Perturbative Quantum Chromodynamics:

Evidence that α_s Runs

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Research Note 258 (Revised Version) February, 1992

ABSTRACT

We consider the theoretical predictions of perturbative Quantum Chromodynamics (QCD) for the quantities $R_{e^+e^-}$, R_τ and $\Gamma^{Z^o}_{hadronic}$. Using $\alpha_s(M_\tau) = .34^{+.04}_{-.05}$, we find that $\alpha_s(34\text{GeV}) = .140^{+.006}_{-.008}$ in agreement with the value extracted from experiment, $\alpha_s(34\text{GeV}) = .148(22)$, and $\alpha_s(\text{Mz}) = .119(5)$, in excellent agreement with the experimental value from LEP, $\alpha_s(\text{Mz}) = .120(7)$. We show that using only the above value for α_s (M_τ) as input and <u>no free parameters</u> we can predict α_s (μ) for $M_\tau \leq \mu \leq 200$ GeV. Our result is in agreement with all known experimental values. Thus it can be seen that α_s truly runs, in agreement with perturbative QCD.

Recently we obtained a new result[1] for

$$R_\tau = \Gamma(\tau \to \nu_\tau + \text{hadrons})/\Gamma(\tau \to e\bar{\nu}\nu) \tag{1}$$

Our result was based on the new corrected value[2,3] for

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \tag{2}$$

in four-loop order.

Our result for the perturbative part of R_τ was

$$R_\tau^{\text{Pert}} = 3\left[1 + \frac{\alpha_s}{\pi} + 5.20\left[\frac{\alpha_s}{\pi}\right]^2 + 26.38\left[\frac{\alpha_s}{\pi}\right]^3\right] \tag{3}$$

Using eq. (3) and the non–perturbative and weak interaction contributions and the experimental value,

$$R_{\text{expt}} = 3.65(13) \tag{4}$$

we obtained the result for the strong-coupling constant at M_τ, the mass of the τ lepton,[1,4]

$$\alpha_s(M_\tau) = .34^{+.04}_{-.05} \tag{5}$$

We now use the relation for α_s in terms of the QCD parameter Λ,

$$\alpha_s(\mu) = \frac{2\pi}{\beta_0 L}\left\{1 - \frac{\beta_1 \ln 2L}{2\beta_0{}^2 L} + \frac{1}{4L^2\beta_0{}^4}\cdot \left[\beta_1^2 \ln^2 2L - \beta_1^2 \ln 2L + \beta_2\beta_0 - \beta_1^2\right]\right\} \tag{6}$$

where β_o, β_1, β_2 are the coefficients of the QCD β function,

$$\beta_o = 11 - 2N_f/3$$

$$\beta_1 = 102 - \frac{38N_f}{3}$$

$$\beta_2 = \frac{2857}{2} - \frac{5033}{18} N_f + \frac{325}{54} N_f^2 \tag{7}$$

and $L = \ln\mu/\Lambda$ and N_f is the number of fermions (quarks).

For $N_f = 3$ we obtain from eq (5)

$$\Lambda^{(3)}_{\overline{MS}} = 380 \begin{Bmatrix} +70 \\ -90 \end{Bmatrix} \text{MeV} \tag{8}$$

We now evolve up to $N_f = 4$ and obtain

$$\Lambda^{(4)}_{\overline{MS}} = 320 \begin{Bmatrix} +70 \\ -80 \end{Bmatrix} \text{MeV} \tag{9}$$

Finally, evolving up to $N_f = 5$ we obtain

$$\Lambda^{(5)}_{\overline{MS}} = 220 \begin{Bmatrix} +55 \\ -60 \end{Bmatrix} \text{MeV} \tag{10}$$

and, hence,

$$\alpha_s(34\text{GeV}) = .140 \begin{Bmatrix} +.006 \\ -.008 \end{Bmatrix} \tag{11}$$

These results should be compared with our values obtained from a direct comparison of the theoretical and experimental values[5] of

$$R = \frac{\sigma(e^+e^- \to \text{ hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \tag{12}$$

$$\alpha_s(34 \text{ GeV}) = .148(22) \tag{13}$$

and

$$\Lambda^{(5)}_{\overline{MS}} = 290 \begin{Bmatrix} +275 \\ -175 \end{Bmatrix} \text{ MeV} \tag{14}$$

It can be seen that eqs (11) and (13) are beautifully consistent. Moreover the values of $\Lambda^{(5)}_{\overline{MS}}$ in eqs (10) and (14) agree as well. Note, however, that the error in eq (10) is much smaller than that in eq (14).

