To be published in
Quark-Gluon Plasma 2,
edited by R.C. Hwa,
(World Scientific, Singapore, 1995)

OITS-570

FERMILAB
LIBRARY

Fermilab Library
0 1160 0047491 0

OITS-570

SCALING PROPERTIES OF QUARK-HADRON PHASE TRANSITION

RUDOLPH C. HWA

Institute of Theoretical Science and Department of Physics
University of Oregon, Eugene, OR 97403, USA

CONTENTS

1. **Introduction**

2. **Scaling Behaviors in the Ginzburg-Landau Theory**
   - 2.1 Factorial Moments
   - 2.2 Second-Order Phase Transition
   - 2.3 Experimental Verification of ν
   - 2.4 Inhomogeneity
   - 2.5 First-Order Phase Transition
   - 2.6 Remarks

3. **Cluster Production in the Mixed Phase**
   - 3.1 The Problem
   - 3.2 Cluster Growth in Heavy-Ion Collisions
   - 3.3 Self-Organized Criticality
   - 3.4 Cellular Automaton
   - 3.5 Scaling Behavior
   - 3.6 Modified CA
   - 3.7 Phenomenology

4. **General Comments**

References

NAME | I.D. No. | M.S.

1. Introduction

The problem of identifying unambiguous signatures of quark-gluon plasma is usually centered around the primordial state, when the quarks are deconfined and presumed thermalized.[1] The complementary problem concerning the final stage of the plasma, as it undergoes phase transition to hadrons, has been largely ignored, except in theoretical studies by lattice gauge calculations on the nature of the phase transition.[2] The usual explanation is that one should investigate the system while it is hot, dense, and distinctive, using leptons and photons as probes. By the time hadrons are formed, the plasma will have cooled so much that all of its characteristic properties will have disappeared. The theme of this review is to present the opposite view.

Despite many years of study of the various leptonic and photon signals from the early stages of the plasma, there has not been any that are free of ambiguities due to competing processes. In quark-hadron phase transition (PT), on the other hand, there are behaviors that are unique, whose signatures cannot be mimicked by any other process. Possible drawbacks are that those signatures may be too weak to be easily identified and that the observed behaviors, even if successfully identified, are not characteristic of the plasma in its robust state. Those reservations, though not unreasonable, deserve some comments.

The signatures in question are associated with the large-scale fluctuations expected in a PT. They are carried by the hadrons produced at the end of the life time of the plasma. If those hadrons form a gas surrounding the plasma, then the signals of PT are likely to be significantly reduced by the thermalization process in the hadronic gas. However, the existence of such a hadron gas has not been proven, theoretically or experimentally, for nuclear collisions at very high energies, such as at LHC, which is where the baryon-free central region is most likely to be found. If the PT is followed by free streaming, then the hadrons are the couriers of the physics of the transition. Whatever the real situation may turn out to be, theoretical studies of the hadronic properties should in any case be pursued and suitable observables proposed so that one knows what to expect under the best of circumstances. Without such knowledge one would not even know what signal is weakened by thermalization, if indeed there is a sheath of hadron gas around the plasma.

As for the second issue concerning the plasma in its waning rather than primordial state, the former is not any less interesting than the latter. If condensed-matter physics is to serve as a guide, critical phenomenon has been one of the most vibrant areas of physics in modern times. The extension of that study to the quark-hadron interface is not only very natural, but imperative. Universal scaling behaviors are common when condensed matter undergo second-order phase transition. Do they exist in heavy-ion collisions? If so, what are they? What observable hadronic properties does the system exhibit, if the quark-hadron PT is first order? Clearly, a quark

and gluon system in transition to hadrons involves many aspects of physics that are interesting, but largely unexplored; in that sense the death of a plasma offers as much insight as when it is young, if not more so. The detection of any of the characteristic features of PT is enough to imply that a quark-gluon plasma has existed.

In this paper the attempts thus far to find answers to the above questions are reviewed. One of the primary problems in the phenomenology of quark-hadron PT is the search for appropriate observables. Quantities such as hadron correlations are obviously important. At the microscopic level lattice gauge calculations cannot treat global properties adequately, while at the macroscopic level hydrodynamical calculations treat average quantities with deliberate intention to ignore the fluctuations. What is needed is a treatment of the problem at an intermediate level that can relate some generally accepted description of PT to some hadronic observables that can convey the special characteristic of critical behavior.

In Sec. 2 we review the studies done in the framework of the Ginzburg-Landau (GL) description of PT.[3] Both second and first order PT will be considered, in that order. Experimental verification of the theoretical prediction will be presented, not in heavy-ion collisions (for which the appropriate experiments are still some years in the future), but in quantum optics. In Sec. 3 a very novel approach to quark-hadron first-order PT will be discussed. Thermal equilibrium will not be assumed, and cellular automata will be used to simulate the transition process in detail. What emerges is a behavior that can be related to self-organized criticality. It is a connection that is very new in the physics of heavy-ion collisions.

In both approaches to the problem scaling behaviors of different observables are found; furthermore, they are universal in that they are insensitive to the details of the experimental conditions under when the plasma is created. It is this universality that provides the hope that the scaling behaviors are generic in the physical problem and can be used as signatures of quark-gluon plasma.

2. Scaling Behaviors in the Ginzburg-Landau Theory

Among the formalisms that have been developed in the past for phase transition, the one most suitable for hadron production is the phenomenological theory of Ginzburg and Landau (GL),[3] which correctly described the behavior of superconductivity before the emergence of the BCS theory. The application of the GL theory to particle production was first considered over twenty years ago,[4] and then revived in more recent years by several groups [5,6,7], each for a different purpose, and none for a quantitative prediction of multiplicity fluctuations related to PT. In a series of papers initiated by Ref. 8 the focus was placed on the scaling properties of factorial moments F_q of the multiplicity distribution derived in the GL formalism. It was found not only that F_q possesses an unusual scaling behavior, but also that the behavior is similar for both first and second order PT, apart from some numerical difference in the scaling exponent. The integration into a common description of observable behaviors for both orders of PT is unusual, but may not be limited to heavy-ion collisions. It is

this line of investigation that we review in this section.

2.1. Factorial Moments

The use of normalized factorial moments F_q to study multiplicity fluctuation in varying scale of resolution was originally suggested by Białas and Peschanski. [9] If the phase space in which the particles are detected is partitioned into M cells, each of size δ^d where d is the dimension of the space, then F_q for particles detected in any one of the cells is defined by

$$F_q = f_q / f_1^q \quad , \tag{1}$$

where

$$f_q = \sum_{n=q}^{\infty} \frac{n!}{(n-q)!} P_n \quad . \tag{2}$$

Here n is the number of particles detected in the cell in an event and P_n is the probability distribution for n, determined experimentally by averaging over many events. For improved statistics F_q can also be averaged over all bins

$$\langle F_q \rangle_M = \frac{1}{M} \sum_{j=1}^{M} F_{q,j} \tag{3}$$

where j refers to the jth cell. In the theoretical consideration below the averaging in (3) will not be done explicitly on the assumption that P_n is independent of j.

An important property of F_q, noted in Ref. 9, is that if P_n is a convolution of the dynamical distribution D with the Poissonian $\mathcal{P}_n$, i.e.,

$$P_n = \int dt\, \mathcal{P}_n(t)\, D(t) \,. \tag{4}$$

where

$$\mathcal{P}_n(t) = \frac{t^n}{n!} e^{-t} \quad , \tag{5}$$

then f_q is just the simple moment of $D(t)$:

$$f_q = \int dt\, t^q\, D(t) \,. \tag{6}$$

Thus F_q is an effective measure of the fluctuation of $D(t)$. Power-law dependence of F_q on the cell size δ has been referred to as intermittency,[9] found theoretically in branching processes,[10,11] and observed experimentally in e^+e^-, lh, hh and AA collisions.[12]

The question here is whether F_q exhibits special features, when quark-gluon plasma is created in heavy-ion collisions and the hadrons detected have gone through

quark-hadron PT.

2.2. Second-Order Phase Transition

In the GL theory of second-order PT the free energy of the system is $F[\phi] = \int d^d r \mathcal{F}[\phi]$, with the free-energy density being

$$\mathcal{F}[\phi] = a\,|\phi(\mathbf{r})|^2 + b\,|\phi(\mathbf{r})|^4 + c\,|\partial\phi/\partial\mathbf{r}|^2 \tag{7}$$

where ϕ is the order parameter; a, b and c are parameters, the latter two of which are positive and nearly constant in the neighborhood of the critical temperature T_c, but a changes sign as

$$a = a'(T - T_c) \tag{8}$$

with $a' > 0$. In applying this formalism to quark-hadron PT, the geometrical space of a cylindrical plasma, when expressed in terms of the spatial rapidity $\eta = \frac{1}{2}\ln\left[(t+z)/(t-z)\right]$, azimuthal angle φ, and radius r, is mapped to the momentum space in the variables: $y = \frac{1}{2}\ln\left[(p_0 + p_3)/(p_0 - p_3)\right]$, φ, and p_T, so that experimental cells in the momentum space can be related implicitly to the r-cells in (7).[13] If a subset of the three variables is considered, the dimension d is then reduced from 3 accordingly.

In identifying $\phi(\mathbf{r})$, it is assumed that the system is in a pure coherent state $|\phi\rangle$, and that $\phi(\mathbf{r})$ is the corresponding eigenvalue of the annihilation operator for the hadrons. In that way the average hadron multiplicity in a volume V becomes [13]

$$\langle n\rangle_V = \int_V d^d r\,|\phi(\mathbf{r})|^2 \tag{9}$$

so that $|\phi(\mathbf{r})|^2$ is the hadron density at $\mathbf{r}$, which in the absence of fluctuations is zero for $T > T_c$, but is nonzero for $T < T_c$, as an order parameter should behave. This behavior of $|\phi(\mathbf{r})|^2$ is the property of (7) and (8), since the first two terms of $\mathcal{F}[\phi]$ describe a stable minimum at $\phi = 0$ when $a > 0$, but the minimum moves to

$$|\phi_0| = -a/2b \tag{10}$$

when $a < 0$. That is the behavior of a second-order PT.

It is known that a pure coherent state has a Poissonian distribution of particle multiplicity

$$\mathcal{P}_n[\phi] = \frac{1}{n!}\left(\int_V d^d r|\phi|^2\right)^n \exp\left[-\int_V d^d r|\phi|^2\right] \quad . \tag{11}$$

The observed multiplicity distribution P_n is, however, the thermodynamical average of $\mathcal{P}_n[\phi]$, when ϕ is allowed to fluctuate in accordance to what the GL free energy prescribes:

$$P_n = \frac{1}{Z}\int \mathcal{D}\phi\,\mathcal{P}_n[\phi]e^{-\int d^d r\mathcal{F}[\phi]} \quad , \tag{12}$$

where $Z = \int \mathcal{D}\phi\, e^{-\int d^d r \mathcal{F}[|\phi|]}$. Eq. (12) is the basic equation for the phenomenology of produced hadrons in the GL description of quark-hadron PT.[8]

If the last term of (7) is neglected for the moment so that $|\phi|^2$ may be regarded as independent of $\mathbf{r}$ [see Sec. 2.4 for not neglecting that term], then the integration of $\mathcal{F}[\phi]$ over all space in (12) may be restricted to the cell volume $V = \delta^d$ and becomes trivial. Thus (12) becomes a Poisson transform of the general form of (4). It then follows from (6) that

$$f_q = Z^{-1}\, \delta^{dq}\, \pi \int_0^\infty d|\phi|^2 |\phi|^{2q} e^{-\delta^d (a|\phi|^2 + b|\phi|^4)} \quad , \tag{13}$$

where the exponent in Z is also $-\delta^d\,(a|\phi|^2 + b|\phi|^4)$. The integral in (13) is of the form

$$J_q(\alpha) = \int_0^\infty dt\, t^q\, e^{2\alpha t - t^2} \tag{14}$$

where

$$\alpha = \sqrt{x/2} \quad , \qquad x = a^2\delta^d/2b \quad . \tag{15}$$

Substituting (13) into (1) yields

$$F_q = J_q J_q^{-q} J_0^{q-1} \quad , \tag{16}$$

from which follows the important result that $F_q(x)$ is a function of x only, instead of depending on a, b, d, and δ, separately. In a heavy-ion collision the temperature decreases as the system expands, so the values of a and b would vary with time accordingly, but not in ways known in detail theoretically. Fortunately, the dependence of F_q on x alone renders unnecessary the determination of a and b. It is straightforward to calculate $F_q(x)$ for $x > 0$ corresponding to $T < T_c$ with hadron condensate. It turns out that $F_q(x)$ does not have a power-law dependence on x. [8,13] The subject would have been dropped as uninteresting, if it were not for an observation that revealed a simple and elegant property of F_q.

By partial integration of (14), $J_q(\alpha)$ can be shown to satisfy the recursion relation

$$J_q(\alpha) = \alpha J_{q-1}(\alpha) + \frac{q-1}{2} J_{q-2}(\alpha) \quad , \tag{17}$$

while by explicit evaluation one has

$$J_0(\alpha) = \frac{\sqrt{\pi}}{2} e^{\alpha^2} (1 + \mathrm{erf}\alpha) \quad , \tag{18}$$

$$J_1(\alpha) = \frac{1}{2} + \alpha J_0(\alpha) \quad . \tag{19}$$

Furthermore, (14) also implies

$$\frac{dJ_q(\alpha)}{d\alpha} = 2J_{q+1}(\alpha) \quad . \tag{20}$$

Thus one can calculate from (16) the function $\beta_q(\alpha)$, defined by

$$\beta_q(\alpha) = \frac{d \ln F_q}{d \ln F_2} \quad , \tag{21}$$

by taking the ratio of $d \ln F_q/d\alpha$ to $d \ln F_2/d\alpha$. The result is shown in Fig. 2.1.[13] Evidently $\beta_q(\alpha)$ is insensitive to the value of α. In a plot of $\ln F_q$ vs $\ln F_2$ the small variations of the local slopes would not be perceptible globally, and a straight-line fit for each q would be excellent. This is shown in Fig. 2.2. Thus there exists a power-law behavior [8]

$$F_q \propto F_2^{\beta_q} \quad , \tag{22}$$

which we shall refer to as F-scaling. This behavior can be checked by varying δ, the only variable under experimental control.

