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NON-GAUSSIAN DENSITY FIELDS CAN PRODUCE A GAUSSIAN SACHS-WOLFE EFFECT

Robert J. Scherrer
Department of Physics, The Ohio State University
Columbus, OH 43210 USA

Robert K. Schaefer.
Bartol Research Institute, University of Delaware
Newark, DE 19716 USA

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The Sachs-Wolfe temperature fluctuations produced by primordial density perturbations are proportional to the potential field ϕ, which is a weighted integral over the density field δ. Because of the central limit theorem, ϕ will tend to be Gaussian even when δ is non-Gaussian. We generate two highly non-Gaussian density fields with a Zel'dovich power spectrum and show that the resulting potential fields have a nearly Gaussian distribution. These results indicate that even if the temperature distribution observed by COBE is Gaussian, such an observation cannot definitively rule out non-Gaussian initial conditions for large-scale structure.

The recent COBE observations of Sachs-Wolfe fluctuations in the microwave background place severe constraints on possible models for large-scale structure.[1-2] One of the unresolved questions regarding the primordial density fluctuations which gave rise to large-scale structure is whether these density fluctuations were Gaussian, i.e., whether the N-point distribution of primordial density fluctuations is a multivariate Gaussian distribution. Gaussian models have the virtue of simplicity, and a great deal is known about their properties.[3] On the other hand, a large number of non-Gaussian models have been proposed recently, such as global texture[4] and non-standard inflation models.[5]

We argue here that the COBE results will not provide a decisive test of whether or not the initial density field was Gaussian. We mention first the trivial possibility that the density field could be non-Gaussian on small scales but Gaussian on the length scales probed by COBE. Such density fields are produced, for example, by a Poisson distribution of objects; the density field is non-Gaussian when smoothed on scales smaller than the mean separation of objects, but Gaussian when smoothed on scales larger than this mean separation.[6]

We can, however, make a stronger assertion: the potential field will tend to have a Gaussian distribution when smoothed on a given length scale even when the density field is non-Gaussian on this same scale. The Sachs-Wolfe fluctuations probed by COBE are directly proportional to the potential field $\phi(\mathbf{r})$ on the surface of last scattering. This potential field is related to the field of density fluctuations $\delta(\mathbf{r})$ via

$$\nabla^2\phi(\mathbf{r}) = 4\pi G a^2 \bar{\rho}\delta(\mathbf{r}), \tag{1}$$

where a is the cosmological scale factor and $\bar{\rho}$ is the mean density. In integral form, we can write this relationship as:

$$\phi(\mathbf{r}) = -Ga^2\bar{\rho}\int \frac{\delta(\mathbf{r}')d^3\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}. \tag{2}$$

Thus, the potential field is a weighted integral over the density field. The central limit theorem indicates that this integral will generally be driven to a Gaussian distribution even when the distribution of δ is non-Gaussian. A similar phenomenon has been noted in the case of the velocity field, which can also be expressed as an integral over the density field; the distribution of velocities tends to be Gaussian even then the underlying density field is non-Gaussian.[7]

To test this hypothesis, we have generated two highly non-Gaussian density fields with a Zel'dovich power spectrum. We begin by setting down an uncorrelated field $\Delta(\mathbf{r})$ on a 128^3 lattice with a non-Gaussian one-point distribution $p(\Delta)$. There are no correlations between the values of Δ on different lattice sites, so the power spectrum for Δ is $P(k) =$ a constant. We Fourier transform $\Delta(\mathbf{r})$ to obtain $\hat{\Delta}(\mathbf{k})$, and then we take $\hat{\delta}(\mathbf{k}) = k^{1/2}\hat{\Delta}(\mathbf{k})$ and $\hat{\phi}(\mathbf{k}) = k^{-3/2}\hat{\Delta}(\mathbf{k})$. (Multiplicative constants are irrelevant for our purposes). We then inverse Fourier transform $\hat{\delta}(\mathbf{k})$ and $\hat{\phi}(\mathbf{k})$ to obtain the density field $\delta(\mathbf{r})$ and the corresponding potential field $\phi(\mathbf{r})$. To eliminate lattice effects, we smooth both of these fields with a Gaussian window function $W(r) = e^{-r^2/2r_0^2}$, with $r_0 = 3$ cell lengths. This procedure is somewhat similar to that used by Weinberg and Cole[8] to generate a family of non-Gaussian density fields with fixed power spectra. Our procedure produces a density field $\delta(\mathbf{r})$ with a slightly different one-point distribution than the original uncorrelated field Δ, but by choosing $p(\Delta)$ to be highly non-Gaussian, we find that $p(\delta)$ is also non-Gaussian, with a form which is fairly similar to the original $p(\Delta)$. In any case, this procedure produces a non-Gaussian density field δ with a Zel'dovich power spectrum, along with the corresponding potential field ϕ.

