On West's asymptotic estimate of perturbative coefficients of $R(s)$ in e^+e^- Annihilation

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Abstract

A comment on the recent asymptotic estimate of the perturbative coefficients of $R(s)$ in e^+e^- annihilation is given. A possible explanation of the disagreement with the results of exact calculations is presented.

Recently, West has suggested an alternative method to estimate perturbative coefficients of $R(s)$ [1]. This estimate gives good agreement with the result of four-loop calculations [2, 3], for $N_f = 5$. This is a very important result, even if it's precision is only 20% [1]. However, a few critical comments were published (see [4, 5, 6]) with the conclusion that the agreement with the exact result of [2, 3], for $N_f = 5$, is accidental.

The R-ratio is given by the perturbative calculation in the following form:

$$R(s) = r_0 \left(1 + r_1 \frac{\alpha_s(s)}{\pi} + r_2 (\frac{\alpha_s(s)}{\pi})^2 + r_3 (\frac{\alpha_s(s)}{\pi})^3 + ...\right) \tag{1}$$

The renormalization scheme-scale ambiguity problem for $R(s)$ was studied in [7]. In particular, we found $R(s)$ in explicit scheme-scale dependent form:

$$\begin{aligned} R^t(s) = & \\ & 3\sum_f Q_f^2 \left[1 + \left(\frac{\alpha_s^t}{\pi}\right) + \left(\frac{\alpha_s^t}{\pi}\right)^2 [(1.9857 + 2.75t) - N_f(0.1153 + 0.1667t)]\right. \\ & + \left(\frac{\alpha_s^t}{\pi}\right)^3 [(-6.6369 + 17.2964t + 7.5625t^2) - N_f(1.2001 + 2.0877t + 0.9167t^2) \\ & + N_f^2(-0.0052 + 0.0384t + 0.0278t^2)] - \left(\sum_f Q_f\right)^2 \left(\frac{\alpha_s}{\pi}\right)^3 1.2395 \; + \; O(\alpha_s^4) \end{aligned} \tag{3}$$

where t fixes the prescription: $\int d^4p \longrightarrow \int d^{4-2\epsilon}p(\mu^2 e^{(-t+O(\epsilon))})^\epsilon$. By changing t we get different MS-type schemes. Obviously, $t = 0$ corresponds to the $\overline{MS}$-scheme [8]. Using the intermediate results of our calculation [2], let's rewrite r_2 and r_3 in the following form with obvious notations (see also eq.(7) of ref. [2]):

$$r_2(t, N_f) = [2.6042 - 0.1528N_f]_{Z_{3,-1}} + [-0.6185 + 2.75t + N_f(0.0375 - 0.1667t)]_{\Pi_{2,0}};$$

$$\begin{aligned} r_3(t, N_f) = & [-15.1377 + 0.9731N_f - 0.1112N_f^2]_{Z_{4,-1}} + [139.0693 - 26.1317t + 15.1250t^2 \\ & + N_f(-18.004 + 3.1953t - 1.8333t^2) + N_f^2(0.5803 - 0.0977t + 0.0556t^2)_{\Pi_{3,0}} \\ & + [-1.4337 + 6.3750t + N_f(0.1780 - 0.7917t)]_{\Pi_{2,0}} + [-129.1347 + 37.0531t - 7.5625t^2 \\ & + N_f(15.6527 - 4.4913t + 0.9167t^2) + N_f^2(-0.4743 + 0.1361t - 0.0278t^2)_{\Pi_{2,1}} \end{aligned}$$

Here we neglected the term proportional to $\left(\sum_f Q_f\right)^2$ and the π^2 -term is included in the scheme invariant part. First, it should be stressed that three-

and four-loop results are scheme dependent while the result of [1] with unclear scheme dependence was compared to the $\overline{MS}$ result of [2, 3]. Scheme dependent contributions to $r_3(t, N_f)$ coming from $\Pi_{i,j}$ have an absolute minimum (+0.28) at $t = 0$ ($\overline{MS}$-scheme) and N_f=5. On the other hand, scheme independent Z-contributions give the following results for $N_f = 5$: $r_2^{z_3,-1}(5) = 1.840$; $r_3^{z_4,-1}(5) = -13.2$. West's estimate gives correspondingly: $r_2 = -1.841$ and $r_3 = -13.4$. For $N_f \neq 5$ there is still a large disagreement. It seems that, neglected saddle-point corrections could give large contributions (see [4]), which are suppressed by the N_f dependent factors at $N_f = 5$. For example, such a possibility is $\mid \beta_2/n^2\beta_0^3 \mid$, which has a minimum (zero) in $N_f \in (5, 6)$.

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