From Fig. 2.1 one can examine the dependence of β_q on q and α. It is an amazing fact that $\beta_q(\alpha)$ satisfies very accurately the formula

$$\beta_q(\alpha) = (q-1)^{\nu(\alpha)} \tag{23}$$

where the values of $\nu(\alpha)$ fall within the narrow range

$$\nu = 1.305 \pm 0.010 \tag{24}$$

for $0.3 < \alpha < 2$. Since the variation of ν is less than 0.8%, it is justified to regard ν as a constant and refer to it as the scaling exponent with the value $\nu = 1.304$.[8] The fit of β_q by $(q-1)^\nu$ is shown in Fig. 2.3, in which the values of β_q determined from the slopes in Fig. 2.2 are shown as dots.[8] The data from heavy-ion collisions [14,15] are also shown for comparison; the corresponding value of ν is around 1.55. It is reasonable to conclude from that figure that current data do not suggest any PT, at least not of the type considered here.

We see that from the general properties of GL follows a universal scaling behavior, where ν is independent of a, b, d and δ. The universality makes the result particularly attractive, since experimentally there is no way to control the temperature around the critical point, and theoretically there is no knowledge of the values of the GL parameters. The most interesting aspect of the result is that the input is totally general, as the GL theory is applicable to a wide variety of physical processes, the heavy-ion collisions being only a very late comer. The F-scaling property has escaped notice for forty years, and the numerical value of ν could have been discovered long ago.

2.3. Experimental Verification of ν

The use of $\nu = 1.304$ as a criterion for quark-hadron phase transition is a novel variation from the signature of the primordial plasma. So far we can only show that it is a necessary consequence of the GL type PT, but not a sufficient condition. On

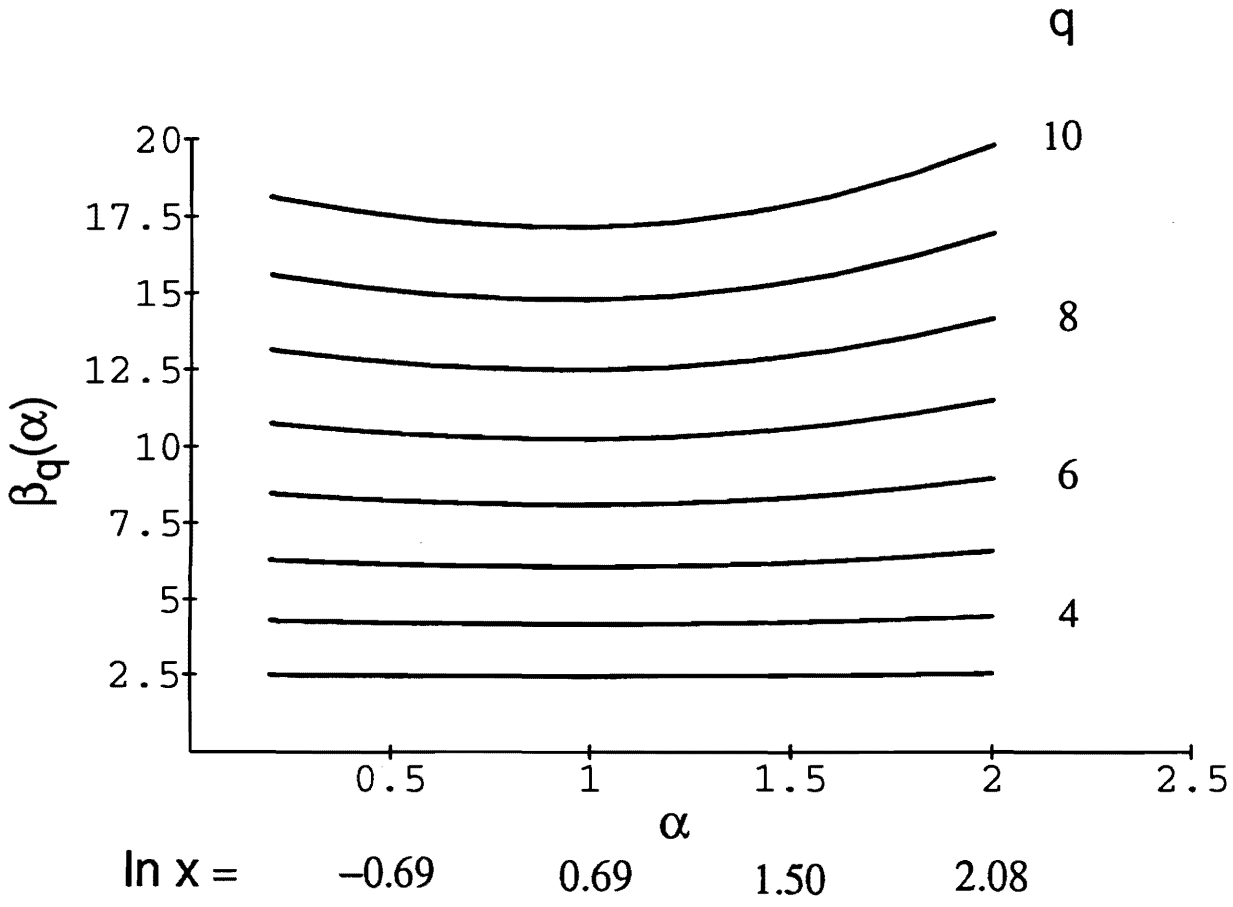
q
10
8
6
4
20
17.5
15
12.5
10
7.5
5
2.5
$\beta_q(\alpha)$
0.5
1
1.5
2
2.5
α
ln x =
−0.69
0.69
1.50
2.08

Fig. 2.1

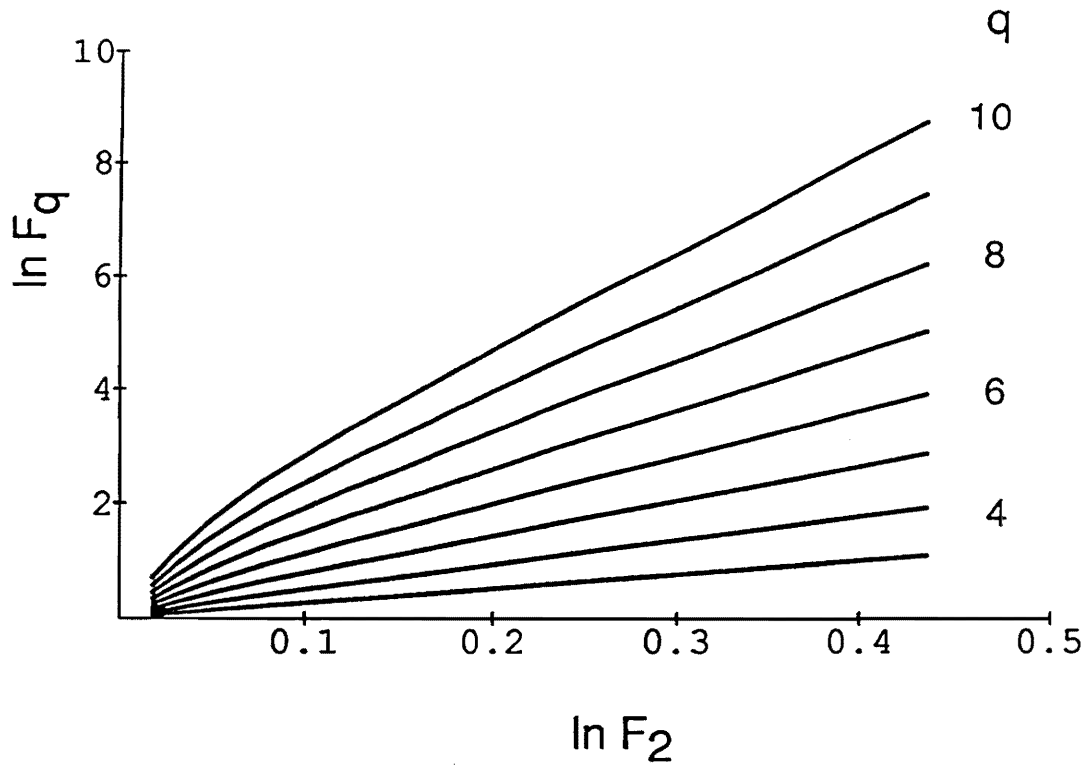
q
10
8
6
4
ln F_q
10
8
6
4
2
0.1
0.2
0.3
0.4
0.5
ln F_2

Fig. 2.2

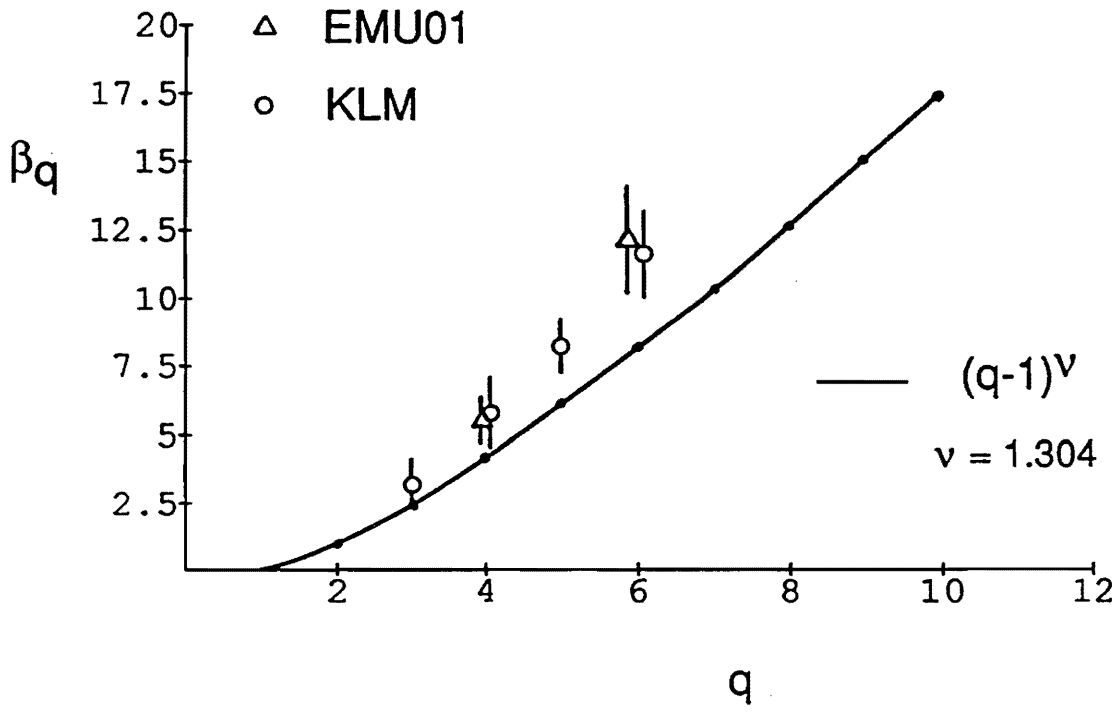
EMU01
KLM
β_q
20
17.5
15
12.5
10
7.5
5
2.5
$(q-1)^\nu$
$\nu = 1.304$
2
4
6
8
10
12
q

Fig. 2.3

the experimental side the verification of the theoretical prediction on ν for heavy-ion collisions is not imminent, since the best chances are for a baryon-free region where the GL description may be adequate for the simple hadronization process. That would require much higher energy, such as at CERN-LHC currently under planning. However, since, as we have noted, the GL theory is very general, there should be other areas of physics that exhibit second-order PT, where F-scaling should be looked for and the value of ν checked.

Such an area was indeed found in quantum optics. It has been known for a long time that the threshold behavior in lasers and nonlinear optics involves the onset of instabilities of certain modes of oscillation in the optical system. These instabilities near threshold are intimately related to the general phenomena of phase transitions. Indeed the phenomenology of threshold behavior in the single-mode laser is described very well by the GL theory of second-order PT.[16] It is therefore highly pertinent to investigate photon number fluctuations in lasers at the threshold of lasing and check whether F-scaling exists and what the scaling exponent may be. In contrast to heavy-ion collisions where experimental control of the temperature is lacking, the photon count problem in laser transition has a pump parameter that is under excellent control in the experiment.

Without entering into a discussion of laser physics here, it is only necessary to state that the steady-state distribution for the complex field amplitude E of the single-mode laser is given by [17]

$$P(E) = \frac{1}{Z_p} e^{-U(E)} \quad , \tag{25}$$

where Z_p is the normalization constant, and

$$U(E) = -\frac{1}{2} p |E|^2 + \frac{1}{4} |E|^4 \quad . \tag{26}$$

Here p is the pump parameter that characterizes the operating point of the laser, $p < 0$ below threshold, and $p > 0$ above. In a photoelectric counting experiment the photocount distribution $P_n(\tau)$ is measurable, and gives the probability of detecting n photons in time τ. If the counting interval τ is short compared to the correlation time of the laser, $P_n(\tau)$ is given by [18]

$$P_n(\tau) = \int dE\, dE^* \frac{(\eta\tau |E|^2)^n}{n!} e^{-\eta\tau |E|^2} P(E) \quad , \tag{27}$$

where η is the detector efficiency.

In a laser experiment it is possible to work with a homogeneous system. Thus (26) fits in exactly the general form of (7) with the gradient term not contributing. In that sense lasing is a second-order PT from no photon production below threshold to photon production above threshold. Since the photocount distribution $P_n(\tau)$ in (27) is perfectly analogous to the hadron multiplicity distribution P_n in (12), the laser

experiment provides an easier way to check F-scaling and the scaling exponent ν. There are some experimental complications involving detector dead-time corrections and background corrections, for the details of which the reader is referred to Ref. 19, where the first experiment on the subject was reported. Instead of varying the time interval τ, which is the counter part of δ in heavy-ion collisions, the pump parameter p is varied, since F_q depends on only one overall variable x [see (15)]. The result on F_q is shown in Fig. 2.4, which clearly verifies F-scaling. From the straight-line fits of the data points the slopes β_q are shown in Fig. 2.5; they agree remarkably well with the theoretical prediction $\beta_q = (q-1)^\nu$, $\nu = 1.304$, represented by the solid line.

The confirmation of the F-scaling and the scaling exponent ν, which is independent of the pump parameter p and the detector efficiency η, is not only an advance in laser physics, suggesting the possibility of similar behavior in other more complicated laser systems, hitherto overlooked. It gives crucial support to the proposal that the scaling behavior of F_q can be used as a diagnostic test of whether a second-order quark-hadron PT exists in heavy-ion collisions.

