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The total cross section for off-shell photons at high energies*

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Abstract

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THE TOTAL CROSS SECTION FOR OFF-SHELL PHOTONS AT HIGH ENERGIES

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Virtual photon scattering at high energy provides a clean probe of low x dynamics in QCD. I study the total cross section for this process at present and future e^+e^- colliders, and discuss the perturbative dependence of the result on the energy s and on the photon virtualities.

QCD dynamics at low x is one of the frontiers of the theory of strong interactions [1]. Applications of this dynamics to deep inelastic scattering have been the object of many investigations, particularly since experimental tests of the low x behaviour started to be performed at HERA [2]. Lately, studies of low x dynamics in the context of photon-photon collisions have attracted new interest [3-6]. In this paper I focus on the total cross section for virtual photon scattering, following the lines of the calculation in Ref. [5]. This cross section presents some theoretical advantages as a probe of small x dynamics compared to the structure functions for deep inelastic scattering off a proton (see for instance the discussion in Ref. [2]) or a (quasi)-real photon (see for example Ref. [3]), essentially because it does not involve a non-perturbative target. Unlike protons or quasi-real photons, virtual photon states can be described through perturbative wave functions. In some respects the off-shell photon cross section presents analogies with the process of scattering of two quarkonia (or "onia"), which has been proposed as a gedanken experiment to investigate low x phenomena [7]. In this case, non-perturbative effects are suppressed by the smallness of the onium radius. In the case of virtual photons the size of the wave function is controlled by the photon virtuality, instead of the heavy quark mass. It is an interesting feature of investigations at e^+e^- colliders that this size can be tuned by measuring the momenta of the outgoing leptons. On the other hand, experimental studies may prove to be difficult due to the smallness of the available rates. An estimate of the number of events one may expect at LEP200 and a next linear collider is given below.

Consider the scattering of two transversely polarized virtual (space-like) photons $\gamma^*(q_A)$, $\gamma^*(q_B)$, with virtualities $q_A^2 = -Q_A^2$, $q_B^2 = -Q_B^2$, in the high energy region where $s = (q_A + q_B)^2$ is much larger than Q_A^2, Q_B^2. We suppose that the photon virtualities are in turn large with respect to the QCD scale Λ_{QCD}^2, so that the process occurs at short distances (much smaller than $\Lambda_{QCD}^{-1} \sim 1$ fm) and QCD perturbation theory applies. The perturbative expansion of the total cross section in the high energy region has the form

$$\sigma(\gamma^* \gamma^*) \sim \sigma^{(0)} \left[1 + \sum_{k=1}^{\infty} a_k \, (\alpha_S \, L)^k + \ldots \right] , \quad L = \ln(s/Q_A \, Q_B) . \tag{1}$$

$\sigma^{(0)}$ is the cross section in the Born approximation, and is constant at large s, $\sigma^{(0)} \sim \alpha^2 \alpha_S^2 f(Q_A^2, Q_B^2)$. To higher orders in the strong coupling α_S, logarithmic corrections in the energy s appear, due to multiple gluon exchange. This effect is known as the QCD pomeron, or BFKL effect [8].

The summation of these corrections can be best given in terms of the moments σ_N of the cross section. Define

$$\sigma_{\gamma^* \gamma^*}(s, Q_A^2, Q_B^2) = \int_{a - i\infty}^{a + i\infty} \frac{d N}{2\pi i} \times e^{N L} \, \sigma_N(Q_A^2, Q_B^2) , \tag{2}$$

where the N-integral runs parallel to the imaginary axis and to the right of any singularities in σ_N. Powers of logarithms in s appear as multiple poles at $N = 0$ in the moment space. The summation of these poles to the accuracy $(\alpha_S^2/N) \times (\alpha_S/N)^k$, for any k, is accomplished by the formula

$$\sigma_N(Q_A^2, Q_B^2) = \frac{1}{2\,\pi\, Q_A \, Q_B} \int_{1/2 - i\infty}^{1/2 + i\infty} \frac{d\gamma}{2\pi i} \times \left(\frac{Q_A^2}{Q_B^2} \right)^{\gamma - 1/2} \frac{V(\gamma)\, V(1-\gamma)}{N - \bar{\alpha}_S \, \chi(\gamma)} , \tag{3}$$