Next we evolve α_s up to M_Z, the mass of the Z boson and obtain[6]

$$\alpha_s(M_Z) = .119(5) \tag{15}$$

This should be compared with the recent result, which is an average of the four LEP collaborations from the ratio of hadronic to leptonic Z-decay widths and hadron distributions, presented by H. Wachsmuth.[7]

$$\alpha_s(M_Z) = .120(7) \tag{16}$$

The agreement between eqs (15) and (16) is incredibly good!

In figure 1 we plot $\alpha_s(\mu)$ vs μ for $M_\tau \leq \mu \leq 200$ GeV. It should be emphasized that once $\alpha_s(M_\tau)$ is fixed by eq (5) <u>there are no free parameters</u> used to obtain this figure. The shaded region is the allowed region obtained by using the upper and lower bounds from

evolving α_s from M_τ to 200 GeV. The values predicted for 100 GeV $\leq \alpha_s \leq$ 200 GeV are insensitive to the choice of the top-quark mass.[8]

The independently obtained experimental values with error-bars are also shown in the figure. The values are tabulated in TABLE I along with our predictions. It can be seen that the agreement between theory and experiment is excellent. Figure 2 shows the same results for $M_\tau \leq \alpha_s \leq$ 100 GeV more clearly. α_s (5 GeV) was obtained from the ratio $\Gamma(\Upsilon \rightarrow \gamma gg)/\Gamma(\Upsilon \rightarrow ggg)$ and α_s(10 GeV) was obtained from deep inelastic muon-carbon scattering. The value of α_s (17.3 GeV) comes from pp collisions at the CERN ISR and α_s (80.6 GeV) = α_s (Mw) comes from the ratio W $\rightarrow$ 1 jet/W $\rightarrow$ 0 Jets.

In conclusion, we have shown that α_s truly runs, in agreement with perturbative QCD.

It is a pleasure to thank M. Margano, E. Braaten, S. Stone and P. Lepage for helpful discussions, and the Aspen Winter School, where these results were first presented. This work was supported by the U.S. Department of Energy under Grant No. DE-FG05-84ER40215.

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(6) See ref. 4. Their result is $\alpha_s(M_Z) = .120(7)$, in agreement with ours.

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(8) We obtain $\Lambda^{(6)}_{\overline{MS}} = 93^{+31}_{-29}$ MeV for top-quark masses between 100 GeV and 180 GeV. See W. Marciano, Phys. Rev. D29, 580 (1984).

