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SCALING PROPERTIES OF HADRONIC OBSERVABLES IN PHASE TRANSITIONS

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Abstract

Quark-hadron phase transition is considered for both second and first order cases. In the former case the Ginzburg-Landau theory is used to derive a scaling law for the normalized factorial moments of hadron multiplicities. A scaling exponent $\nu = 1.304$ is obtained, and has subsequently been verified by an experiment on photocount near laser threshold. In the case of first-order transition the problem of cluster growth is considered. It is suggested that a scaling distribution in the cluster sizes would be a novel signature of quark-gluon plasma.

1 Introduction

The problem of identifying unambiguous signatures of quark-gluon plasma is usually centered around the primordial state when the quarks are deconfined and presumed thermalized. While such efforts should certainly be pursued despite ambiguities due to competing processes, we propose to study the problem in a total different realm. We regard the quark-hadron phase transition (PT) to be not only a more interesting physics problem but also a phenomenon that can possible be inferred from the experimental data of heavy-ion collisions, provided that the right measurables are investigated and the appropriate signals are recognized.

A way of viewing the various approaches to the problem is to contrast the volume effect from the surface effect. The former refers to the signals emanating from the plasma proper, while the latter refers to observables sensitive to the PT that occurs

near the surface, where temperature is low enough for the transition to hadrons to take place. Hadronic signature of PT may be subject to blurring by possible final state interaction, the existence and extent of which are unknown in nuclear collisions at very high energy, it being a question of the existence of the so-called hadron gas phase. To make any progress in understanding the effect of the latter complication, it is necessary first to know the nature of a clear signal from PT without a hadron gas.

Current understanding of quark-hadron phase transition from latttice gauge calculations does not give a definitive answer on whether it is of first or second order. We shall consider both cases, the latter first since it is more tractable analytically. We shall find a scaling behavior that is characterized by an exponent ν, whose validity will be shown to be more general than the plasma problem originally intended. For the case of first-order PT, the problem is more complex; however, we have a computer simulated result to illustrate the main idea that we want to advance.

2 Second-Order Phase Transition

In condensed matter physics it is well known that a system at the critical point exhibits large fluctuations. If there is a similar phenomenon in quark-hadron phase transition, the fluctuations that one looks for can only be found in the hadron multiplicities. Present theoretical studies of quark-hadron PT do not deal with multiplicities: at the microscopic level lattice gauge calculations cannot treat global properties adequately, while at the macroscopic level hydrodynamical calculations treat average quantities with deliberate intention to ignore the fluctuations. What we must do is to consider the problem at an intermediate level where hadron multiplicity is a variable in the theoretical framework as well as a measureable quantity whose properties can be predicted.

Among the formalisms that have been developed in the past for critical phenomena, the one most suitable for our use is the phenomenological theory of Ginzburg and Landau (GL) [1], which correctly described the behavior of superconductivity before the emergence of the BCS theory. The application of the GL theory to particle production has been considered before [2]-[5], but each for a different purpose, and none for a quantitative prediction of multiplicity fluctuations related to PT. We report here our attempt to study intermittency in the GL theory and our discovery of the scaling exponent ν, even though there is no intermittency in the strict sense [6]-[8].

The GL free energy density is

$$\mathcal{F}[\phi] = a|\phi|^2 + b|\phi|^4 + c|\partial\phi/\partial z|^2 \tag{1}$$

where ϕ is an order parameter, and z is a spatial variable representing a point in d-dimensional space. The parameters a, b, and c characterize the system in the neighborhood of the critical point. Generally, b and c are positive and approximately constant near T_c, while a changes sign as

$$a(T) = a'(T - T_c) \tag{2}$$

with $a' > 0$. The first two terms of $\mathcal{F}[\phi]$ describe a stable minimum at $\phi = 0$ when $a > 0$, but the minimum moves to

$$|\phi_0| = -a/2b \tag{3}$$

with indeterminant phase when $a < 0$. This is the behavior of a second-order PT where the order parameter is zero in one phase ($T > T_c$) and nonzero in the other ($T < T_c$).