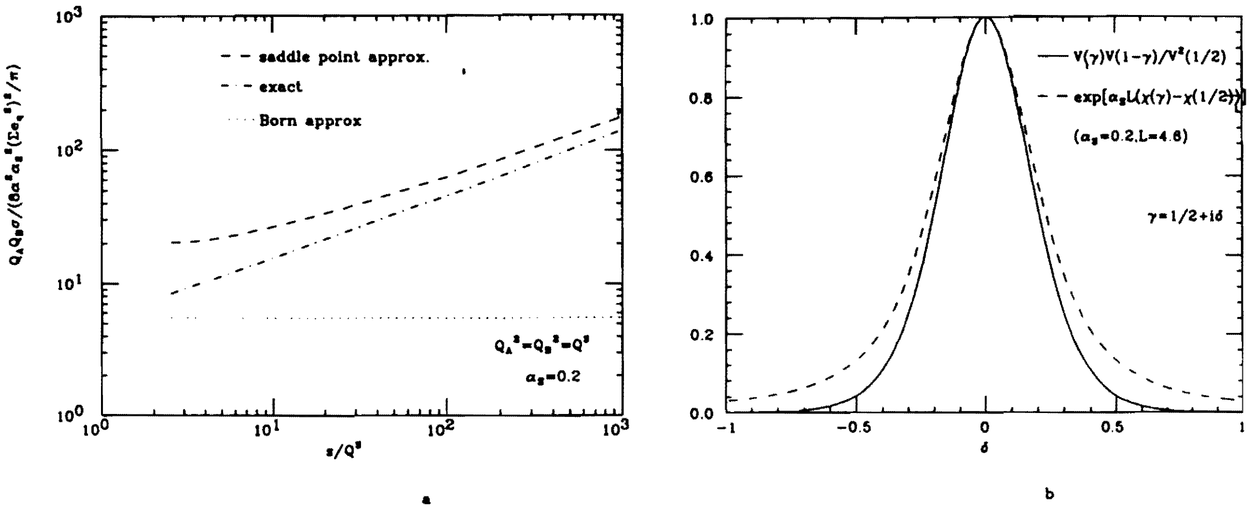

Figure 1: (a) The $\gamma^* \gamma^*$ cross section at high energies; (b) the BFKL pomeron factor and the wave function factor (normalized to the saddle point) along the integration contour.

where $\bar{\alpha}_S = \alpha_S C_A/\pi$ ($C_A = 3$), and χ and V are known functions of γ. Their explicit expressions can be found in Ref. [5]. χ is a universal function that characterizes the BFKL pomeron, whereas V is associated with the virtual photon wave function.

The dependence of the resummed $\gamma^* \gamma^*$ cross section on the energy s and the photon virtualities is perturbative, and can be read from Eqs. (2),(3). Fig.1a shows the s/Q^2-dependence of σ for $Q_A = Q_B = Q$ and a fixed value of α_S. In the asymptotic limit $s \gg Q_A Q_B$ the γ-integral is dominated by saddle point approximation around $\gamma = 1/2$. Then the cross section rises like s^λ, with $\lambda = \bar{\alpha}_S \chi(1/2)$, and with a normalization pre-factor controlled by the squared value of the function $V(\gamma)$ at $\gamma = 1/2$. The corrections to the saddle point approximation can be mainly traced back to the fact that the function $V(\gamma)$ itself is rather sharply peaked around $\gamma = 1/2$. This is illustrated in Fig. 1b, where we see that for $\alpha_S = 0.2$ and $s/Q^2 = 10^2$ the width of the pomeron factor $\exp[\bar{\alpha}_S L \chi(\gamma)]$ is still not small compared to the width of the wave function factor $V(\gamma)\,V(1-\gamma)$. This effect accounts for most of the shift in the normalization of the cross section between the asymptotic and exact evaluation of the leading logarithmic sum.

