (2) and (4) with $\delta = d$ (and $a_+ = m_R$ independent of κ, while $a_-(\kappa) = 4m_D^3(\ln\kappa^2)^{2d}/\kappa^2$). The former confirms the leading log scaling behaviour of the bare mass $m_0(\kappa) \sim m_R(\ln\kappa)^{-d}$, while the latter suggests that m_D^3 is the renormalised value of a divergent nonperturbative quantity which scales (perturbatively) as the operator $\bar\psi\psi$ (i.e. as $(\ln\kappa)^d$; $m\bar\psi\psi$ is clearly a renormalisation group invariant, so $\gamma_{\bar\psi\psi} = -\gamma_m$).

1.2. Criticism

Lane's argument appears vulnerable to two types of criticism; its use of the rainbow/ladder approximation (do higher order terms change the leading asymptotic behaviour?) and its reliance on the Landau gauge (is the asymptotic behaviour gauge dependent?).

The first of these criticisms was addressed in part by Lane himself[1]; if we include terms of $O(g^{2(n+1)}), n > 0$ in either the gauge field propagator $D^{ab}_{\mu\nu}$ or the vertex function Γ^b_ν (or both), the asymptotic forms (2) and (4) are both modified by corrections which are down by $(\ln p^2)^{-n}$ (modulo $\ln\ln p^2$) with respect to the leading term. So asymptotic freedom guarantees the validity of the rainbow approximation in the deep Euclidean region. Furthermore, one may model nonperturbative behaviour of the gluon propagator $D^{ab}_{\mu\nu}(q^2)$ by modifying its form arbitrarily in the infrared. If restricted to a region $q^2 \lesssim M^2$ such modifications can only affect the asymptotic form of $\Sigma(p)$ significantly if they are singular as $q^2 \to 0$. Appropriate such singularities are a $1/q^4$ behaviour or a $\delta^4(q^2)$ term (considered by some to model confinement); these give corrections to (2) and (4) which are down by $1/p^2$. Vertex modifications will be considered shortly. The contribution of instantons to $\Sigma_-(p)$ falls as $1/p^6$ (i.e. down by $1/p^4$) as $p^2 \to \infty$.

We next address the second potential criticism. For an abelian theory, the vertex $\Gamma_\nu(p,q)$ should satisfy the Ward identity

$$(p-q)^\nu\Gamma_\nu(p,q) = S^{-1}(p) - S^{-1}(q) = (\not p - \not q) + \Sigma(p) - \Sigma(q). \tag{9}$$

This constraint fixes Γ_ν up to an essentially arbitrary transverse piece Γ^T_ν (such that $q_\nu\Gamma^T_\nu = 0$) which however cannot be responsible for gauge dependence in Σ (in covariant gauges $D^\xi_{\mu\nu}\Gamma^T_\nu$ is independent of the gauge parameter ξ). We have checked numerically that the use of vertices satisfying (9) (for example the Ball–Chiu vertex[3]) in the Schwinger–Dyson equation (1) does not change the asymptotic form (7) of $\Sigma_\pm(p)$, and that this behaviour is then independent of the gauge parameter ξ. Of course for the nonabelian theory Γ^b_ν should satisfy not (9) but the full Slavnov-Taylor identity;

$$(p-q)^\nu\Gamma^a_\nu(p,q)\big(1 + b(p-q)\big) = \big(\lambda^a + B^a(p,q)\big)S^{-1}(p) - S^{-1}(q)\big(\lambda^a + B^a(p,q)\big),$$

where b is the ghost self energy and B^a is the proper ghost-fermion vertex. However not only are b and B^a of higher order in g^2, but they are soft in the deep Euclidean region $p^2, q^2, (p-q)^2 \gg M^2$, and thus of no consequence for the asymptotic behaviour of Σ. An easy way to see this is to observe that in Lane's argument the nonabelian nature of the gauge interaction manifests only through the scale dependence of

the coupling $g(\kappa)$ in (7), which behaviour is itself independent of ξ when the Ward identity (9) is satisfied.