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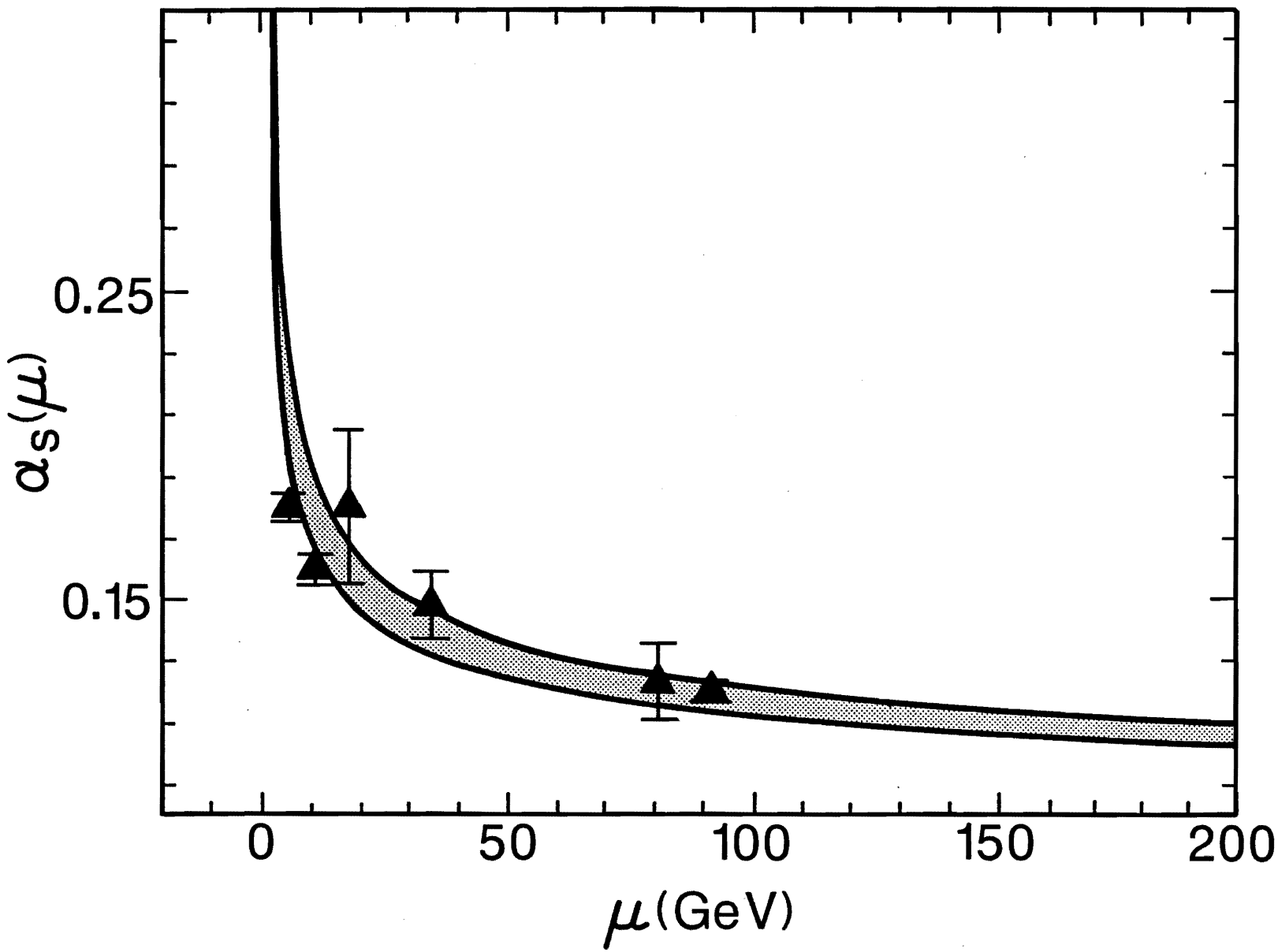
TABLE I. Experimental and Predicted Values for $\alpha_s(\mu)$