2.4. Inhomogeneity

The considerations in the preceding two subsections treat only the case where the $c|\partial\phi/\partial\mathbf{r}|^2$ term in (7) can be neglected. While that can be arranged in the laser problem by considering a homogeneous laser, it is necessary to understand the effect of that term in the general case. In the following we give first a quick way of seeing its effect without detailed calculation, and then the results of an actual calculation based on mean-field type approximation.

It is well known that when spatial dependence is introduced into the problem by the gradient term, the correlation length is

$$\xi = (c/|a|)^{1/2} \tag{28}$$

If $b = 0$, the correlation function behaves as

$$C(\mathbf{r}_1 - \mathbf{r}_2) \propto e^{-|\mathbf{r}_1-\mathbf{r}_2|/\xi}/|\mathbf{r}_1 - \mathbf{r}_2| \quad . \tag{29}$$

To estimate the effect of this correlation, let a drastic approximation be made to simplify the consideration. Let $C(\mathbf{r}_1 - \mathbf{r}_2)$ be represented by a function shaped as a step-function, such that all points separated by a distance less than ξ are fully correlated; otherwise, there is zero correlation. Thus in the discretized form of the functional integral in (12), where the $\mathbf{r}$-space is partitioned into M cells of size δ^d, all ϕ_i are required to be equal for any cell i within a block of size ξ^d centered at the cell under study. All cells not in the block give rise to contributions that cancel between the numerator and denominator of (12), and need not be considered any further. This approximation of strong correlation for $|\mathbf{r}_i - \mathbf{r}_j| \leq \xi$ leads to f_q that is essentially in the same form as (13) for the $c = 0$ case, except that the δ^d factors are replaced by ξ^d. That means that for $\delta > \xi$ the effects of the gradient term may

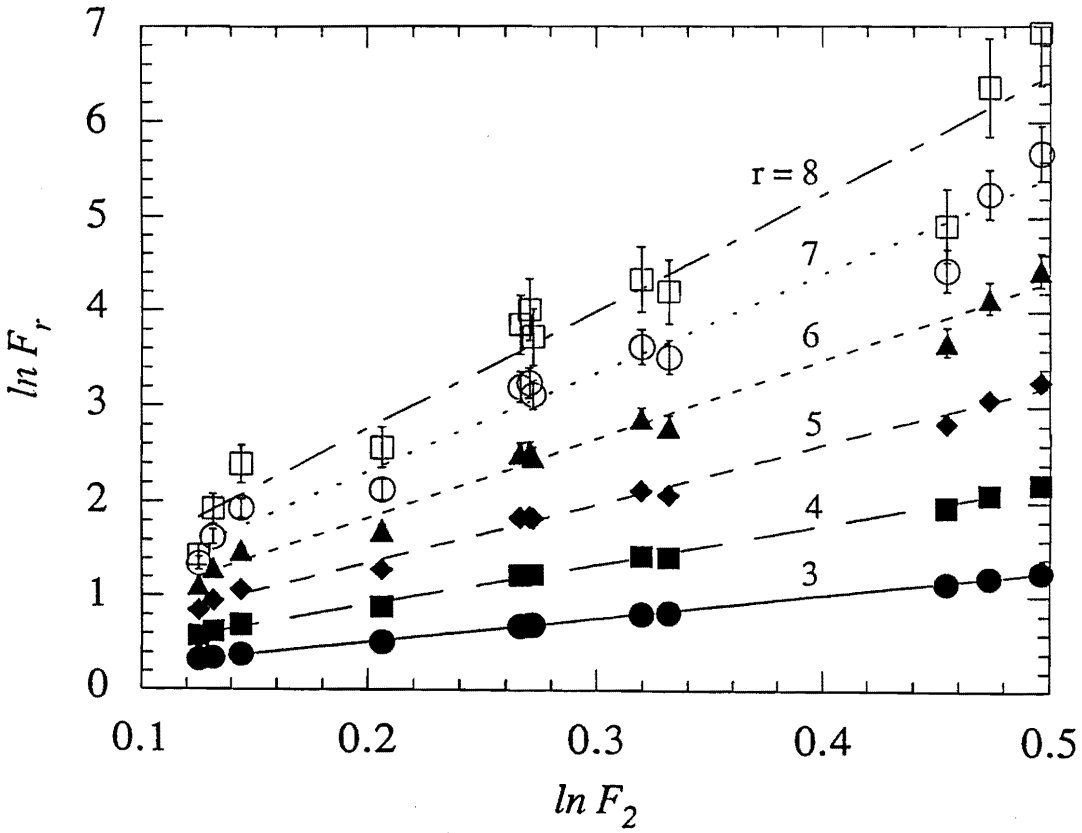
7
6
5
4
3
2
1
0
0.1
0.2
0.3
0.4
0.5
ln F_r
ln F_2
r = 8
7
6
5
4
3

Fig. 2.4

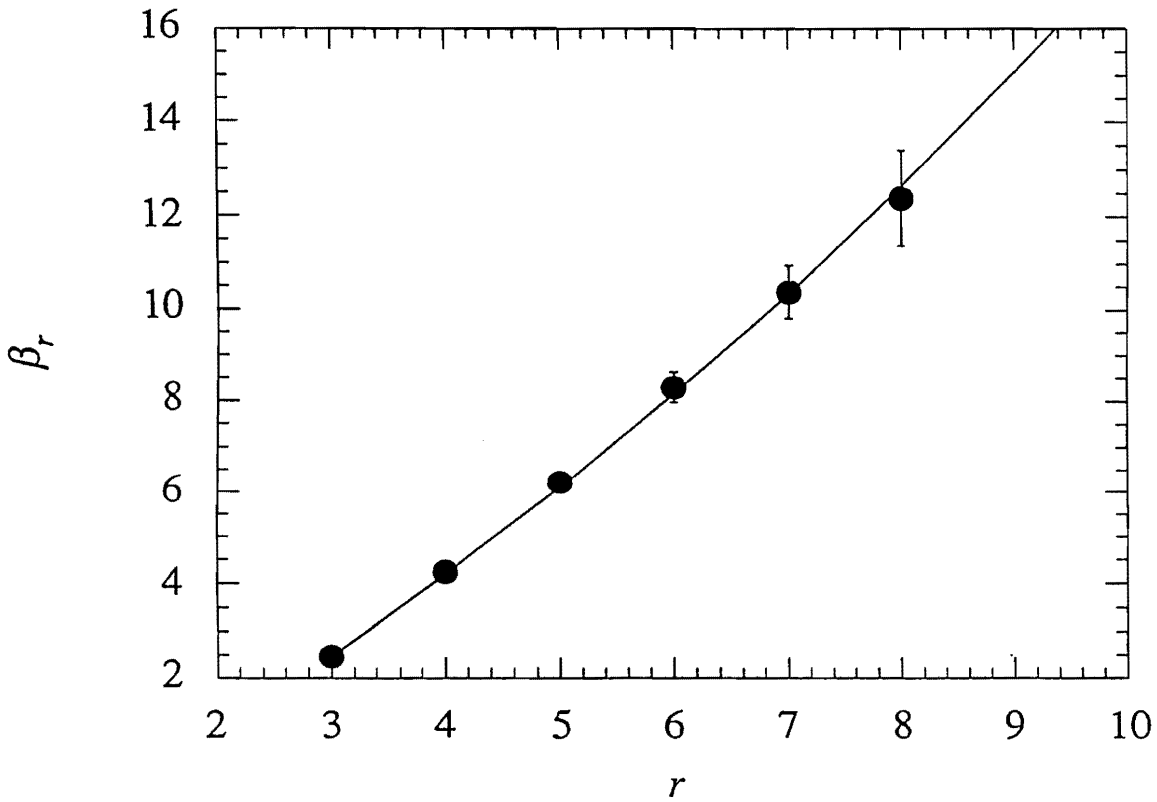
16
14
12
10
8
6
4
2
2
3
4
5
6
7
8
9
10
β_r
r

Fig. 2.5

be ignored, but when δ is decreased to below ξ, the effective cell size for the $c \neq 0$ case is equivalent to ξ^d in the $c = 0$ case. Thus the results obtained for the $c = 0$ case are still applicable, so long as δ is not allowed to become smaller than ξ. When the experimental δ is decreased below ξ, the result of the $c = 0$ case (with δ replaced by ξ), should be used. Of course, that is the physical meaning of having nontrivial correlations with a range of ξ. In terms of the x variable it means that x should not be decreased to below $x_\xi = a^2\xi^d/2b$. It turns out that excluding the $x < x_\xi$ region does not invalidate the result of Sec. 2.2, since the region that exhibits F-scaling is for larger x.[8,13] Hence, the general properties of F-scaling and the numerical value of ν are retained, when the gradient term in the free-energy density is accounted for in the approximation considered here.

A more quantitative analysis has been done by taking the gradient term into account explicitly in the calculation of F_q.[20] Since the full problem is difficult to treat, an approximation is made in the spirit of mean-field theory. The focus is on the multiplicity fluctuation from event to event in one cell only, so the $\partial\phi/\partial\mathbf{r}$ term in (7) can be treated in terms of incremental changes of ϕ in that cell relative to ϕ_j of the neighboring cells, where j labels the cell number of the neighbors. The approximation is to assign to ϕ_j the same background field ϕ_0 for all j, where $|\phi_0|^2 = -a/2b$, the value at which the GL potential has its minimum for $a < 0$. Thus the emphasis is on the fluctuation of ϕ at the cell under examination from the mean ϕ_j of the neighboring cells, rather than on the fluctuations of ϕ_j from ϕ_0 themselves. Because of the statistical factor $\exp(-F[\phi])$, fluctuations of ϕ in all cells are around the most probable value $|\phi_0|$; the derivative term $c|\partial\phi/\partial\mathbf{r}|^2$ gives a measure of the inhibition on the degree of those fluctuations. The approximation decouples all cells without losing the effect of the derivative term.

It can be shown [20] that in this approximation F_q depends on only two variables, x and γ, where x is as defined in (15), and

$$\gamma = \pi \left(\frac{cd}{|a|}\right)^{1/2} \left(\frac{a^2}{2b}\right)^{1/d} \quad . \tag{30}$$

Thus the dependence on c is transmitted to the dependence on γ. Fig. 2.6 shows that F-scaling remains to be valid for any fixed γ; moreover, the scaling behavior is insensitive to γ. If the slopes of the log-log plot are fitted by

$$\beta(\gamma) = (q-1)^{\nu(\gamma)} \quad , \tag{31}$$

the resultant scaling exponent $\nu(\gamma)$ depends on γ as shown in Fig. 2.7.[20] Evidently, the variation is mild. For $0 < \gamma < 10$, the value of ν is within the range 1.316 ± 0.012. With this result one should feel assured that the gradient term in the GL free energy can be ignored without serious effect on the scaling behavior of F_q.

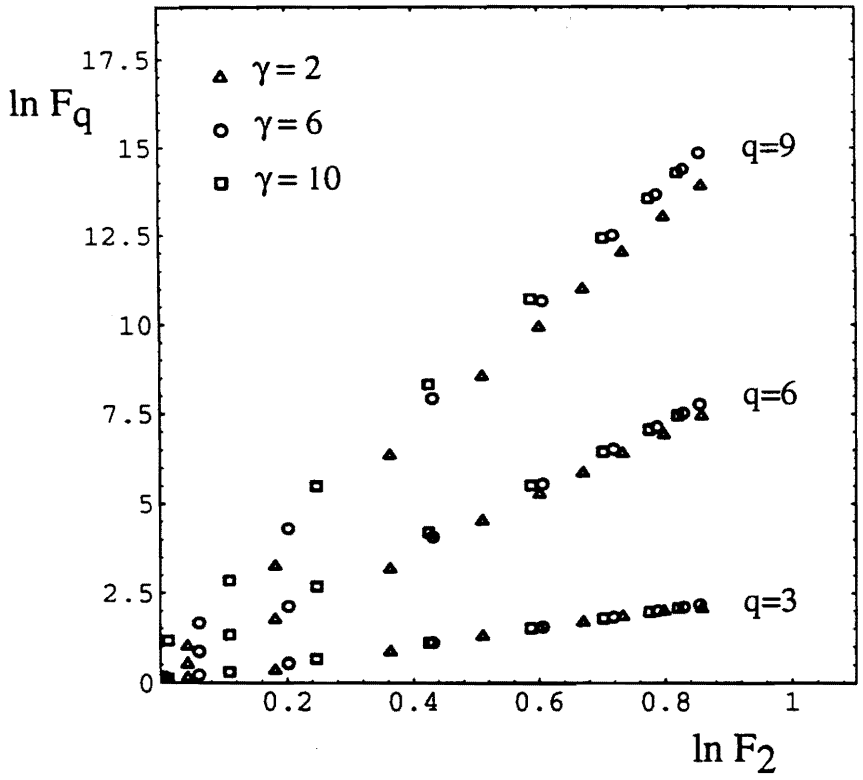
ln F_q
17.5
15
12.5
10
7.5
5
2.5
0
0.2
0.4
0.6
0.8
1
ln F_2
$\gamma = 2$
$\gamma = 6$
$\gamma = 10$
q=9
q=6
q=3

Fig. 2.6

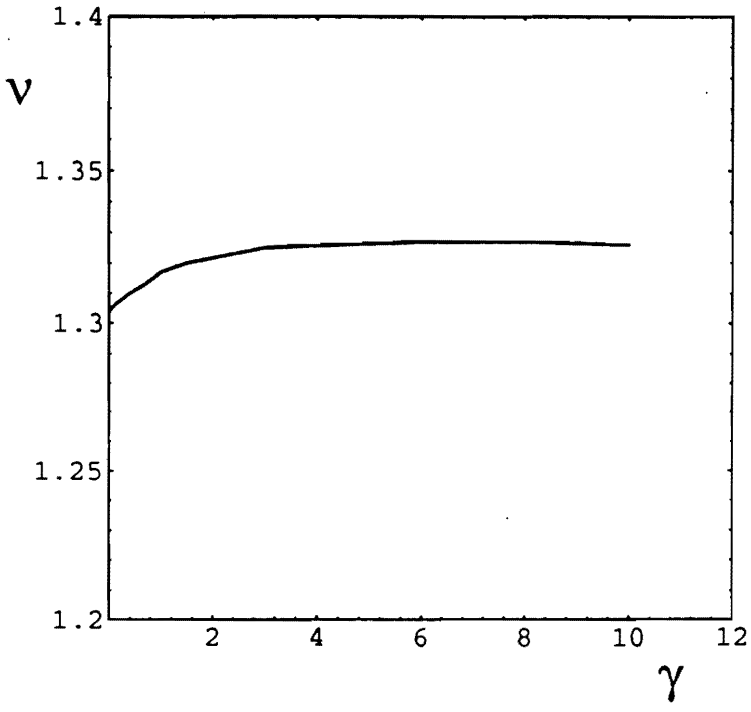
ν
1.4
1.35
1.3
1.25
1.2
2
4
6
8
10
12
γ

Fig. 2.7

2.5. First-Order Phase Transition

Lattice gauge calculations indicate that the QCD PT may be second order or a weak first order, depending upon the number of quarks in the problem.[21,22] If there are only two massless quarks, the transition is second order, but if there are three massless quarks, then it is first order. For a realistic strange quark mass the situation is somewhere in between, and may be near the tricritical point.[23] Since the GL formalism is also applicable to the study of first-order PT, it is of some interest to investigate the effect on the scaling behavior discussed above, when the PT changes from second order to first order.