We may thus conclude that Lane's asymptotic forms ((2) and (4) with $\delta = d$) are not only unmodified by higher order or nonperturbative corrections, but are also independent of the choice of gauge[4].

1.3. Running Couplings

Another 'improvement' to the rainbow approximated Schwinger–Dyson equation (5) is to promote the coupling constant $g^2(\kappa)$ to a running coupling $\bar{g}^2(q^2)$ inside the momentum integral, which is thereby rendered finite even for the irregular solution (2). Justification of this prescription (which amounts to a regularisation of the loop integral) is highly nontrivial[5], and only works when $q^2 \gg p^2$ or (by reversing the order in which the integrations are performed) $M^2 \ll q^2 \ll p^2$. The use of the running coupling makes little sense when $q^2 \lesssim M^2$, or $(q-p)^2 \lesssim M^2$; fortunately these regions are not important for determining the asymptotic behaviour of $\Sigma(p)$ (§*1.2* above). Now, with such a prescription, the result is always Lane asymptotics[3]. Other prescriptions, which employ $\bar{g}^2(p^2)$ for $q^2 \ll p^2$, may be found in the literature[6]; they have not been derived from first principles, and always yield Politzer asymptotics. (Miransky assumes that the fermion-fermion scattering kernel behaves as $\bar{g}^2((p-q)^2)/(p-q)^2$, whereas $g^2(\kappa)/(p-q)^2$ is obtained from the renormalisation group[1].)

2. Asymptotics from OPEs

2.1. Politzer's argument[2]

Consider the operator product expansion for the fermion propagator in the chiral limit;

$$S(p) = \int d^4x\, e^{ip.x} \langle 0|T\bar{\psi}(x)\psi(0)|0\rangle \underset{p^2\to\infty}{\sim} C_1(p) + C_{\bar{\psi}\psi}(p, g(\kappa))v(\kappa) + \cdots \tag{10}$$

where $v(\kappa) \equiv \langle 0|\bar{\psi}\psi|0\rangle_\mu$. Now $S(p) \sim 1/\not{p} + \Sigma(p)/p^2 + \cdots$, while since the fermion field is not renormalised at leading order, $C_1 \sim 1/\not{p}$. Using the renormalisation group on $C_{\bar{\psi}\psi}$, and noting that $C_{\bar{\psi}\psi}(p, \bar{g}^2(p^2)) \sim 3C_2(r_f)\bar{g}^2(p^2)/p^2$, we thus find that

$$\Sigma(p) \underset{p^2\to\infty}{\sim} \frac{3C_2(r_f)\bar{g}^2(p^2)}{p^2} \exp\Big(-\int_\kappa^{p^2/M^2} \frac{d\kappa'}{2\kappa'}\, \gamma_{(\bar{\psi}\psi)}(\kappa')\Big)\, v(\kappa). \tag{11}$$

Taking for the anomalous dimension $\gamma_{(\bar{\psi}\psi)}$ the perturbative anomalous dimension $\gamma_{\bar{\psi}\psi} = -\gamma_m$ of the local operator $\bar{\psi}\psi$ (as given to leading order by (3)) yields the regular asymptotic form (4) with $\delta = d-1$; Politzer asymptotics. A similar argument away from the chiral limit ($m_R > 0$) gives irregular asymptotics (2).