Our problem in applying this formalism to quark-hadron PT in heavy-ion collisions is firstly to identify the order parameter and secondly to calculate an observable quantity. For the former we draw an analogy between pion production and photon production in lasers, where $|\phi|^2$ is the photocount rate [9]. Indeed, in the laser problem $\phi(z)$ is the eigenvalue of the annihilation operator whose eigenstate is the coherent state $|\phi\rangle$. This formalism was first adopted in [2] to treat particle production. We follow the same procedure and recognize a pure coherent state as having a Poissonian distribution of particle multiplicity n in a volume V

$$\hat{P}_n[\phi] = \frac{1}{n!}\left(\int_V dz|\phi|^2\right)^n exp\left[-\int_V dz|\phi|^2\right] \tag{4}$$

The observed multiplicity distribution P_n is, however, the thermodynamical average of $\hat{P}_n$, when ϕ is allowed to vary in accordance to what the GL free energy prescribes:

$$P_n = \frac{1}{Z}\int \mathcal{D}\phi \hat{P}_n[\phi] e^{-\int dz \mathcal{F}[\phi]} \tag{5}$$

where $Z = \int \mathcal{D}\phi e^{-\int dz \mathcal{F}[\phi]}$. Thus the dynamical component of the fluctuation in n is totally controlled by $\mathcal{F}[\phi]$. A measurement of that fluctuation should therefore reveal the central features of the PT. Eq. (5) may alternatively be regarded as the starting point of our approach without the derivation from coherent states; in that approach one simply assumes that P_n is a convolution of the statistical fluctuation described by $\hat{P}_n$ and the dynamical fluctuation specified by the Boltzmann factor involving $\mathcal{F}[\phi]$.

To establish a closer connection between fluctuations and $\mathcal{F}[\phi]$, let us consider the factorial moments

$$f_q = \sum_{n=q}^{\infty} \frac{n!}{(n-q)!} P_n = \frac{1}{Z}\int \mathcal{D}\phi \left(\int_V dz|\phi|^2\right)^q e^{-\int dz \mathcal{F}[\phi]} \tag{6}$$

If V is taken to be a d-dimensional cell of δ on each side, i.e. δ^d, then f_q depends on a, b, c, d, δ and q. Some of the integration factors can be eliminated if we consider the normalized factorial moments

$$F_q = f_q / f_1^q \tag{7}$$

In the simple case $c = 0$, which we examine first, it turns out that F_q depends on only one variable x [6, 7]

$$x = \frac{a^2}{2b}\delta^d \quad , \tag{8}$$

besides q, when $a < 0$, the hadron phase. The question then is whether F_q exhibits intermittency [10], i.e. power-law behavior in x

$$F_q(x) \propto x^{-\varphi_q} \tag{9}$$

when x is small. Since $F_q(x)$ can be exactly calculated, the answer can be determined unambiguously. It is found that there is no extensive region of x in which F_q behaves as in (9), except in the limit $x \to 0$ for which φ_q has the uninteresting value 0 [6].

The absence of intermittency in the usual sense does not mean the lack of scaling behavior in general. With the Ochs-Wosiek plot [11] in mind, we examined the dependence of F_q on F_2 as shown in Fig. 1. Evidently, there is linear behavior in the log-log plot for every q, implying the scaling law

$$F_q \propto F_2^{\beta_q} \quad . \tag{10}$$

To a very good degree of accuracy [8], β_q is independent of x. This is important because we do not know the parameters a, b and c at all for quark-hadron PT. What is even more amazing is the fact that β_q can be described by a very simple formula [6]

$$\beta_q = (q-1)^\nu \tag{11}$$

with a fixed exponent ν; it satisfies the trivial constaints at $q = 1$ and 2. As Fig. 2 indicates, all the values of β_q shown can be very well fitted by (11) with

$$\nu = 1.304 \quad . \tag{12}$$

Thus we have discovered a numerical constant that is a consequence of the Ginzburg-Landau theory of second-order PT. It is not derived from any numerical input, since it is independent of a, b, d and δ (for the case of uniform ϕ, i.e., $c = 0$). The universality of ν is what makes this result particularly attractive. Since there is no way to control temperature around the critical point in heavy-ion colliisons, there is no way to tune the parameters in the GL free energy. Nevertheless, the universality of ν renders that deficiency unimportant. It implies that despite our ignorance about those parameters

we can use the explicit value of ν to serve as a criterion to determine whether the hadrons produced are a result of the type of PT that we have considered here.

In general, c in (1) is not zero. The kinetic term makes the problem much more complicated. In [7, 8] we considered some approximations and found that the result is not affected too much by the $c|\partial\phi/\partial z|^2$ term. The scaling law (10) is still valid, and (11) is still good with

$$\nu = 1.316 \pm 0.012 \tag{13}$$

for all values of c.

Current data on multiparticle production in pp and AA collisions can all be described by (10) and (11). In Fig. 2 are shown some data points from nucleus-emulsion experiments [12, 13] with $\nu = 1.55 \pm 0.12$. The NA22 hh data [14] can be well fitted by (10) and (11); the result on the y distribution gives the value $\nu = 1.45 \pm 0.04$ [15] (note the small error). Higher dimensional analyses as well as in the Q^2 variable give higher values of ν [15]. We may conclude that current experiments on hadronic and nuclear collisions show no indication of PT according to our criterion, which is not a surprise.