For first-order PT the coefficient of the quartic term in $\mathcal{F}[\phi]$ in (7) becomes negative, so the next order term in the expansion must be considered. Leaving out the gradient term, the free energy density becomes

$$\mathcal{F}[\phi] = a|\phi|^2 + b|\phi|^4 + k|\phi|^6 \quad , \tag{32}$$

which may be rewritten in the form $\mathcal{F}[\phi] = (a^3/k)^{1/2} t f(t)$, where

$$f(t) = 1 - 2(1+g)t + t^2 \quad , \tag{33}$$

$$t = \sqrt{\frac{k}{a}}\,|\phi|^2 \quad , \qquad g = -\left(1 + \frac{b}{2\sqrt{ak}}\right) \quad . \tag{34}$$

When g is positive, i.e., $b < -2\sqrt{ak}$, $f(t)$ has a minimum lower than 0, so the minimum of $\mathcal{F}[\phi]$ jumps from $\phi = 0$ to a finite value of $|\phi|$, as g increases through zero. That corresponds to first-order PT.

The parameters a, b, and k are not known for quark-hadron PT, let alone their dependences on T. Whether hadrons are formed at T_c or below it depends on how rapidly the latent heat released can be removed; in the latter case g would be positive. In view of such complications it is sensible to regard g as a parameter and investigate the scaling behavior as a function of g.

The procedure is essentially the same as before. It is straightforward to show that [24]

$$F_q = J_q J_1^{-q} J_0^{q-1} \quad , \tag{35}$$

where

$$J_q = \int_0^\infty dt\, t^q\, e^{-xtf(t)} \quad , \tag{36}$$

$$x = \delta^d \left(a^3/k\right)^{1/2} \quad . \tag{37}$$

Thus F_q is a function of x and g only. Interestingly, the result shows that F-scaling is still valid in this case, as shown in Fig. 2.8, for $0 \le g \le 0.6$. For each value of g, the slope satisfies

$$\beta_q(g) = (q-1)^{\nu(g)} \quad , \tag{38}$$

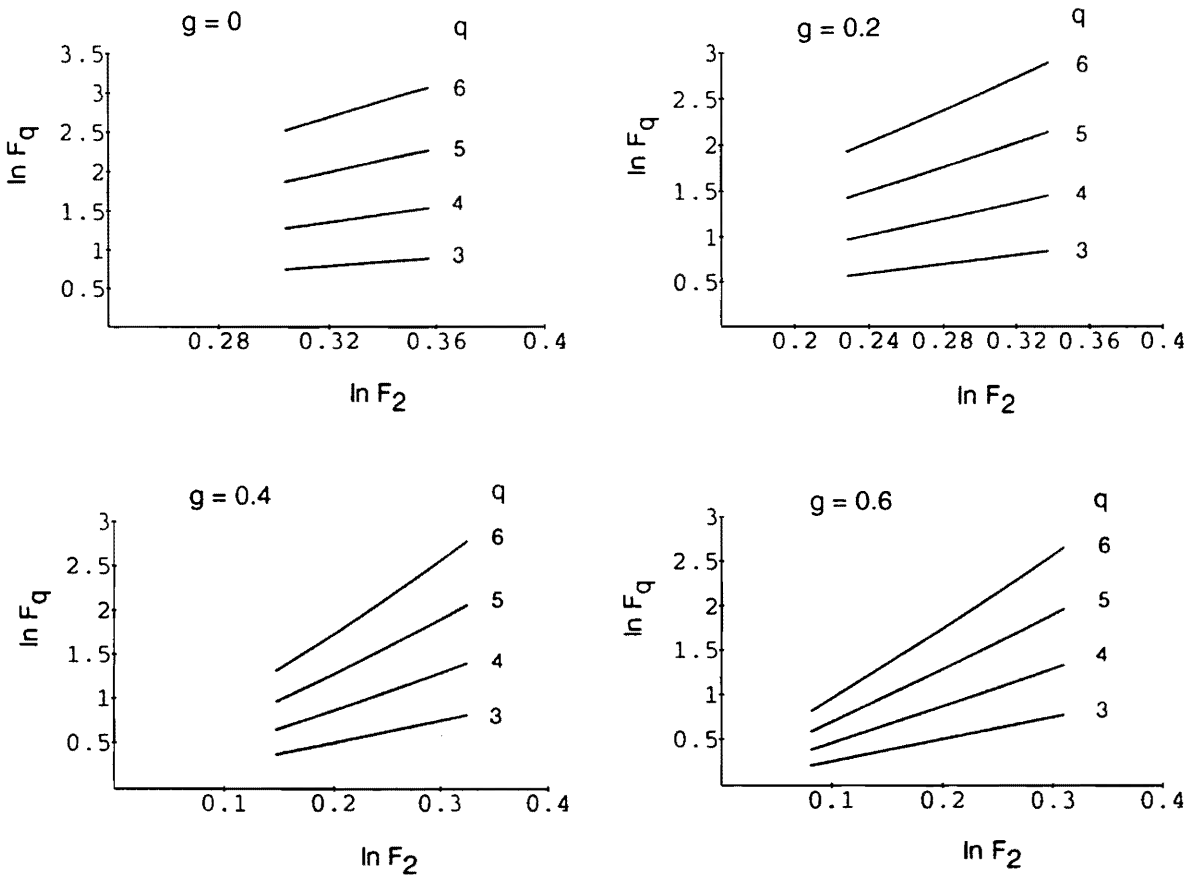

Fig. 2.8

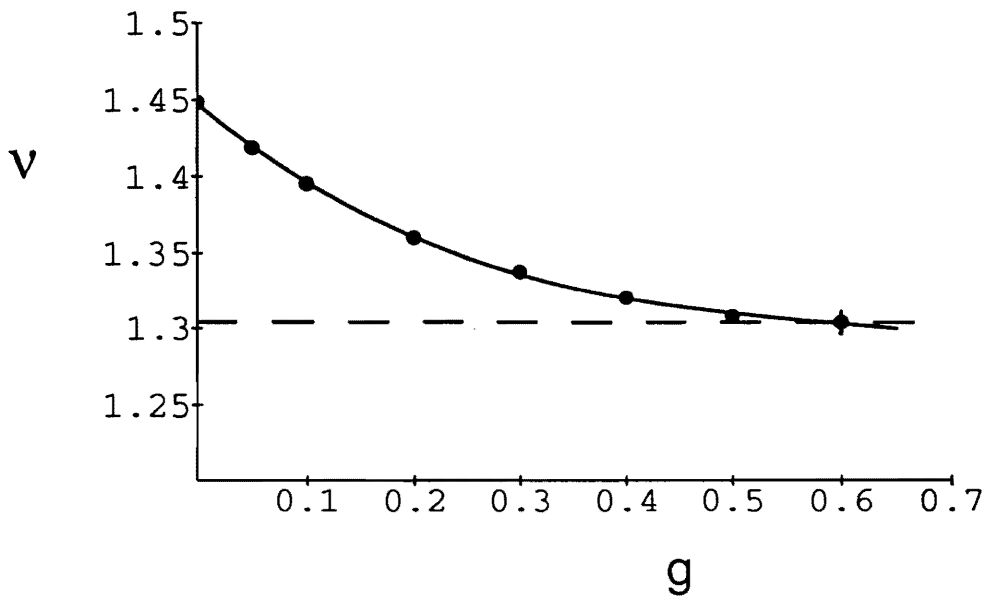

Fig. 2.9

so that the scaling result can be summarized by the g-dependence of ν, as shown in Fig. 2.9.[24] The solid line is a fit of the dots determined from the fits of $\beta_q(g)$ by (38). The dashed line is for $\nu = 1.304$.

The result is remarkable in the sense that a unifying feature about first- and second-order PT has been found. Fluctuations of particle multiplicity as measured by F_q exhibit similar scaling behaviors, and the scaling exponent ν changes from a value of 1.45 that is distinctly first-order at $g = 0$ to nearer 1.3 at higher g, a value that has been characteristic of second-order.

The subject has also been considered recently in Refs. 25-27. In Ref. 26 it is found that β_q shows no difference between first- and second-order PT, contrary to the conclusion exhibited in Fig. 2.9. That result has subsequently been retracted in Ref. 27, where agreement with Refs. 8,24 is found. For first-order PT, which requires $a > 0$ and $b < -2\sqrt{ak}$ in (32), a particular path in the a–b plane is considered in Ref. 27, namely: fixing $b < 0$ and allowing a to decrease in accordance to (8) with T kept above T_c. In following that path one encounters a discontinuity in the minimum $|\phi|$, as a is decreased beyond the point $a = b^2/4k$. Since the PT occurs at $g = 0$, the jump in the minimum of $tf(t)$ can be seen from (33) to go from $t = 0$ to $t = 1$. From (34) one sees that $|\phi|^2_{\min} = \sqrt{a/k} = -b/2k$, when that happens. Thus if $-b$ is not small, the PT may be regarded as being strong first order. However, if the path in the a-b plane is not along a fixed-b line at large negative b, but along a fixed-a line at small positive a, with b decreasing from positive to negative values, then hadronization at large g (where the solid and dashed lines in Fig. 2.9 merge) corresponds to weak first order, a scenario made possible by supercooling that leads to PT at $g > 0$.

2.6. Remarks

What have been investigated in this section are the general consequences of the GL description of PT. Nothing specific about quarks and hadrons has actually been put in, except that the form of the free-energy density $\mathcal{F}[\phi]$ can be inferred from the field theory of quarks undergoing PT. Of course, the identification of the order parameter with hadron density is crucial. Nevertheless, the formalism is sufficiently general to allow for the order parameter to be identified with the photocount rate of lasers, and immediately the subject becomes relevant to nonlinear optics at the threshold of lasing. The discovery of F-scaling in quantum optics and the verification of $\nu = 1.304$ give hope to observing the same behavior in heavy-ion collisions, when such experiments are performed at high enough energies.

The relevance of this subject can be extended to other fields. First of all, even in quantum optics it is possible to find laser systems where lasing threshold behaves like first-order PT. It is therefore of great interest to have experiments done in that field to verify the results in Sec. 2.5. If we move to free-electron lasers, which is quite unlike lasers in quantum optics, there has been no study on whether they behave as having a PT at threshold. Experimental evidence for or against F-scaling would be

very interesting. One can also go into condensed-matter physics, where PTs of all sorts are abound. Metal-insulator transition would be relevant, since one can measure the electric current at the threshold and calculated F_q. If diverse areas of physics can show F-scaling with the same scaling exponent, then we shall have found a truly universal behavior that cuts across the boundaries separating the various fields of physics.

Finally, coming back to heavy-ion collisions, our assumption has been that thermalization of quarks and gluon is achieved early in the formation of the plasma so that the temperature is a good local variable to describe the expanding and cooling system and that the GL theory is applicable at PT. However, the geometry of the problem has not been introduced into the problem in any essential way. It becomes an important aspect of the problem, if there is a mixed phase where quarks and hadrons can coexist, as in the first-order PT. The space-time complications introduced in the mixed phase cannot be handled by the GL formalism, especially if a thermalized system is a bad approximation. The problem must then be investigated by use of a totally different method, which is the subject of the next section.

3. Cluster Production in the Mixed Phase

3.1. The Problem

In the foregoing section we have considered the problem of hadron production in a thermalized quark-gluon plasma for which the GL formalism is a good description. The order parameter is related to the hadron density. The dynamical aspect of the heavy-ion collision is not emphasized. The spatial variables are integrated over in the free energy, and there is no time evolution. We now focus on the space-time properties of the PT process.

If the PT is of second order, there is no mixed phase. The part of the plasma that is cooled to T_c and below condenses to hadrons and leaves the system. It occurs at the surface of the expanding cylinder, and no complication is expected in the space-time development of the hadronization process. On the other hand, if the PT is of first order, then there is a mixed phase in which quarks (and gluons) and hadrons can coexist in a finite spatial volume over a finite period of time. What happens to those hadrons in the mixed phase as they move toward the surface of the plasma is the problem to be examined now. Their characteristics as they emerge from the plasma can provide observable clues about the nature of the PT.

It should be mentioned at the outset that the hadrons to be studied here are not the ones that reach the detector; the former are at the transition temperature, the latter at $T = 0$. What happens between the surface of the plasma and the detector is still controversial. Whether there exists a hadron gas surrounding the plasma, or there is free streaming, is not an issue that has a general consensus. What effects chiral transition has on the hadron masses are also debatable. Our attitude on those issues is totally open. They are to be addressed in time, but for now our focus is only

on the properties of the hadrons at the outer boundary of the mixed phase, i.e., on the plasma surface. After those properties are derived, some remarks will be made at the end of this section to indicate what the likely observable consequences might be.

In treating the problem, the method to be adopted shall be general enough to be applicable even to a system not in thermal equilibrium, so long as there is a region where quarks and hadrons can coexist. We shall continue to call it the mixed region, although it has a specific meaning in equilibrium statistical physics. In the treatment the notion of temperature is not a necessary ingredient. Indeed, if the quark-gluon system produced in heavy-ion collisions is not thermalized, almost all previous treatments of the subject would become invalid, but what is to be discussed below should on the whole remain valid.