2.2. Criticism

Compared to Lane's argument, Politzer's is rather insecure. The anomalous dimension $\gamma_{(\bar{\psi}\psi)}$ is defined as

$$\tfrac{1}{2}\gamma_{(\bar{\psi}\psi)}(\kappa) \equiv -\frac{\partial}{\partial \ln \mu^2} \ln\langle 0|\bar{\psi}\psi|\mathrm{Phys}\rangle_\mu = -\frac{\partial \ln v(\kappa)}{\partial \ln \kappa}, \tag{12}$$

where $|\text{Phys}\rangle$ is any *physical* state of appropriate (vacuum) quantum numbers within the Hilbert space on which the S-matrix is constructed. For an asymptotically free gauge theory (like QCD) these states are not directly related to the elementary degrees of freedom ('quarks' and 'gluons') fluctuating about the perturbative vacuum, but are the bound states ('mesons' and 'baryons') built upon the true nonperturbative vacuum state, in which various condensates (for example $\langle 0|F_{\mu\nu}F_{\mu\nu}|0\rangle_\mu$ and $\langle 0|\bar\psi\psi|0\rangle_\mu$) are nonzero. Since these condensates are nonperturbative in origin, they have essential singularities in the coupling constant, and cannot be expanded in powers of $g^2(\kappa)$ even as $\kappa \to 0$. Consequently whenever operators which can couple to such condensates appear in an operator product expansion they may "have a large anomalous dimension that is due to the bound state itself, rather than from the ultraviolet divergences of perturbation theory" [7].

To show that this is indeed what happens here, we evaluate $v(\kappa)$ directly; in the chiral limit, modulo meson and baryon loops,

$$\begin{aligned}\langle 0|\bar\psi\psi|0\rangle_\Lambda &= -n_c \int^\Lambda \frac{d^4p}{(2\pi)^4}\,\mathrm{tr}\,S(p)\\ &= -\frac{n_c}{16\pi^2}\mathrm{tr}\int_0^{\Lambda^2} p^2dp^2\,\frac{\Sigma(p^2)}{p^2+\Sigma(p^2)^2} \underset{\Lambda^2\to\infty}{\sim} -\frac{n_cn_f}{4\pi^2}m_D^3\left(\left(\ln\frac{\Lambda^2}{M^2}\right)^{\delta+1}+\cdots\right),\end{aligned} \tag{13}$$

using the regular asymptotic form (4). Renormalising at a scale μ^2, we must take $Z_{\langle\bar\psi\psi\rangle}(\Lambda/\mu, g) \sim (g^2\ln\Lambda/\mu)^{1+\delta}$, and, noting that $\kappa \equiv \mu^2/M^2$,

$$\ln(-v(\kappa)) \equiv \ln(-\langle 0|\bar\psi\psi|0\rangle_\Lambda) - \ln Z_{\langle\bar\psi\psi\rangle}(\Lambda/\mu, g(\kappa)) \underset{\kappa\to\infty}{\sim} (1+\delta)\ln\ln\kappa,$$

so that according to the definition (12),

$$\tfrac{1}{2}\gamma_{\langle\bar\psi\psi\rangle}(\kappa) \underset{\kappa\to\infty}{\sim} -(\delta+1)/\ln\kappa. \tag{14}$$

Inserting this in (11) gives back the behaviour (4) , with $m_D^3 = -\frac{3C_2(r_f)}{4\beta^0}\lim_{\kappa\to\infty}\frac{v(\kappa)}{(\ln\kappa)^{1+\delta}}$. So the operator product expansion analysis is self consistent, *for all values of* δ; to determine δ requires further information. Politzer asymptotics ($\delta = d-1$) result only when we make the additional (unjustifiable) assumption that $\gamma_{\langle\bar\psi\psi\rangle} = \gamma_{\bar\psi\psi}$.

One may also obtain (14) by taking $|\text{Phys}\rangle = |S(k=0)\rangle$ (since the amputated Bethe–Saltpeter amplitude for the scalar bound state S is also $\varphi(p^2) = \Sigma(p^2)/f_p$ at $k = 0$, by chiral symmetry), and thus any S-wave state with an even number of pseudoscalar Goldstone bosons, since their couplings to S, and the propagator for the (virtual) S, are all ultraviolet finite, and thus scale independent. This is as it should be; according to current algebra, $\langle 0|\bar\psi\psi|0\rangle = f_P^{-2}\langle P|\bar\psi\psi|P\rangle$, for example.