It would be more satisfying if the predicted value of ν can be verified by experiment. Unfortunately, any heavy-ion experiment that can possibly lead to phase transition will have to wait for the completion of RHIC or LHC, which will be many years from now. Yet, fortunately there is another realm of physics in which an experiment can readily be carried out to test our theory. That is the production of photons at the threshold of lasing in quantum optics. It is known that the behavior of the laser system near threshold is analogous to a second-order PT describable by the GL theory [16]. Thus the fluctuation of the photon number should be as discussed above; in particular, the moments F_q of the number of photons emitted near threshold should satisfy (10) and (11). An experiment on this has been carried out recently [17]. The result for a homogeneous laser ($c = 0$) yields a value of ν in perfect agreement with (12), as shown in Fig. 3. With the verification of our scaling prediction and its universality property, we now feel more emboldened to propose that the scaling exponent should be used as a criterion to test in heavy-ion collisions the proximity of the quark-gluon system to the GL type of PT.

3 First-Order Phase Transition

If the PT is first order, then there is a mixed region of quarks and hadrons. Those hadrons are not the same as the ones that end up in the detector since they are at finite temperature T_c. Furthermore, the hadrons in the mixed region can stick together and grow. The growth process can depend on many factors. In cosmic PT the hadronic bubble can grow to a very large size, the dynamics being determined to a large extent

by the surface tension and the pressure difference at the quark-hadron interface. In heavy-ion collisions all scales concerning QCD, hadron mass, cluster size, and even nuclear radius are of the same order of magnitude. In that case one certainly cannot justify hydrodynamical treatment of the bubble boundary. Indeed, the bubbles need not be spherical, since dendretic structure of hadronic clusters cannot be ruled out on the basis of energy and pressure considerations at the quark-hadron interface. The problem seems untractable and very different from that of the second- order PT discussed above.

Despite the lack of a formalism that can be used to treat the problem quantitatively, we can approach the subject at the qualitative level first and ask some leading questions. If hadronic clusters are formed in the quark system, do they change in size, and if so, what mechanisms influence the change? What aspect of the PT can best characterize the transition? Is it observable?

The usual reason for bubble growth is that the negative volume term overcomes the positive surface term in the free energy difference from the quark to hadron phase. This is a consideration of the statics of the problem concerning pressure difference, and can provide a basis for the study of the dynamical change of the average size of the bubbles. Since the conventional approach does not give a realistic treatment when the bubbles are not large and are irregular in shape, we have to abandon asking the conventional questions, but ask instead whether the fluctuations of the cluster size from the average can reveal other interesting aspects about PT relatively independent of what controls the shape and average size of the clusters. Since the hadronic clusters are subject to collisions by quarks and gluons in the mixed region, they behave like Brownian particles performing random walks around a drift motion toward the boundary. They can therefore collide in their random motion, whereupon the clusters can coalesce, breakup, or scatter elastically. Successive coalescence would lead to large clusters — rare but possible. Thus the shape of the cluster-size distribution $P(S)$ can reveal the dynamical process that the clusters go through before emerging from the plasma.

We have put these ideas in a cellular automaton and simulated the cluster growth process [18]. We found that for various values of the coalescence and breakup probabilities we can get scaling behavior in $P(S)$, i.e.

$$P(S) \propto S^{-\sigma}\,, \tag{14}$$

except when S gets too large and the finite-size effect causes $P(S)$ to deviate from (14). This is shown in Fig. 4. On the basis of this result we suggest that the detection of cluster sizes should be emphasized in future experiments on heavy-ion collisions. The experimental discovery of a scaling distribution such as (14) would undoubtedly stimulate a great deal of excitement in the field.

There is, however, a possible hurdle having to do with the spreading in momentum space of the hadrons originating from a cluster. Although there are experimentalists

who regard it as their job to identify clusters, it is nevertheless interesting to ask how much the spreading can be. We have considered a quenching-bunching mechanism that may limit the spreading [19]. Assuming that there is no hadron gas surrounding the plasma, the sudden immersion of a cluster of hadrons at T_c into the cold vacuum at $T = 0$ may be referred to as quenching. After the cluster is thrust into the cold vacuum, the breaking of chiral symmetry is brought to completion by the $T = 0$ new environment, so the quark mass reaches its full constituent quark mass at 330 MeV from a value many times lower at T_c. Since the cluster is now in the vacuum, the only way the particles in it can become the usual hadrons with physical masses is that the quarks gain their masses at the expense of the thermal energy of the particles in the cluster, there being no other agent available to supply the needed energy. The ρ meson and other vector particles may never attain their physical masses. The loss of the relative kinetic energy of the final hadrons in the cluster is what we mean by bunching. At midrapidity the hadrons are mainly pions and kaons, which according to the bunching mechanism should therefore form clusters of particles close together in phase space [19].