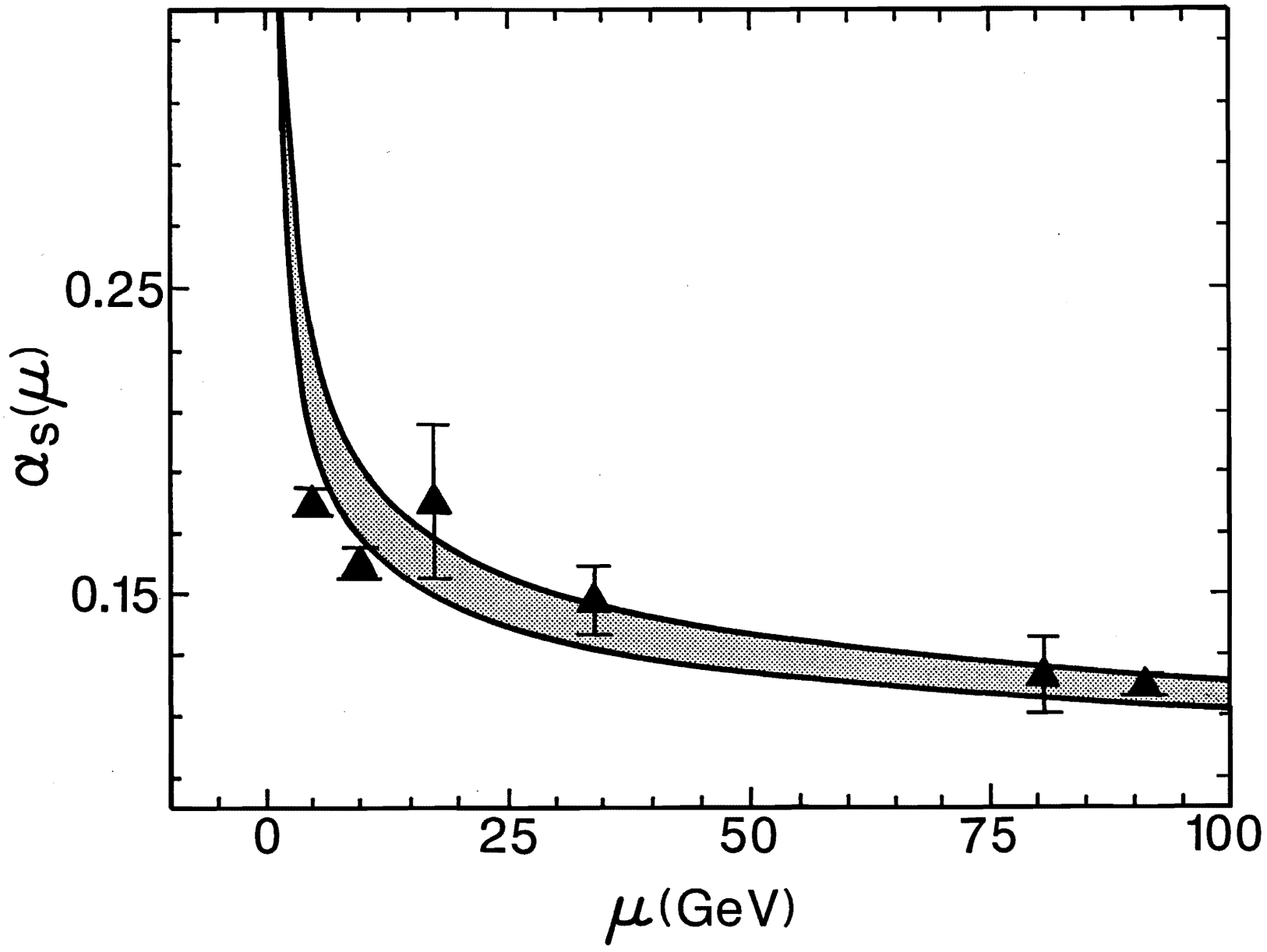
μ(GeV)	Theoretical Values for $\alpha_s(\mu)$	Experimental Values for $\alpha_s(\mu)$
1.784	input	$.34^{+.04}_{-.05}$ ref. 1
5.0	$.219\ ^{+.015}_{-.019}$	.179(9) ref. 9
10.0	$.180^{+.010}_{-.012}$	.160(10) ref. 10
17.3	$.160^{+.007}_{-.010}$	.18(5) ref. 11
34.0	$.140^{+.006}_{-.008}$	.148(22) ref. 5
80.6	$.121^{+.005}_{-.006}$	.123(25) ref. 12
91.2	$.119^{+.004}_{-.006}$	.120(7) ref. 7
180.0	$.108^{+.004}_{-.005}$	

Figure Captions

Figure 1: $\alpha_s(\mu)$ vs μ for 1.784 GeV $\leq \mu \leq$ 200 GeV. The shaded region is the theoretical prediction. The points, with error-bars, are the results from various experiments.

Figure 2: $\alpha_s(\mu)$ vs μ for 1.784 GeV $\leq \mu \leq$ 100 GeV. The shaded region is the theoretical prediction. The points, with error bars, are the results from various experiments.


0.25
0.15
$\alpha_s(\mu)$
0
50
100
150
200
μ(GeV)


0.25
0.15
$\alpha_s(\mu)$
0
25
50
75
100
μ(GeV)