As an example, we have chosen two extremely non-Gaussian forms for $p(\Delta)$. We first take the distribution for $p(\Delta)$ to be a gamma function: $p(\Delta) \propto \Delta e^{-\Delta}$. The distribution of the density field is shown in Fig. 1a. This distribution closely resembles the original gamma function; it deviates strongly from a Gaussian and has large positive skewness. The corresponding distribution for the potential field is shown in Fig. 1b. While it clearly retains a slight imprint of the original gamma distribution, it is nearly indistinguishable from a Gaussian. For our second example, we take the symmetric delta-function distribution defined by $\Delta(\mathbf{r}) = \pm\delta_0$, where δ_0 is a constant, and the value is chosen randomly in each cell. The resulting distribution of $\delta(\mathbf{r})$ is shown in Fig. 2a. Again, while the original delta function distribution is smeared out, the resulting density distribution is strongly non-Gaussian and resembles the original distribution. The corresponding distribution for the potential (Fig. 2b) is essentially indistinguishable from a Gaussian.

These examples indicate that the distribution of the potential field tends to be Gaussian even when the distribution of the density field is strongly non-Gaussian. Our examples

are not physically motivated; we have deliberately chosen density fields which are much more non-Gaussian than in most physically-interesting models. Because of the central limit theorem, we expect our results to apply to nearly all non-Gaussian density fields: the potential field will tend to be more Gaussian than the underlying density field.

We must add two caveats to this conclusion. It is certainly possible to generate non-Gaussian potential fields by construction.[5,9] Secondly, we have only shown that the one-point distribution function for the potential field is Gaussian. This is not sufficient to prove that the full N-point distribution is Gaussian, and it is possible to construct non-Gaussian fields in which the one-point distribution is Gaussian.[10–11] The procedure for going from δ to ϕ does not alter the phases of the Fourier components, so any phase correlations in the initial density field are also present in the potential field. The difference is that the potential field more strongly weights the long-wavelength Fourier modes, whose phases are less strongly correlated than the phases of the short-wavelength modes.

Our results indicate that even if the temperature field seen by COBE turns out to have a one-point Gaussian distribution, this observation by itself cannot definitively prove that the underlying density field is Gaussian. On the other hand, a non-Gaussian distribution of temperatures would indicate either a strongly non-Gaussian density field, or a non-primordial origin for the temperature fluctuations.

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FIGURE CAPTIONS

Fig. 1: (a) The distribution of densities in a model generated by Fourier-transforming an uncorrelated gamma distribution and forcing the power spectrum to a Zel'dovich form. Solid curve is a Gaussian distribution with the same mean and standard deviation. (b) The distribution of potentials for the same model. Solid curve is a Gaussian distribution with the same mean and standard deviation.

Fig. 2: (a) The distribution of densities in a model generated by Fourier-transforming an uncorrelated bimodal delta-function distribution ($\Delta = \pm\delta_0$) and forcing the power spectrum to a Zel'dovich form. Solid curve is a Gaussian distribution with the same mean and standard deviation. (b) The distribution of potentials for the same model. Solid curve is a Gaussian distribution with the same mean and standard deviation.