The photon-photon cross section can be measured in e^+e^- collisions in which the outgoing leptons are tagged. The cross section for the electron-positron scattering process is obtained by folding the $\gamma^* \gamma^*$ cross section with the flux of photons from each lepton. To get an estimate of the cross section available to study BFKL effects at $e^+ e^-$ colliders, we consider the rate

$$\sigma = \int_{\mathcal{R}} \frac{dx_A}{x_A} \frac{dx_B}{x_B} \frac{dQ_A^2}{Q_A^2} \frac{dQ_B^2}{Q_B^2} \times \left(\frac{\alpha}{2\pi}\right)^2 x_A P_{\gamma/e^+}(x_A)\; x_B P_{\gamma/e^-}(x_B) \times \sigma_{\gamma^*\gamma^*}(x_A\, x_B\, s, Q_A^2, Q_B^2)\,. \qquad (4)$$

Here x_A, x_B are the longitudinal momentum fractions carried by the photons, P is the flux factor, and the phase space region $\mathcal{R}$ is determined by the following cuts:

i) $Q_A > \overline{Q}$, $Q_B > \overline{Q}$ with $\overline{Q}$ being of the order of a few GeV for the process to be dominated by the perturbative contribution;

ii) $x_A\, x_B\, s > c\, Q_A Q_B$ with c being a parameter large enough in order for the high energy approximation to be valid (see below).

Note that the equivalent photon approximation, on which Eq. (4) is based, is expected to work fairly well in the region of photon virtualities that we are interested in, that is, $Q^2 \gtrsim 1\,\mathrm{GeV}^2$ and much smaller than the total energy. The $e^+ e^-$ cross section also receives contribution from lon-

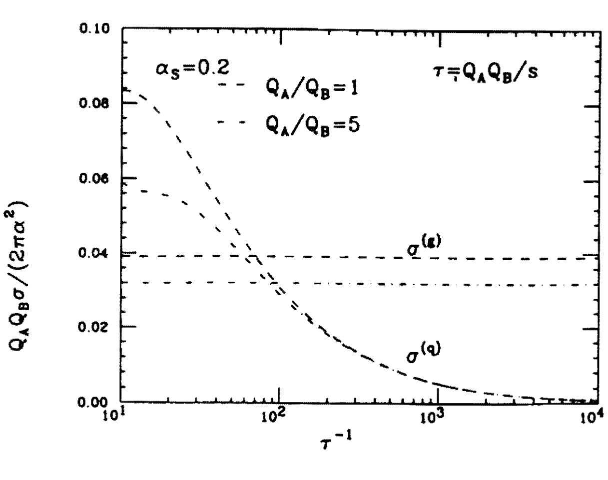

Figure 2: Lowest order contributions to the $\gamma^*\gamma^*$ cross section from gluon exchange ($\sigma^{(g)}$) and quark exchange ($\sigma^{(q)}$).

gitudinally polarized photons, which are not included in Eq. (4) and in the numerical estimates that follow. Calculations in Refs. [5,6] indicate that they contribute an additional correction of the order of about 30 % to the cross section.

An estimate of the value to be chosen for the parameter c can be obtained by comparing the gluon exchange calculation with contributions that are suppressed by a power of s. To do this, we consider the leading order (electromagnetic) contribution to $\gamma^*\gamma^* \to q\,\bar{q}$, occurring via quark exchange ($\sigma^{(q)} \sim 1/s$). In Fig. 2 we plot the corresponding cross section as well as the constant asymptotic value of the lowest order contribution to the cross section (3) in the high energy region. We see that at energies $\sqrt{s}$ about ten times bigger than the photon masses the result from quark exchange goes below that value. This gives an estimate of when the gluon exchange mechanism may start to dominate the cross section. We thus take c to be a number of order 10^2.

To compute the integrated rate (4), we must specify the scale μ^2 at which the coupling α_S is evaluated in the formula (3) for the $\gamma^*\gamma^*$ cross section. A reliable determination of this scale requires a next-to-leading order calculation, which is not available at present. We simply choose the value $\mu^2 = e^{-5/3}\,Q_A\,Q_B$ corresponding to the mean logarithmic virtuality for the gluon propagator, according to the prescription of Ref. [9]. Different choices of this scale are possible. These, as well as corrections of order $\mathcal{O}(\alpha_S)$ to the logarithmic series in Eq. (3), would affect the prediction in the next-to-leading order, which is beyond the present theoretical accuracy. Given the uncertainty of the leading logarithmic approximation, one should regard the numerical results that follow as being accurate only at the order of magnitude level.