3.2. Cluster Growth in Heavy-Ion Collisions

To describe the problem before posing a method of investigation, let us adopt the conventional picture for the collision (head on, say) of two high-A nuclei in that there is rapid longitudinal expansion with slower radial expansion. For the purpose of our consideration it is not important whether and how the system deviates from longitudinal boost invariance. A global description of the whole expanding system is not required. Consider only the problem of cluster production at midrapidity in a very high energy collision, preferrably at LHC, so that one may focus on a thin slice of the expanding cylinder at $y = 0$, and assume that it is a baryon-free region. Using the language of thermal equilibrium in order to set up the scenario, assume further that initially there is a plasma at high temperature, and that it cools due to longitudinal expansion. As time t evolves, the temperature profile $T(r,t)$ will change in such a way that initially T will decrease faster at larger r than at $r = 0$ due to radial diffusion. Again, the exact form of $T(r,t)$ is not crucial. However, for ease in discussing the problem assume that the usual hydrodynamical solution is approximately valid until the end of the mixed phase, i.e., for $r < r_b(t)$, where $r_b(t)$ is the outer boundary of the mixed phase, which decreases with increasing t due to refraction.[28,29,30,31] There is also an inner boundary $r_0(t)$, which also decreases with t at a faster rate, separating the mixed phase from the quark phase at $r < r_0(t)$. Our consideration here is only for the region in the annular ring $r_0 < r < r_b$, which we shall refer to as the M region, for short.[32]

The usual description of the first-order PT in the mixed phase is in terms of the free energy, which for spherical hadronic bubbles has the form

$$\Delta F = \frac{4\pi}{3}\, R^3\, \Delta p + 4\pi R^2\, \sigma, \tag{39}$$

where ΔF is the change in free energy from the quark to hadron phase, Δp is the pressure difference $p_q - p_h$ between the two phases, σ is the surface free energy at the bubble-plasma boundary, and R is the bubble radius. Since $F_h < F_q, \Delta p$ is negative, but σ is positive. Thus nucleation begins only if the initial radius is greater than

the critical radius $2\sigma/|\Delta p|$. Csernai and Kapusta[33] have calculated the nucleation rate and studied the time evolution of a homogeneous plasma as it undergoes PT by bubble formation and growth.

We consider the problem with emphasis on different issues: (a) geometry of the plasma in heavy-ion collisions, (b) modification of (39), (c) fluctuations due to collisional effects, and (d) possible observables related to fluctuations.[32]

For the longitudinally expanding cylinderical plasma produced in a head-on heavy-ion collision, T in the quark phase at $r < r_0$ decreases with t, but is greater than the transition temperature T_t. The region therefore serves as the source of quarks and gluons for the region M where $T \simeq T_t$. In the mixed phase hadronic bubbles are formed releasing latent heat, which drives the longitudinal expansion of the M region without lowering T. For the disk at a fixed y, every small cell in it has a local fluid velocity in the radial direction. In the mixed phase that velocity is roughly constant because the velocity of sound is nearly zero. Thus we shall regard all quarks and bubbles in the M region to drift toward r_b at a constant velocity.

Because of the difficulties in calculating fluctuations, especially in the absence of a reliable theoretical framework, the discussion below will perforce be qualitative, and will be based on the assumption that the above kinetic and geometrical picture is essentially correct in the disk at midrapidity over a period of time during which most of the hadrons are produced.

Concerning bubble formation in the region M, the usual consideration is concerned with the balance of the volume and surface terms as shown in (39). These are the only terms necessary in cosmic PT [34,35] or for any bubble whose radius is orders of magnitude larger than the range of microscopic interaction. In heavy-ion collisions the bubbles cannot be too large, since the mixed phase in the M region can at most be several fm in radial thickness, not more than 20 fm, say. The time scale involved is also short, so a bubble cannot grow beyond a few fm in radius before it is thrusted out of the plasma system. When all scales in the problem are comparable, one cannot ignore the curvature term, linear in R, that can contribute to (39), as well as other terms that become relevant if the bubble is not spherical.[36] For simplicity's sake, let us consider first the spherical case, and write more generally

$$\Delta F = -\alpha R^3 + \beta R^2 - \gamma R. \tag{40}$$

There is no firm determination of the coefficients α, β and γ from QCD that can reliably be used for our problem. If the curvature and volume terms are negative,[36] but all three terms have comparable magnitude, there are then two cases to consider: (a) $\beta^2 < 3\alpha\gamma$, and (b) $\beta^2 > 3\alpha\gamma$.

In case (a) ΔF decreases monotonically with R so a bubble once formed will grow without restraint except by the possible increase of temperature due to the release of latent heat L. As a general rule the rate of bubble formation and growth depends on how fast L can be gotten rid of. Presumably the coefficients α, β and γ depend on T in such a way that the system finds itself in a steady state of hadronization in the time scale considered here.

In case (b) there is a metastable state at R_- and a barrier whose peak at R_+ is not far off, where

$$R_\pm = \frac{1}{3\alpha}[\beta \pm (\beta^2 - 3\alpha\gamma)^{1/2}]\,. \tag{41}$$

If the barrier is crossed by coalescence to be discussed below, the bubble will then grow as in case (a). Thus the distribution of the bubble sizes at $r = r_b$ is a measure not only of β^2 relative to $3\alpha\gamma$, but also of the dynamical mechanisms that can give rise to bubble growth from the metastable state.

It should be remarked that since the bubbles in either case cannot grow very big in the restricted region of the mixed phase, they cannot be regarded as volumes of hadron gas. Thus in our problem the term bubble is a misnomer for a small droplet or nugget of hadrons, even the largest of which may only rarely contain more than a dozen hadrons. The application of hydrodynamics to the study of the kinetics of the quark-hadron boundary of the bubbles, as one does in cosmic phase transition,[35] therefore cannot be justified. Furthermore, the bubbles need not be spherical. The discussion given above is mainly for the purpose of indicating the possibilities that can arise in connection with (40), which is for the simplified case of a spherical bubble. In general, the volume can be like a dendrite with fingers.[36] The growth properties are far more complicated than what can be inferred from (40). Since they have irregular shapes, we shall hereafter call them *clusters*. They need not have rigid structures as they grow, so they are not of the type usually studied in local growth models or diffusion-limited growth models.[37] Having motivated the study of clusters, we shall relinquish (40) or any generalization of it so long as the coefficients α, β, etc., are unknown, and proceed with the assumption that there is some probability p for nucleation.

Turning to the dynamical mechanisms for growth, the most important one is the inelastic collision between clusters that leads to coalescence. For a metastable state in the spherical case mentioned above, the kinetic energy absorbed can put the enlarged cluster over the free-energy barrier. Even if the system is not in equilibrium, for which the use of free energy would be meaningless, coalescence of two encountering clusters can still occur. Moreover, the breakup of a large cluster into smaller ones due to collisions is also possible, such as in the severing of a finger from a dentrite. Thus the stochastic equation describing the evolution of the cluster-size distribution function should have both gain and loss terms. Since the color-neutral clusters are under constant bombardment by quarks and gluons in the mixed region, they are expected to undergo Brownian motion around the average drift motion. Such fluctuations lead to random relative velocities among the clusters when they collide, which in turn influence the coalescence and breakup rates. Herein lies an aspect of the growth process that renders the problem untractable analytically.

The problem is analogous to that of many random walkers, generated randomly at all points in space and time in region M with some fixed initial size; they can increase or decrease in size, or die, as walkers meet. In addition to these successive random processes, each cluster independently undergoes a steady nonlinear growth process.

These dynamical features have been put into a cellular automaton that simulates the cluster growth process.

The essence of a cellular automaton is to summarize the dynamics by a set of simple rules and by repeated application extract interesting pattern in the output whose characteristics transcend the details of the rules used. Although this approach to the problem is susceptible to the criticism that the rules adopted are also an oversimplification of the phase transition process, the focus is shifted from finding an analytical description of the problem on the basis of ignoring certain complications to focusing on those complications in the framework of some simple rules.

3.3. Self-Organized Criticality

Before proceeding to the formulation of the problem, let us take a short digression and have a brief look at a related subject in modern statistical physics. The traditional approach to critical phenomena is to take a thermal system to its critical point and study its scaling behavior in that neighborhood. The critical exponents, such as α for specific heat

$$C_V \propto |T - T_c|^{-\alpha} , \tag{42}$$

have been the objects of intensive study in past years both theoretically and experimentally.[38] In more recent years it has been recognized that there are many systems which exhibit criticality without the tuning of any parameters, such as the lowering of T to T_c. The dynamics of those systems are such that they approach by themselves a critical state, which has been given the name "self-organized criticality" (SOC).[39] The first case studied was the sandpile problem; it has remained to be a primary example of SOC even today.

In the sandpile problem grains of sand are dropped randomly over a table of definite size, and a sandpile is allowed to build up with zero height at the table's edge, since the sand grains are allowed to fall off the table. A sand grain falling on the pile in the interior can tumble down toward the edge when the local height differs from its neighbors in some specified way. It can either fall down the pile a certain distance and stops, or it can initiate an avalanche, resulting in many grains of sand falling off the table, or it can have any situation in between, all depending on the local heights and the rules for tumbling. The aim of the problem is not so much to model accurately a realistic sandpile as to use the sandpile model to illustrate a more interesting phenomenon of deeper significance.

Instead of formulating the problem analytically, which is difficult, the problem was first posed by use of cellular automata.[39] Subsequently, other rules were considered for the same problem, and essentially the same results were obtained.[40] They are the following. The sandpile is first built up to a critical slope; thereafter each additional grain of sand dropped leads to an avalanche of size S (number of grains falling off the table), which can vary from 0 (no fall-off) to a large number. Repeated application

of random dropping of sand (one grain at a time) leads to a distribution of S

$$P(S) \propto S^{-\alpha} \tag{43}$$

where α is some positive number. Except for the finite-size effect (due to the finiteness of the table-to-grain size ratio), this type of scaling behavior generally emerges, relatively independent of the details of the rules. Since the result involves large fluctuations in S without any characteristic scale, it suggests that the system is in some critical state. The system evolves to that state without the tuning of any parameters externally. Thus the sandpile problem illustrates a general SOC phenomenon of large-scale fluctuations in the outcome of an evolution process. Other examples are earthquakes, forest fires, etc.[41]

It will become self-evident in the following in what sense SOC is relevant to the problem of cluster production. The hadron clusters are like the sandpile avalanches, emitted by the plasma in all sizes. The aim is to look for scaling laws like (43), and investigate their universality. As in SOC, the experimentalists have no control of the quark-gluon system once it is formed in a heavy-ion collision; it evolves on its own without external tuning of any parameters. The discovery of a universal scaling law would then be of fundamental importance.

3.4. Cellular Automaton

In the sandpile and forest-fire problems [39,40,41], the rules are extremely simple; consequently, the models are not very close to reality. Nevertheless, they are interesting because each of them exemplifies a class of problems that exhibit interesting scaling behaviors. Our problem here is, however, different. We are interested in a specific problem: that is, the discovery of observable signatures of quark-hadron phase transition in heavy-ion collisions. Thus our rules cannot be so simple as to mutilate the physical problem that we wish to address. Since the relevant dynamics that controls the nature of fluctuations at the transition point is not known very well, there is some freedom in the choice of the rules within the space-time structure outlined in Sec. 3.2. Within the confines of that framework one should adopt rules as simple as possible to describe cluster growth, and search for generic results that are insensitive to the parameters used in the rules, or better still detail aspects of the rules themselves.

In Ref. 32 the problem is considered in 1-dimension. It is a drastic simplification of the realistic problem that provides a first glimpse of the type of results that emerged. Subsequently, the problem has been generalized to 2 dimensions [42] with new features that are absent in 1D. The most prominent one is that the clusters can have various shapes. On the other hand, the breakup of clusters that is simple to implement in 1D has in 2D so many possibilities that it is not included in Ref. 42 in order to limit the number of rules. We describe here the procedure considered in Ref. 42, deferring to a later subsection more recent attempts to modify it so as to include breakup.

In 2D a thin slice of the expanding cylinder at midrapidity has an annular ring in which the quarks (and gluons, which we shall suppress mentioning for the sake of

brevity) and hadrons coexist. Consider a wedge of that annular ring and map it to a 2D square lattice of size $L \times L$ initially, as depicted in Fig. 3.1. That is the M region whose evolutionary behavior in space and time is to be tracked. We impose periodic boundary condition in the vertical y direction, and regard the first column at $x = 1$ as the inner boundary of the ring, and $x = x_y(t)$ as the outer boundary (depending on y) that changes with time t, starting with $x_y(0) = L$ for all y. The time dependence of $x_y(t)$ will be a result of the process of cluster emission from the plasma.

The rules for the the cluster formation process are as follows.

(a) *Single-site nucleation.* Initially, all sites are unoccupied, representing quark phase. Let each site have the probability p of being occupied in each time step, representing a local hadron phase. The initial hadron is thus of size S_0, which is slightly less than the intersite distance, so that neighboring sites can separately become newly occupied without overlap. For simplicity let single-site nucleation (SSN) be denoted by $S_0 = 1$. At the next time step only the unoccupied sites can become occupied with probability p at each site again.

(b) *Growth.* If a newly occupied site is a nearest neighbor of an old occupied site, regard it as a growth process, and require the two to be bonded to form a cluster (or an extension of a cluster, if the old site is already part of an existing cluster). All sites in a cluster are bonded.

(c) *Average drift and Random walk* All hadrons drift toward the outer boundary at the same velocity at any given time, simulating the radial expansion. This drift motion in the M region is represented by the requirement that all occupied sites $s_i(t) = (x_i, y_i)$ are to move to $s_i(t+1) = (x_i + 1, y_i)$ at the next time step, assuming no fluctuations. On top of that motion the clusters take random walks around their average drift as in Brownian motion. Thus with fluctuation, $s_i(t+1)$ can be one of the following four possible sites, (x_i+2, y_i), (x_i+1, y_i+1), (x_i+1, y_i-1), and (x_i, y_i), with equal probability. All the other sites in the cluster move together as a rigid body. All vacated sites become unoccupied.