Fig. 1 (a)

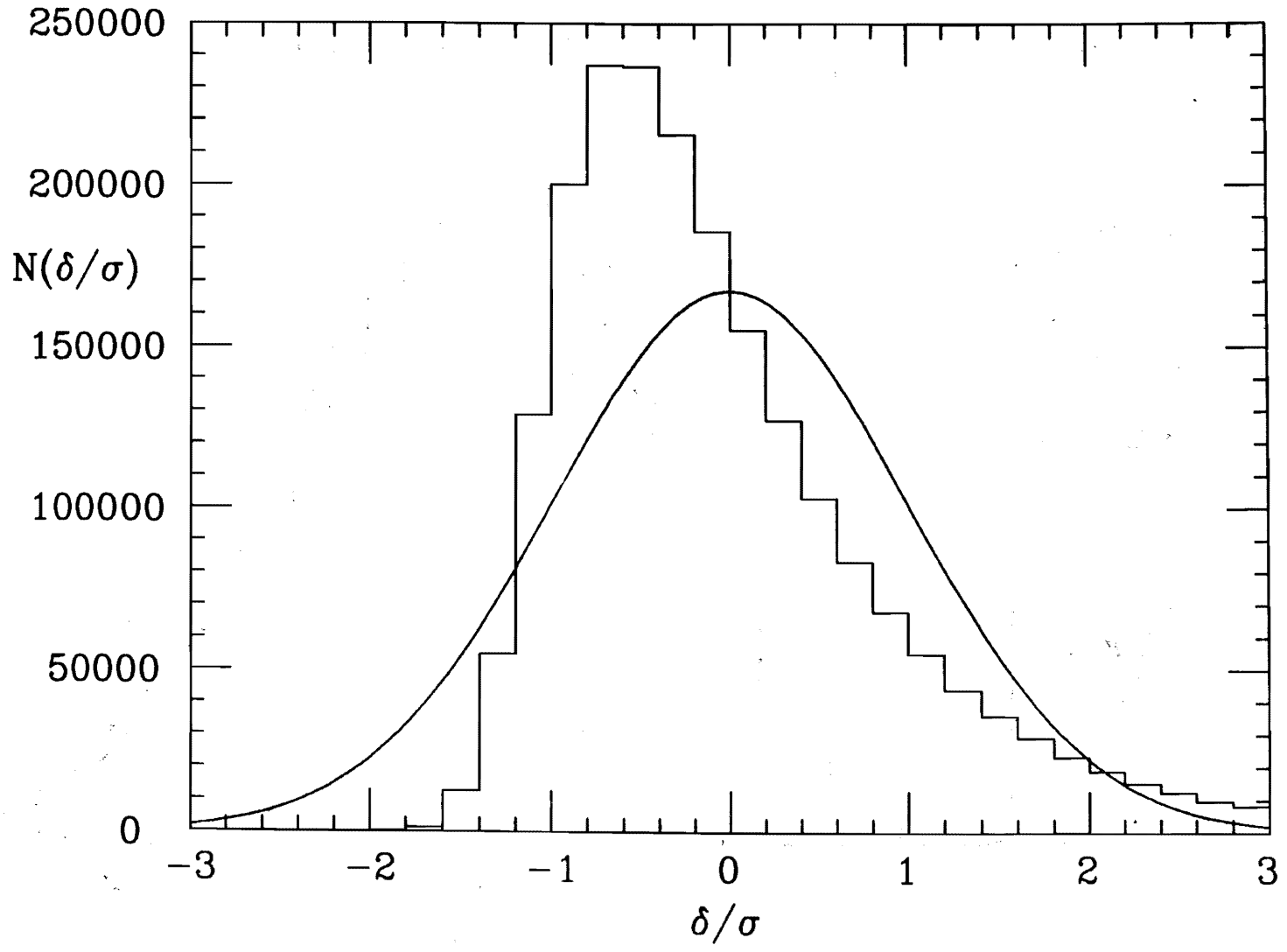

Fig. 1(b)