Performing the integrations in Eq. (4) with this choice of μ^2 and with the cuts $\overline{Q}^2 = 10\ \mathrm{GeV}^2$, $c = 10^2$, we obtain $\sigma \simeq 0.6\,\mathrm{pb}$ at LEP200 energies ($\sqrt{s} = 200\,\mathrm{GeV}$), and $\sigma \simeq 3\,\mathrm{pb}$ at the energy of a future next linear collider ($\sqrt{s} = 500\,\mathrm{GeV}$) [5]. While this looks rather marginal at LEP200 (about 300 events for a value of the luminosity $L = 500\,\mathrm{pb}^{-1}$), it appears that at a next linear collider measuring off-shell photon scattering could be a viable way of studying QCD pomeron effects (about 10^5 events for $L = 50\,\mathrm{fb}^{-1}$). Similar conclusions have been reached in the analysis of Ref. [6].

It will be of interest to compare experimental results against, on one hand, the perturbative predictions for large Q_A and Q_B and, on the other hand, a simple Regge model that should apply for small Q_A and Q_B. For on-shell photons, Regge factorization gives $\sigma_{\gamma\gamma} \approx \sigma_{\gamma p}\,\sigma_{\gamma p}/\sigma_{pp}$. Assuming the values $\sigma_{\gamma p} \approx 0.1$ mb, $\sigma_{pp} \approx 40$ mb, one gets $\sigma_{\gamma\gamma} \approx 250$ nb. For virtual photons with small Q_A and Q_B, the fall-off of the cross section can be estimated from vector meson dominance, as follows

$$\sigma_{\gamma^*\gamma^*} \sim \left(\frac{M_\rho^2}{M_\rho^2 + Q_A^2}\right)^2 \left(\frac{M_\rho^2}{M_\rho^2 + Q_B^2}\right)^2 \sigma_{\gamma\gamma} \ . \tag{5}$$

At large photon virtualities (of the order of a few GeV or bigger) the cross section should go over to the perturbative scaling behaviour in Eq. (3). Note that the large Q_A, Q_B cross section goes down like the square of the photon mass scale, whereas the extrapolation of the vector meson dominance formula (5) to large photon virtualities would give the fall-off $\sigma_{\gamma^*\gamma^*}/\sigma_{\gamma\gamma} \sim (M_\rho^2)^4/(Q_A^2\,Q_B^2)^2$.

Fig. 3 shows a log-log plot of the curves corresponding to the soft and the perturbative formulae (in lowest order) for the Q^2-behaviour of the cross section. The intermediate region in the photon virtualities (Q of the order of 1 GeV) is where the

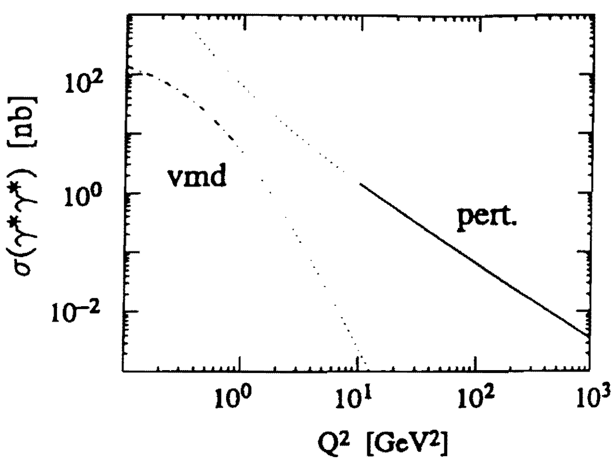

Figure 3: Q^2-behaviour of the vector meson dominance and perturbative cross sections in lowest order, with $Q_A^2 = Q_B^2 = Q^2$.

transition from one regime to the other is expected to occur. The mechanism through which this happens is not theoretically under control at present. It seems likely that experiments at LEP200 will be limited by luminosity to the range of fairly low Q^2 in this graph. However, with a future higher energy linear collider one may be able to explore to higher Q^2 and thus probe experimentally the effects of pomeron exchange in the region where summed perturbation theory should apply.

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