(d) *Coalescence.* As the clusters move, they may collide. A collision occurs during the random walk when there is an overlap of occupied sites belonging to two different clusters. When that happens, the two clusters are treated as one thereafter, with one of the double at the overlap site be taken to occupy the nearest unoccupied site (at random if more than one), and be bonded to the rest of the enlarged cluster.

(e) *Boundaries.* Since the quark phase on the inside of the annular ring supplies the quarks to the mixed region, the quark density in the M region remains constant throughout, including the boundary at $x = 1$. That means that all unoccupied sites remain unoccupied until an occupied site moves there, or a nucleation

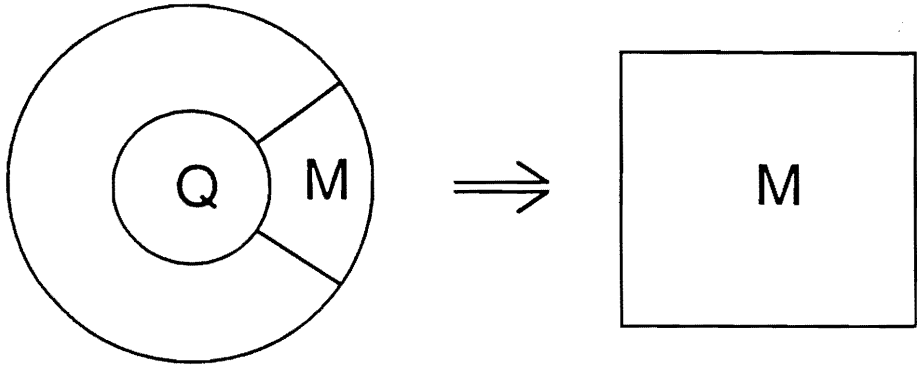
Q
M
M

Fig. 3.1

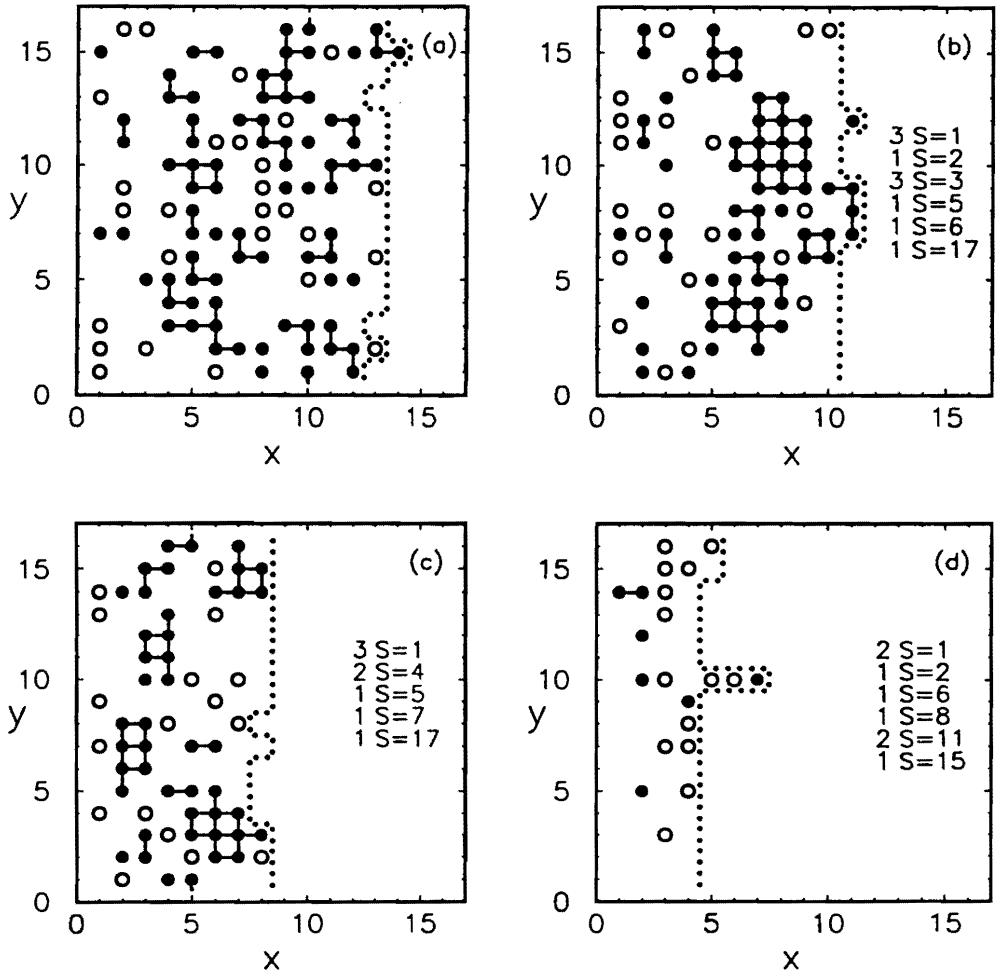
(a)
(b)
3 S=1
1 S=2
3 S=3
1 S=5
1 S=6
1 S=17
(c)
3 S=1
2 S=4
1 S=5
1 S=7
1 S=17
(d)
2 S=1
1 S=2
1 S=6
1 S=8
2 S=11
1 S=15
x
y
0
5
10
15

Fig. 3.2

takes place there. Unoccupied sites do not drift. At the outer boundary hadron clusters can leave the M region, but not the quarks due to confinement. When any site in a cluster of size S breaks through the outer boundary, then the whole cluster is removed from the M region, which is now redefined to have S sites less vertically and one layer less horizontally where the cluster is emitted.

(f) *Evolution.* The above steps are repeated again and again until all clusters are emitted from the plasma. The evolution stops when there are no more unoccupied sites left, and the phase transition is over.

(g) *Cluster-size distribution.* With $n_j(S)$ denoting the number of clusters of size S emitted in the jth event, and $N_j = \sum_S n_j(S)$, calculate the average distribution after $\mathcal{N}_{\rm evt}$ events defined by

$$P(S) = \frac{1}{\mathcal{N}_{\rm evt}} \sum_{j=1}^{\mathcal{N}_{\rm evt}} \frac{n_j(S)}{N_j} \quad . \tag{44}$$

The above rules define cellular automaton CA2.1 [42]. Note that thermal equilibrium has not been assumed. The only essential assumption is that there exists a region in which quarks and hadrons can coexist for a sufficiently long time. Unlike usual lattice calculations, the size L has a physical meaning and cannot be taken to be arbitrarily large. Taking the lattice spacing to very small can be considered, only if the relative ratios of L, S_0, drift step size and random-walk step size remain the same in that limit. But to take the continuum limit is contrary to the ideas of cellular automata (CA). Variations of the ratios of those physical quantities will, however, be considered in the following.

3.5. Scaling Behavior

Using CA2.1 the cluster production process has been simulated in 10^3 events.[42] A typical event would have a time sequence that can be exemplified by four frames illustrated in Fig. 3.2, corresponding to (a) $t = 4$, (b) $t = 5$, (c) $t = 6$, (d) $t = 7$ for $L = 16$ and $p = 0.2$. Solid circles represent occupied sites that have previously been in existence elsewhere; open circles are newly nucleated sites. Linked sites are clusters formed either by growth or by coalescence. The dotted lines are the outer boundaries at each stage of the development. To the right of the dotted lines are indicated the numbers of clusters of particular sizes that have just been emitted.

After averaging over all events the result on the cluster-size distribution $P(S)$ is as shown in Fig. 3.3 for SSN ($S_0 = 1$), $L = 16$ and $0.01 \le p \le 0.5$. Evidently, except for small values of p, $P(S)$ exhibits scaling behavior

$$P(S) \propto S^{-\gamma} \quad , \tag{45}$$

where the scaling exponent γ depends only mildly on p with its value in the range

$$\gamma = 1.86 \pm 0.18 \quad , \tag{46}$$

while p varies in a range that stretches over an order of magnitude from $p = 0.05$ to 0.5. If the lattice size is enlarged to $L = 32$, the linear portions of the scaling curves are extended further without any changes in the slopes (see Fig. 3.4). Furthermore, the scaling behavior remains unchanged, if rule (a) is replaced by double-site nucleation (DSN), i.e., two neighboring unoccupied sites must simultaneously become occupied in one time step to initiate a $S_0 = 2$ cluster. This is shown in Fig. 3.5 for $L = 32$ (the same being true for $L = 16$). The value of γ is still as given in (46). Thus one can conclude that the cluster production process exhibits universal scaling behavior, independent of L, S_0 and essentially also of p.

One can also consider a variation of the drift speed. In CA2.1 the drift step is set equal to one lattice spacing per time step. Doubling the drift step results in Fig. 3.6, [43] which shows that scaling remains valid only for $p > 0.1$. Thus for low nucleation rate there is insufficient growth and coalescence to generate large clusters before they move rapidly out of the M region. But more interesting is the observation that for $p > 0.1$ the scaling behavior is unchanged. If L is doubled also to 32, then scaling is restored for $p > 0.05$ with the same value of γ. This is not a consequence of simply scaling up the lattice spacing, since the random-walk step size is not doubled also. Equivalently, the latter is halved, if L and drift step stay unchanged. Thus the invariance of the scaling behavior implies insensitivity to the temperature of the system, if thermal description is appropriate.

It is also possible to use CA2.1 to check the importance of growth versus coalescence by turning off either one or the other mechanism. It is found that coalescence alone is able to sustain the scaling behavior, but not the other way around.[43] With growth only, no large clusters can be formed even at high p.

The resemblance of (45) and (43) strongly suggests that the quark-hadron PT is another example of SOC. Indeed, we have not tuned any parameter to bring the systems to the critical point. This point is significant for heavy-ion collisions, since there are no experimental means to control the quark-gluon plasma once it is created. The universality of γ is also important, since it implies independence on the experimental conditions under which the plasma is formed, such as nuclear size, collision energy, impact parameter, etc. The only requirement is that the M region of the mixed phrase is sufficiently large, a scenario that is consistent with the recent trend in the theoretical estimate of the time spent in the mixed phase.

3.6. Modified CA

On the basis of CA2.1 it has been shown that the scaling behavior of $P(S)$ is independent of many parameters representing physical features of the system. A legitimate question is on the sensitivity of the results to the particular rules in CA2.1 themselves. It is only when there is independence on the details of the CA can one believe the result to be a general consequence of the dynamical systems.

There are other reasons to want to modify CA2.1. So far no breakup of clusters is allowed, mainly because no simple rule can be added to account for all possible ways

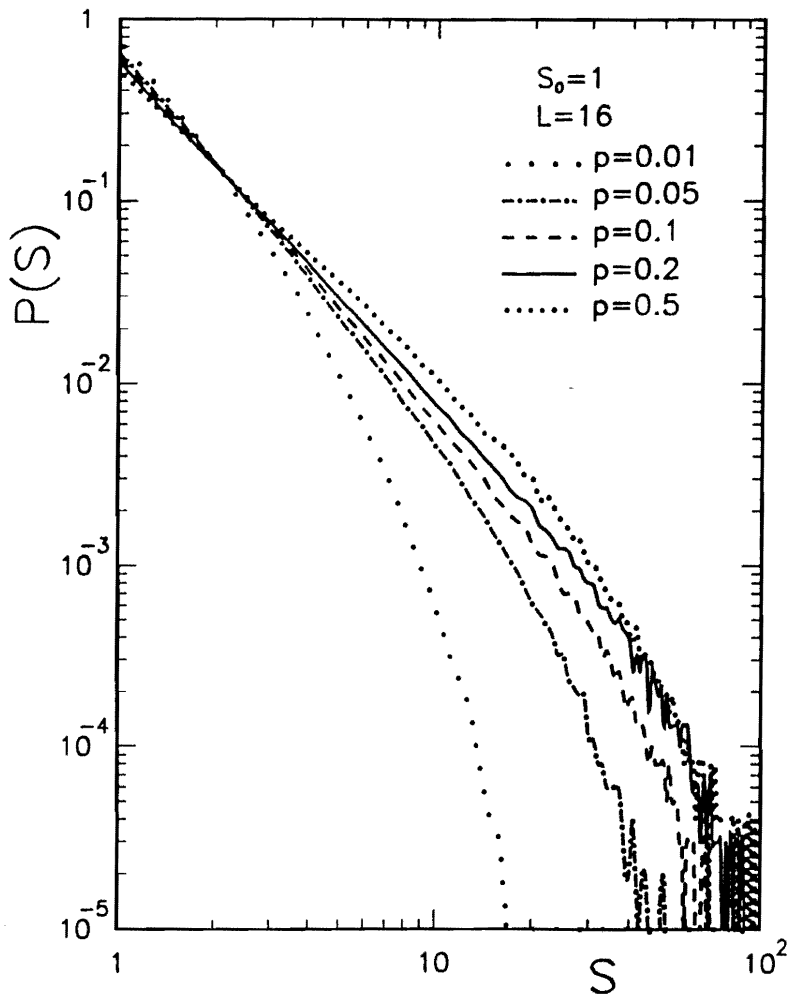
$S_0=1$
L=16
p=0.01
p=0.05
p=0.1
p=0.2
p=0.5
P(S)
S
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
1
10
10^2