2.3. Nonperturbative evolution

Using Lane's argument, $\delta = d$ and ((14) and (3))

$$\tfrac{1}{2}\gamma_{\langle\bar\psi\psi\rangle} \underset{\kappa\to\infty}{\sim} -(d+1)/\ln\kappa \sim -(\beta^0+\gamma^0)g^2(\kappa). \tag{15}$$

Thus $\gamma_{\langle\bar\psi\psi\rangle}$ has a nonperturbative contribution, and $\langle 0|\bar\psi\psi|\text{Phys}\rangle_\mu$ evolves nonperturbatively even as $\kappa \to \infty$; nonperturbative physics normally associated with low momentum scales can indeed contaminate high momentum behaviour in channels in which condensates may form.

It is interesting to compare this rather subtle nonperturbative evolution with that found in strongly interacting nonasymptotically free gauge theories (such as strong QED) in which the asymptotic behaviour of $\Sigma(p, g(\kappa))$ is given by (7) with $g^2(\kappa) \to g_c^2$ such that $\nu(\kappa) \to 0$ as $\kappa \to \infty$. Using the same argument as in §*2.2* we now find

$$\langle 0|\bar{\psi}\psi|0\rangle_\Lambda \underset{\Lambda^2\to\infty}{\sim} -\frac{n_f}{\pi^2} m_D^2 \Lambda,$$

so that $\gamma_{(\bar{\psi}\psi)} = \gamma_{\bar{\psi}\psi} = -\gamma_m = -1$. Since $\gamma_{\bar{\psi}\psi}$ is now hard, new operators (in particular four–fermion operators) become relevant, and the fundamental nature of the theory is radically altered[8]. By contrast, in the asymptotically free case, $\gamma_{\bar{\psi}\psi}$ and $\gamma_{(\bar{\psi}\psi)}$ both vanish as $\kappa \to \infty$, and no new operators become relevant.

In principle the logarithmic scaling (15) of $\langle 0|\bar{\psi}\psi|0\rangle_\mu$ could be checked by a lattice computation, although with present techniques this does not seem to be very practical.

3. Conclusions

Our main conclusion is that while Lane's argument stands up to closer scrutiny, Politzer's does not; $\delta = d$. As a corollary, when a running coupling constant is used in a Schwinger–Dyson or Bethe–Saltpeter equation, its argument should be the loop momentum q^2, rather than $(p-q)^2$, $\max(p^2, q^2)$, or any other such combination, the only evidence for which has always been Politzer's asymptotics[6].

The fermion condensate $v(\kappa)$ and its evolution is rather difficult to access experimentally. The Gell-Mann–Oakes–Renner relation $m_P^2 f_P^2 = -2m_R v_R$ would, if m_R were known, determine the renormalisation group invariant $v_R \equiv -k_v m_D^3$, where k_v is a finite positive constant, calculable modulo the same infrared uncertainties present in the Pagels–Stokar evaluation of f_P[9]. Although v_R may be naturally interpreted as $\sim v(\kappa)/(\ln\kappa)^{1+\delta}$ we see no reason to expect these two quantities to be equal. The experimental implications of the nonperturbative modification (15) of the evolution of $v(\kappa)$ are even less clear; it may be that an explanation for the $\Delta I = \frac{1}{2}$ rule will be one consequence, if the appropriate matrix elements of the effective four-fermion operators have a similarly enhanced nonperturbative evolution.

More dramatic effects may result for operators which couple to condensates but whose leading perturbative evolution is at two loops, and thus rather weak. An example of such is the axial singlet current which couples to the condensate $\int d^4x \, \langle 0|TF\tilde{F}(x)F\tilde{F}(0)|0\rangle$ via the axial anomaly. Nonperturbative evolution then screens out the axial singlet charge at large Q^2 modifying the Ellis–Jaffe sum rule for the first moment of the proton spin structure function $g_1^p(x)$, in just such a way as to provide a natural resolution of the EMC proton 'spin crisis'[10].

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