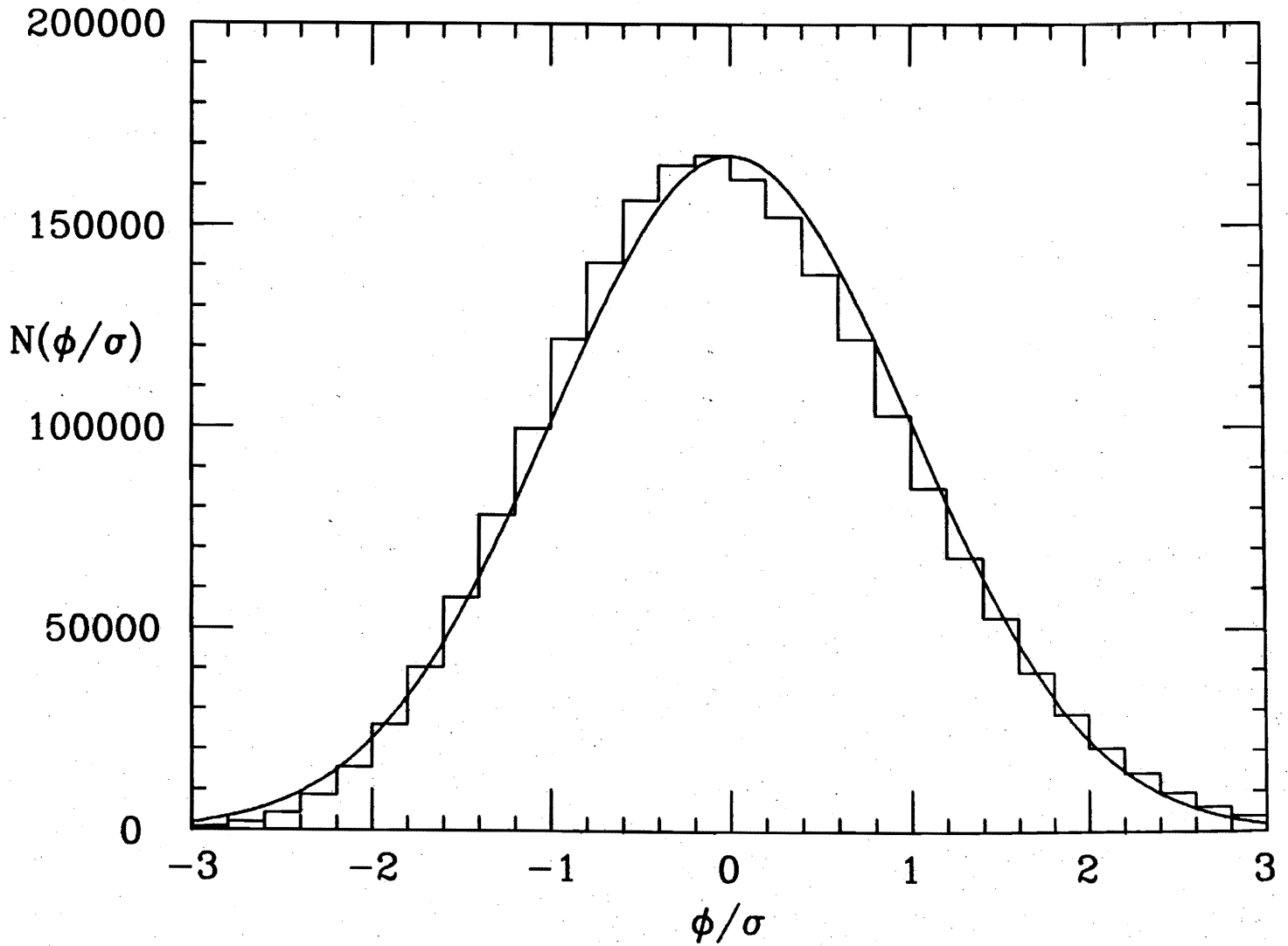

Fig. 2(a)