Fig. 3.3

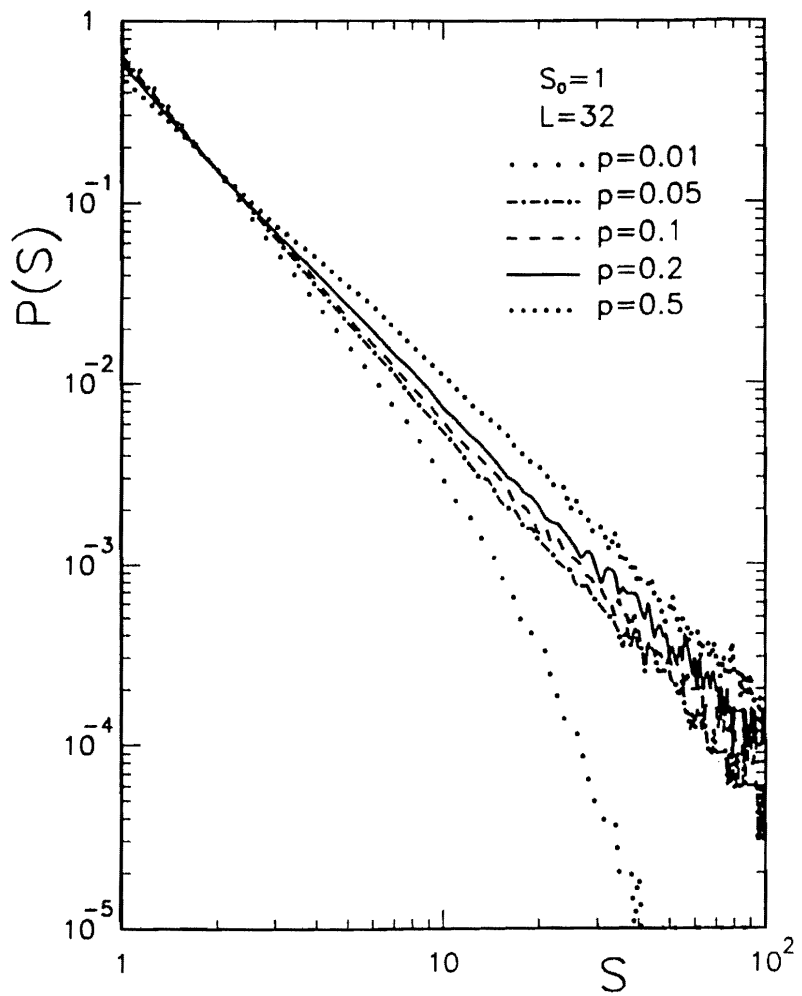
$S_0=1$
L=32
p=0.01
p=0.05
p=0.1
p=0.2
p=0.5
P(S)
S
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
1
10
10^2

Fig. 3.4

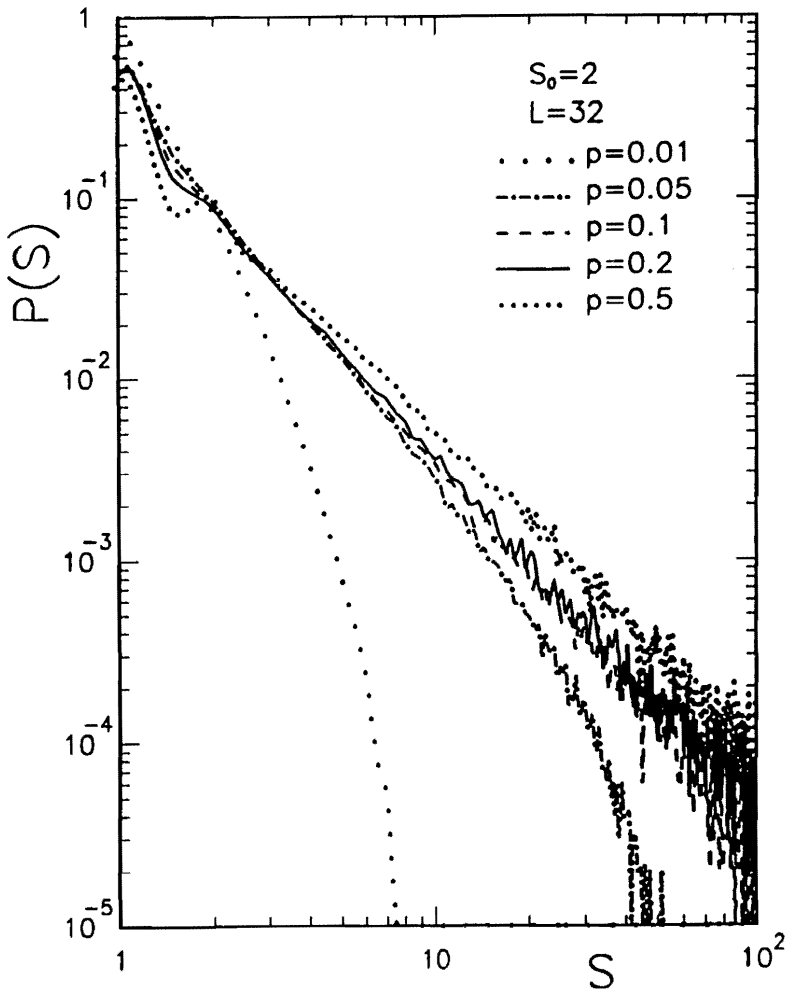
$S_0=2$
L=32
p=0.01
p=0.05
p=0.1
p=0.2
p=0.5
P(S)
S
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
1
10
10^2

Fig. 3.5

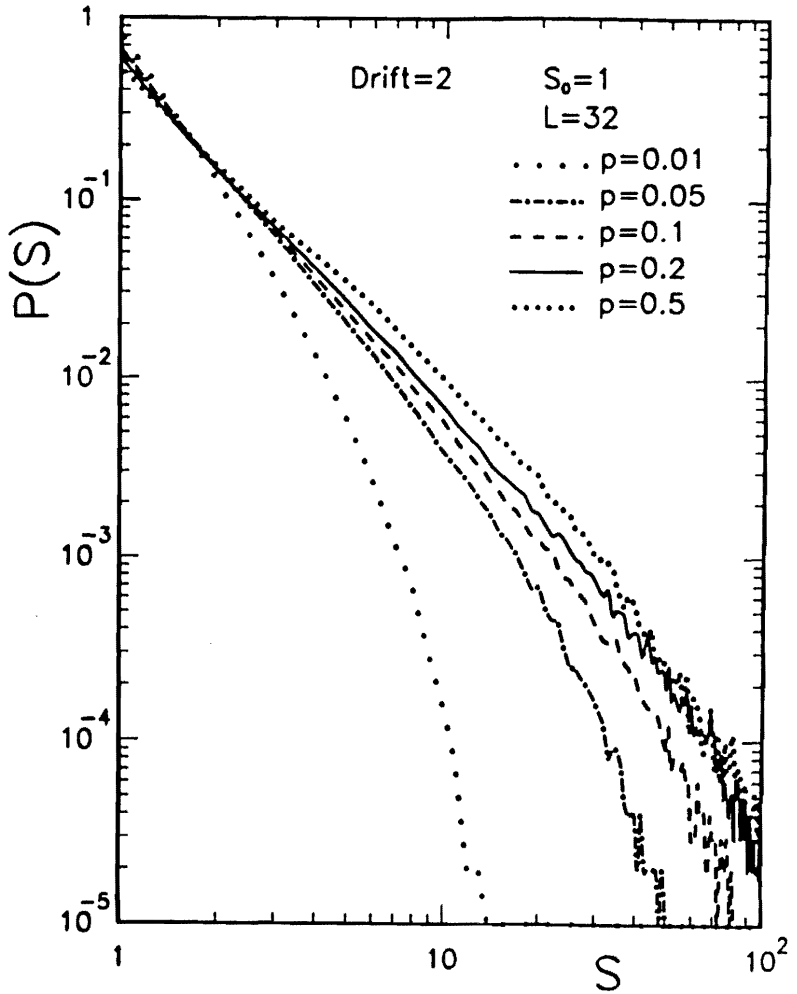
Drift=2
$S_0=1$
L=32
p=0.01
p=0.05
p=0.1
p=0.2
p=0.5
P(S)
S
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
1
10
10^2

Fig. 3.6

that breakup can take place. Furthermore, the clusters in CA2.1 are rigid, while having deformability is more reasonable, especially if the surface tension is large, positive or negative. In CA2.2 both deformation and breakup are allowed.[43]

The first step in modifying CA2.1 is to relax the rigidity of the clusters, whose shapes depend on where along the peripheries the growth and coalescence processes happen to occur. That can be achieved by assigning a circular area to every cluster with an effective radius

$$R(S) = \alpha\sqrt{S/\pi} \quad , \tag{47}$$

where α is a parameter that specifies how dendritic the shape is. If $\alpha = 1$, it is circular; if $\alpha > 1$, it is irregular, or may even have holes. One may think of a cluster as a vertical stack at the center with a range of influence extending to a distance of R away. The stack is always placed at a lattice site, whether or not the effective circle corresponding to (47) covers S discrete sites. In drift and random walk the stack moves in discrete steps as before.

For growth any unoccupied site immediately outside the effective circle, when nucleated, becomes an addition to the stack. For coalescence between two stacks with radii R_1 and R_2, we require the distance between the two stacks to be less than $R_1 + R_2$ in order for the two stacks to be combined into one and be placed at a site closest to the c.m. position.

With this scheme it is not difficult to consider breakup. We use

$$B(S, S_1, S_2) = \beta\left(\sqrt{S_1/S} + \sqrt{S_2/S}\right)^{-1} \tag{48}$$

to describe the probability that a cluster S breaks up into S_1 and S_2 spontaneously. If it follows immediately after a coalescence, the combined two-step process may be regarded as an inelastic collision. The probability $B(S, S_1, S_2)$ favors minimization of circumference-to-area ratio with positive surface tension being assumed implicitly; β is a positive coefficient. The daughter clusters are placed at y_1 and $-y_2$ vertically away from the mother site, where $y_{1,2}$ are the largest integers less than $\alpha\sqrt{S_{2,1}/\pi} + 1$. If either one of the effective circles of S_1 and S_2 overlaps with that of an existing cluster, the breakup is forbidden from occurring in order to avoid a chain reaction of coalescence and breakup.

In Fig. 3.7 the result of CA2.2 is shown for three values of α but with $\beta = 0$. This is a check of CA2.2 compared to CA2.1 without breakup. Note that there is very little dependence on α, and the slope is consistent with (46). Thus the new CA yields essentially the same result as the old, verifying the insensitivity to the details of the rules. When breakup is introduced, there is noticeable dependence on β for small p;[43] however, when p is in the normal range, the dependence on β is minor, as shown in Fig. 3.8 for $p = 0.2$. That is because when there is sufficient nucleation, there are enough clusters in the M region to cause the products of breakup to coalesce readily with others so that the net cluster distribution is essentially unaffected.

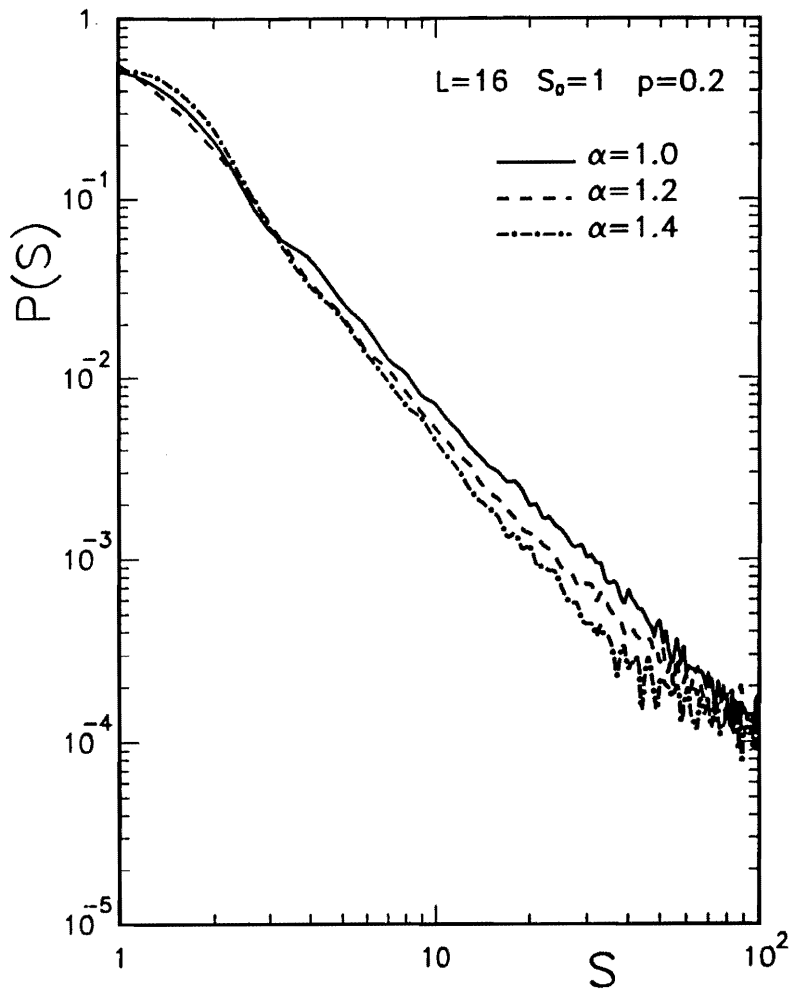
L=16 S₀=1 p=0.2
α=1.0
α=1.2
α=1.4
P(S)
S
1
10
10²
10⁻¹
10⁻²
10⁻³
10⁻⁴
10⁻⁵

Fig. 3.7

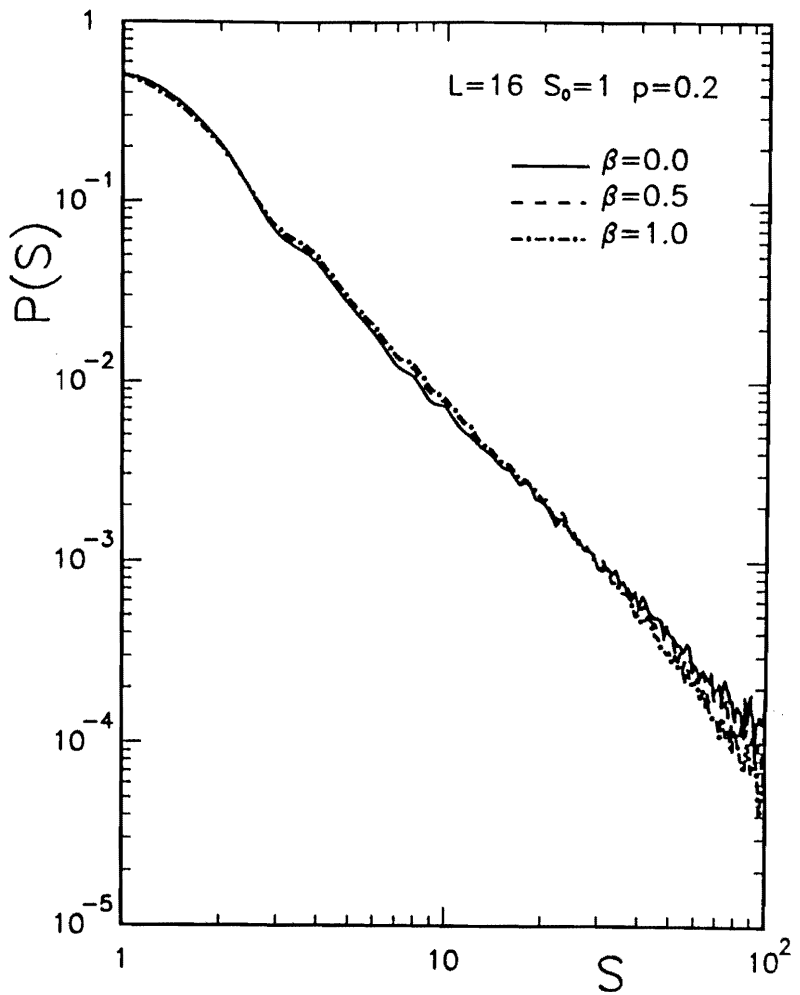
L=16 S₀=1 p=0.2
β=0.0
β=0.5
β=1.0
P(S)
S
1
10
10²
10⁻¹
10⁻²
10⁻³
10⁻⁴
10⁻⁵

Fig. 3.8

The universality feature of the scaling behavior is now extended to include breakup. Furthermore, the independence on the specific rules in a CA to describe the same dynamics gives confidence to the generality of the result.