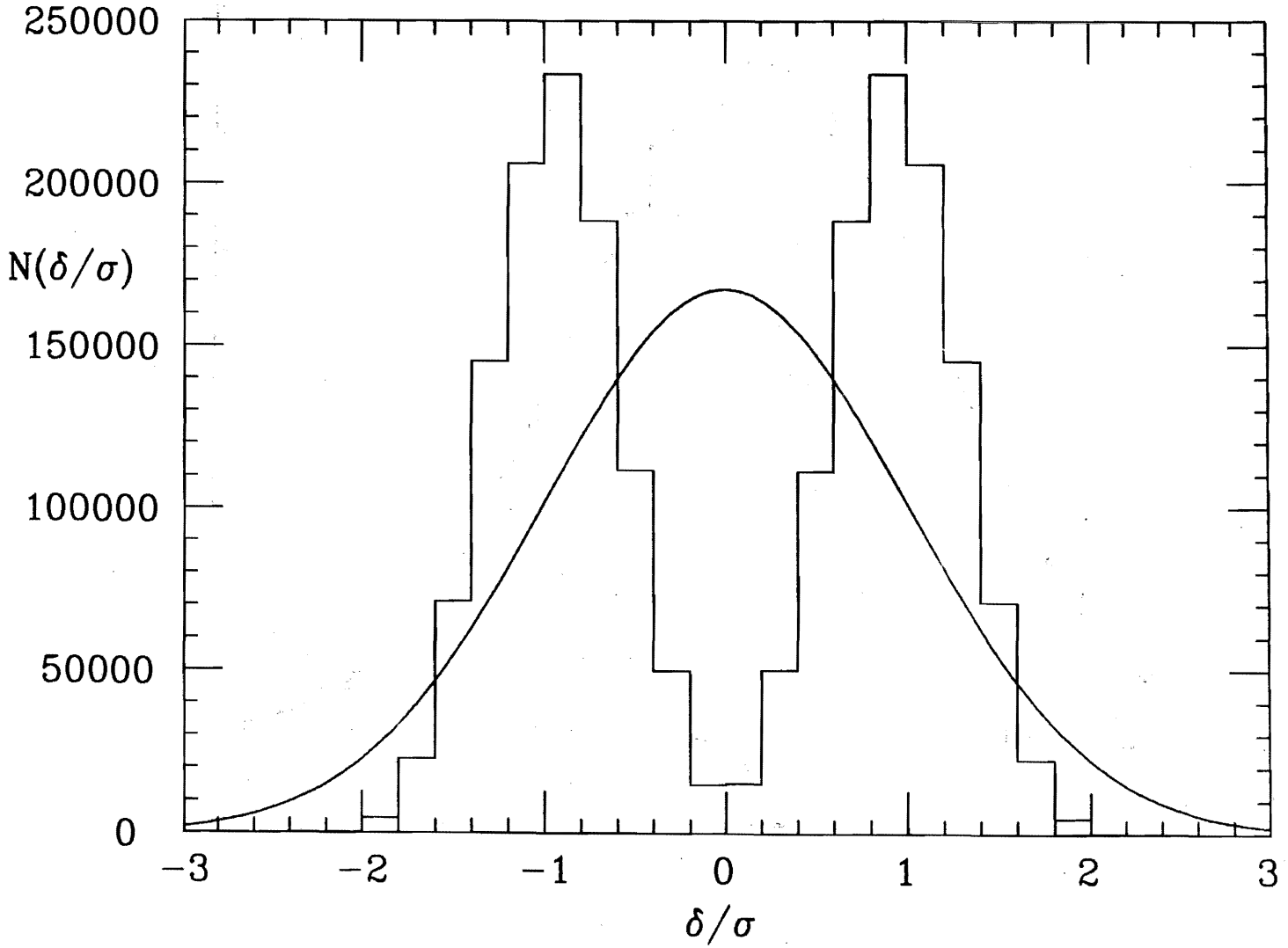
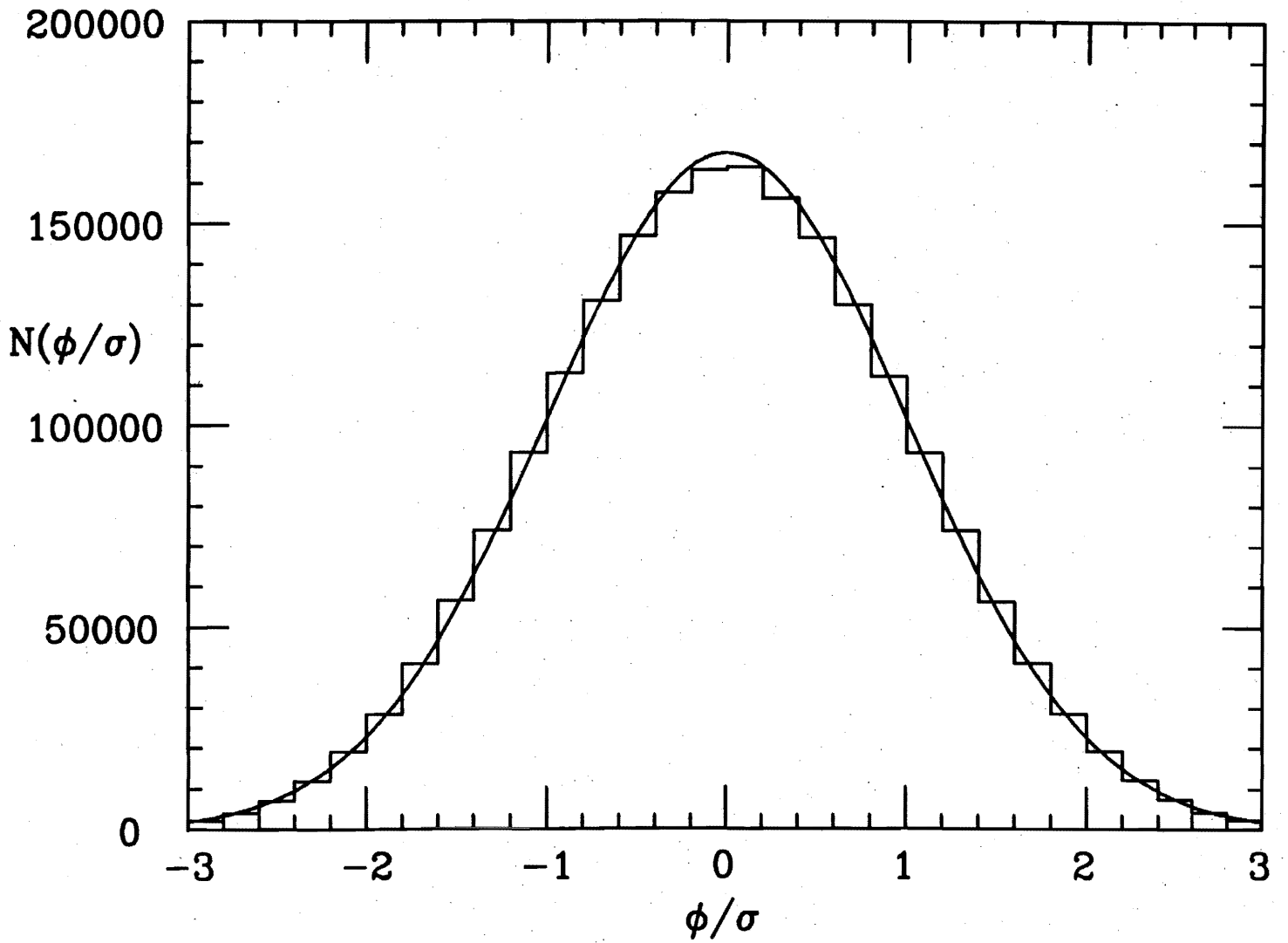
Fig. 2(b)