3.7. Phenomenology

The scaling distribution $P(S)$ discussed in the foregoing subsections refers to hadronic clusters produced at the outer edge of the mixed phase as they emerge from the M region. They are not necessarily what the experimentalists can see at the detector, depending on what transpires in between. Since there is little consensus on the physics issues involved, we give below some speculation on what may happen.

The most important question is whether the clusters will collide with one another after they leave the M region. If they do extensively, then the clusters would be broken up, and the evidences of clustering may be hard to detect. The existence of a hadronic gas phase seems to be mainly an assumption, based perhaps on the physics of heavy-ion collisions at low energies. At LHC energies where a baryon-free central region is most likely to be found, the hadrons that leave the mixed phase have such high p_T that the transverse geometrical space available (increasing quadratically with time) may be large enough to significantly cut down the probability of final-state interaction. If that is true, then free streaming of the hadronic clusters is a likely possibility worthy of further consideration.

It is reasonable to assume that the hadrons within a cluster are in thermal equilibrium, since they are confined to a small volume in the mixed region. After the cluster is ejected from its quark environment, a naive view would suggest that the thermal energy of the hadrons inside the cluster at $\sim T_c$ would cause the hadrons to gain kinetic energies in the vacuum. For $T_c \sim 150$ MeV, the hadrons belonging to the cluster would be spread out in phase space, leading to a diffuse collection of pions at the detector with no characteristic of an identifiable cluster.

However, there is at least one additional piece of physics that would intervene. At T_c, confinement PT turns quarks into hadrons, but the chiral symmetry is still in the process of undergoing a transition. That is, the constituent quark mass does not reach its full value of 330 MeV until T is lowered to zero. Thus it is only some time after the cluster is thrust into the cold vacuum that the breaking of chiral symmetry is brought to completion by the $T = 0$ environment. Since the cluster is now in the vacuum and not in contact with any other, the only way that the quarks can gain their masses is at the expense of the thermal energy of the hadrons in the cluster. The question of the temperature dependence of the hadronic masses is still very controversial, and there is no definitive guidance from lattice gauge theory. An increase of the ρ meson mass, [44] as T is decreased, would be in favor of our argument that the thermal energy in the cluster would be converted to the hadronic mass. Indeed, if m_ρ is very low at T_c, then there may not be enough energy for ρ to attain its physical mass before it decays into two pions. The loss of the relative kinetic energies of the hadrons in the cluster should therefore result in bunching, i.e., the detected particles (mostly pions

and kaons) are close together in phase space. In short, if the hadron masses in the cluster at T_c are not high, energy conservation alone should be enough to imply that the detected hadrons would be bunched.

If at T_c the hadrons in the clusters in the M region have low masses (both strange and non-strange), then it should be far more efficient to generate strange particles inside the clusters than normally is possible in pp collision in the $T = 0$ vacuum. For example, reactions such as $\rho + \rho \rightarrow K + \bar{K}$ would have higher rates at T_c. After those clusters leave the plasma and plunge into the $T = 0$ environment, the strange quarks would be frozen in the system as they gain their constituent quark mass. Thus among the final particles that are detected there should be more K and ϕ mesons, compared to the production rate without plasma formation. Since larger clusters spend more time in the M region to acquire their sizes, there is more time for strangeness creation in them, so the detected hadrons should have a strange/nonstrange ratio that increases with the cluster size. This is a new signature that should be very distinctive.

Finally, we come to the question of how the clusters are to be identified experimentally. First of all, one should look at the produced particles one event at a time, since inclusive distribution adds up all events, a procedure that surely will smear out any evidences of clustering. Even in one event one should minimize integration over any variable. Specifically, p_T should not be integrated over. Thus the p_T space should be divided into many bins, and one should examine the η-φ space in each p_T bin. If one uses hydrodynamics as a guide, one should expect that the clusters emitted earlier have larger velocities on the average than those emitted later on in the phase transition. Since we want to avoid the clusters emitted at different times to overlap, separation by p_T partitioning is a method that should enhance the chances of seeing clustering.

Now we come to the η-φ space, in which clustering is to be found for each p_T bin in an event. That space must be partitioned into many two-dimensional cells of a given size. Clearly, if the cell size is very large, all hadrons belong to one cluster. If the cell size is very small, each hadron is an island by itself. Thus the resolution of the cells must be varied to optimize the identification of the clusters.

When a cluster is found it can be distinguished from that due to a jet by comparing the various p_T sections of the same event. If there is a jet, there are many particles in the jet that are concentrated in a small cone in the η-φ space, but have varying p_T — the towers in the lego plot. Thus in different p_T sections, they will contribute to clustering at the same place in the η-φ space. But no such correlation is expected for clusters produced in quark-hadron PT. Hence, if clustering is found at random locations in the η-φ space from one p_T section to another, that would be the true signature of the clusters discussed here.

Furthermore, there are particles in a jet that have high p_T, say greater than $5\,\mathrm{GeV}/c$. No such high-p_T hadrons are expected in clusters produced in PT. Thus if randomly distributed low-p_T clusters are found, they can hardly be confused by

ambiguities due to other competing production mechanisms.

4. General Comments

Two types of scaling behaviors have been discussed as consequences of quark-hadron phase transition. One is on the factorial moments F_q, which exhibit F-scaling, i.e., $F_q \propto F_2^{\beta_q}$, and $\beta_q = (q-1)^\nu$ with $\nu = 1.304$ for second-order PT and up to 1.45 for first-order. The results are derived from the GL theory, and should be valid for various fields of physics. They have been verified in quantum optics. The other scaling behavior has been found in the cluster-size distribution $P(S) \propto S^{-\gamma}$, where $\gamma \simeq 1.9$, nearly independent of many parameters in the problem. That result has been obtained by use of cellular automata, and suggests that the phenomenon exhibits self-organized criticality.

No attempt has thus far been made to relate the two types of scaling behaviors. As they stand, they describe two complementary aspects of the PT process. In the former there is no time dependence, since the system is assumed to be in a steady state. In the latter finite size and finite time-duration are important parts of the problem. They are orthogonal views of the system, emphasizing different observables.

Clustering refers to the geometrical problem of adherence after the hadrons are formed, not considered in GL. Consequently, the sum over S of all clusters produced in each event should give the total number of hadrons formed in the event, which in turn should be related to the multiplicity n, whose fluctuation from the average is characterized by F_q. On the other hand, the nucleation rate p should be related to the order parameter $|\phi|$ so that the GL description of first-order PT can be introduced into the cellular automata. With appropriate interconnections the two aspects of the PT problem can be married without the anticipation of obvious damages that would be done to the two scaling results already obtained. That is, of course, the next important step to be taken theoretically in this subject.

Experimentally, the detectors should be designed with good resolutions for the hadronic momenta and multiplicities. To measure F_q to high values of q for varying cell sizes requires high statistics data. To identify clusters event by event requires laborious work to examine the particle distribution in the η-φ space for various p_T sections small enough to minimize the overlap of clusters. But perhaps the greatest difficulty is to overcome the prevailing bias against hadronic signatures. It is hoped that the work reviewed here will provide some motivation to venture into this unorthodox method of identifying unusual signs of quark-hadron phase transition.

Acknowledgments

I am grateful to many collaborators and colleagues from whom I have benefitted from extensive discussions and help, particularly D. Belitz, H. Carmichael, T. Hwa, C.S. Lam, J. Pan, M.T. Nazirov and S. Singh. This work was supported in part by U.S. Department of Energy under Grant No. DE-FG06-91ER40637.

REFERENCES

1. D. Kharzeev and H. Satz, this volume.
2. C. DeTar, this volume.
3. V.L. Ginzburg and L.D. Landau, Zh. Eksp. Teor. Fiz. **20**, 1064 (1950). ZETFA
4. D.J. Scalapino and R.L. Sugar, Phys Rev. D **8**, 2284 (1973).
5. P. Carruthers and I. Sarcevic, Phys. Lett. B**189**, 442 (1987).
6. I.Dremin and M.T. Nazirov, in *Fluctuations and Fractal Structure*, Proceedings of the Ringberg Workshop, Ringberg Castle, Germany, 1991, edited by R.C. Hwa, W. Ochs, and N. Schmitz (World Scientific, Singapore, 1992).
7. H.-T. Elze and I. Sarcevic, Phys. Rev. Lett. **68**, 1988 (1992).
8. R.C. Hwa and M.T. Nazirov, Phys. Rev. Lett. **69**, 741 (1992).
9. A. Białas and R. Peschanski, Nucl. Phys. **B273**, 703 (1986); **B308**, 857 (1988).
10. C.B. Chiu and R.C. Hwa, Phys. Lett. B**236**, 466 (1990).
11. J. Pan and R.C. Hwa, Phys Rev. D **46**, 4890 (1992); D **48**, 168 (1993).
12. *Fluctuations and Fractal Structure*, Proceedings of the Ringberg Workshop, Ringberg Castle, Germany, 1991, edited by R.C. Hwa, W. Ochs, and N. Schmitz (World Scientific, Singapore, 1992).
13. R.C. Hwa, Phys Rev. D **47**, 2773 (1993).
14. KLM Collaboration, R. Holynski et. al., Phys. Rev. Lett. **62**, 733 (1989); Phys. Rev. C **40**, 2449 (1989).
15. EMU01 Collaboration, M.I. Adamovich et. al., Phys. Rev. Lett. **65**, 412 (1990).
16. R. Graham and H. Haken, Z. Physik **213**, 420 (1968); **237**, 31 (1970); V. DeGiorgio and M. Scully, Phys. Rev. A **2**, 1170 (1970). ZEPYA
17. H. Risken, in Progress in Optics VIII, ed. E. Wolf (North Holland, Amsterdam, 1970).
18. L. Mandel and E. Wolf, Rev. Mod. Phys. **37**, 231 (1965). RMPHA
19. M.R. Young, Y. Qu, S. Singh and R.C. Hwa, Optics Comm. **105**, 325 (1994). OPCOB
20. R.C. Hwa and J. Pan, Phys. Lett. B**297**, 35 (1992).
21. F. Karsch, in *Quark-Gluon Plasma*, edited by R.C. Hwa (World Scientific, Singapore, 1990).
22. S. Gottlieb, Nucl. Phys. B(Proc. Suppl.) **20**, 247 (1991).
23. F. Wilzcek, Int. J. Mod. Phys. A **7**, 3911 (1992). IMPAE
24. R.C. Hwa, Phys Rev. C **50**, 383 (1994).
25. I.A. Lebedev and M.T. Nazirov, Mod. Phys. Lett. A **9**, 2999 (1994). MPLAE
26. A.K. Mohanty and S.K. Kataria, Phys. Rev. Lett. **73**, 2672 (1994).
27. A.K. Mohanty and S.K. Kataria, BARC-preprint 1995.
28. B.L. Friman, K. Kajantie, and P.V. Ruuskanen, Nucl. Phys. **B266**, 468 (1986); K. Kajantie, M. Kataja, L. McLerran, and P.V. Ruuskanen, Phys. Rev. D **34**, 811 (1986); M. Kataja, P.V. Ruuskanen, L. McLerran, and H. von Gersdorff, ibid. 2755 (1986).

29. X.N. Wang and R.C. Hwa, Phys. Rev. D **35**, 3409 (1987).
30. U. Ornik and R. Weiner, Phys. Lett. **B263**, 503 (1991).
31. J. Alam, D.K. Srivastava, B. Sinha, and D.N. Basu, Phys. Rev. D **48**, 1117 (1993).
32. R.C. Hwa, C.S. Lam, and J. Pan, Phys. Rev. Lett. **72**, 820 (1994).
33. L.P. Csernai and J.I. Kapusta, Phys. Rev. D **46**, 1379 (1992); Phys. Rev. Lett. **69**, 737 (1992).
34. K. Freese and F.C. Adams, Phys. Rev. D **41**, 2449 (1990).
35. K. Enqvist, J. Ignatius, K. Kajantie, and K. Rummukainen, Phys. Rev. D **45**, 3415 (1992).
36. G. Lana and B. Svetitsky, Phys. Lett. **B285**, 251 (1992).
37. T. Vicsek, *Fractal Growth Phenomena* (World Scientific, Singapore, 1989).
38. N. Goldenfeld, Lectures on phase transitions and the renormalization group, (Addison-Wesley Reading, 1992).
39. P. Bak, C. Tang, and K. Wiesenfeld, Phys. Rev. A **38**, 364 (1988).
40. L.P. Kadanoff, S.R. Nagel, L. Wu, and S.M. Zhou, Phys. Rev. A **39**, 6524 (1989); J.M. Carlson, J.T. Chayes, E.R. Grannan, and G.H. Swindle, Phys. Rev. Lett. **65**, 2547 (1990).
41. P. Bak, K. Chen, and C. Tang, Phys. Lett. A **147**, 297 (1990); P. Grassberger and H. Kantz, J. Stat. Phys. **63**, 685 (1991); B. Drossel and F. Schwabl, Phys. Rev. Lett. **69**, 1629 (1992). JSTP B
42. R.C. Hwa and J. Pan, Phys. Rev. C **50**, 2516 (1994).
43. R.C. Hwa and J. Pan, Phys. Rev. C., July 1995
44. G. Brown and M. Rho, Phys. Lett. **B338**, 30 (1994).