Apart from the phenomenological question of how to identify the clusters, there is also the interesting theoretical observation that cluster formation in our problem is analogous to the avalanche in the sand-pile problem [20]. The latter is an example of self-organized criticality. For more discussion on the similarity between the two problems, an interested reader is referred to Ref. [18].

4 Conclusion

We have discussed possible observable consequences of quark-hadron PT. In the case that the PT is second order we have studied the F_q moments in the framework of the Ginzburg-Landau theory and found a scaling behavior characterized by an exponent ν, whose value can be calculated. Its value for a homogeneous medium is 1.304, and has been verified experimentally to be correct for photocount at lasing threshold. We propose that the same ν should be used as a criterion to determine whether hadron production in heavy-ion collisions has gone through a GL type PT.

If the PT is first order, then we suggest that the relevant observable is the cluster size. A simulation of the cluster formation and growth process indicates that one can expect a scaling distribution of the cluster sizes. Experimental investigation of that possibility is therefore strongly urged. Any nontrivial distribution would suggest something unusual and therefore interesting.

Acknowledgments

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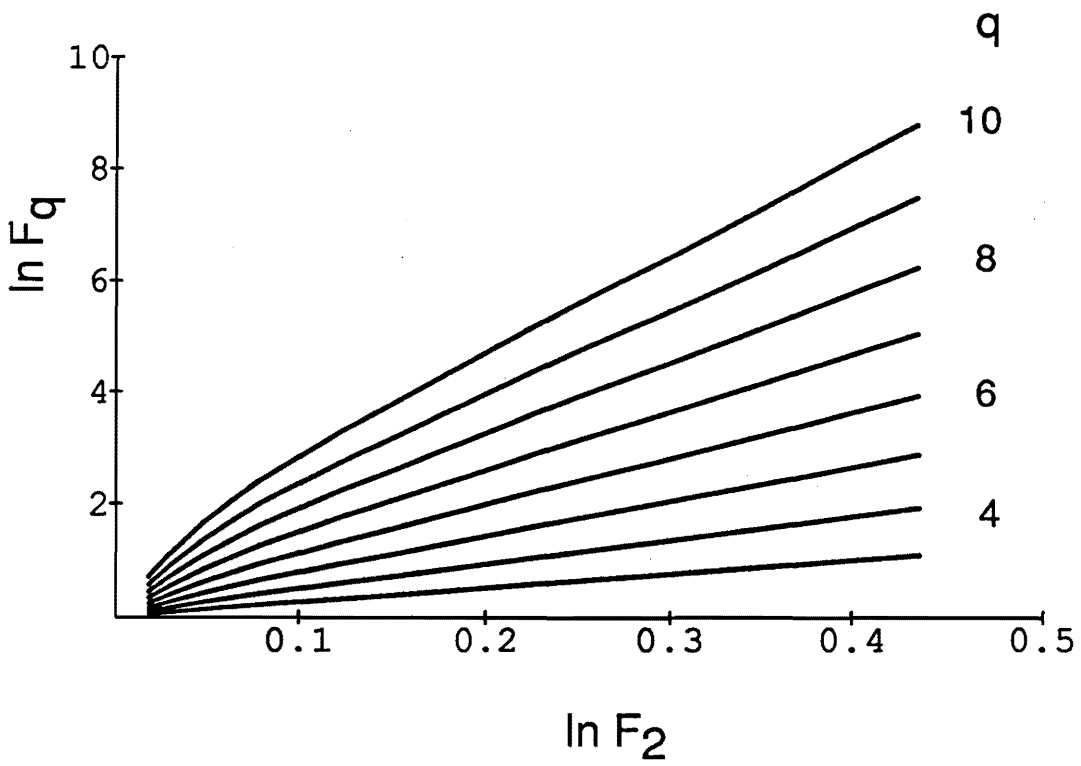
Figure Captions

Fig.1 Scaling behavior of F_q in terms of F_2.

Fig.2 Dependence of β_q on q. Points are from calculation in [6]; data are from [12, 13].

Fig.3 Data on β_q are from laser experiment [17]; the curve is from theory, (11) and (12).

Fig.4 Simulated result on $P(S)$ for various values of b, c and p parameterizing coalescence, breakup and nucleation probabilities.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10
8
6
4
ln F_q
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8
6
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2
0.1
0.2
0.3
0.4
0.5
ln F_2

Fig. 1

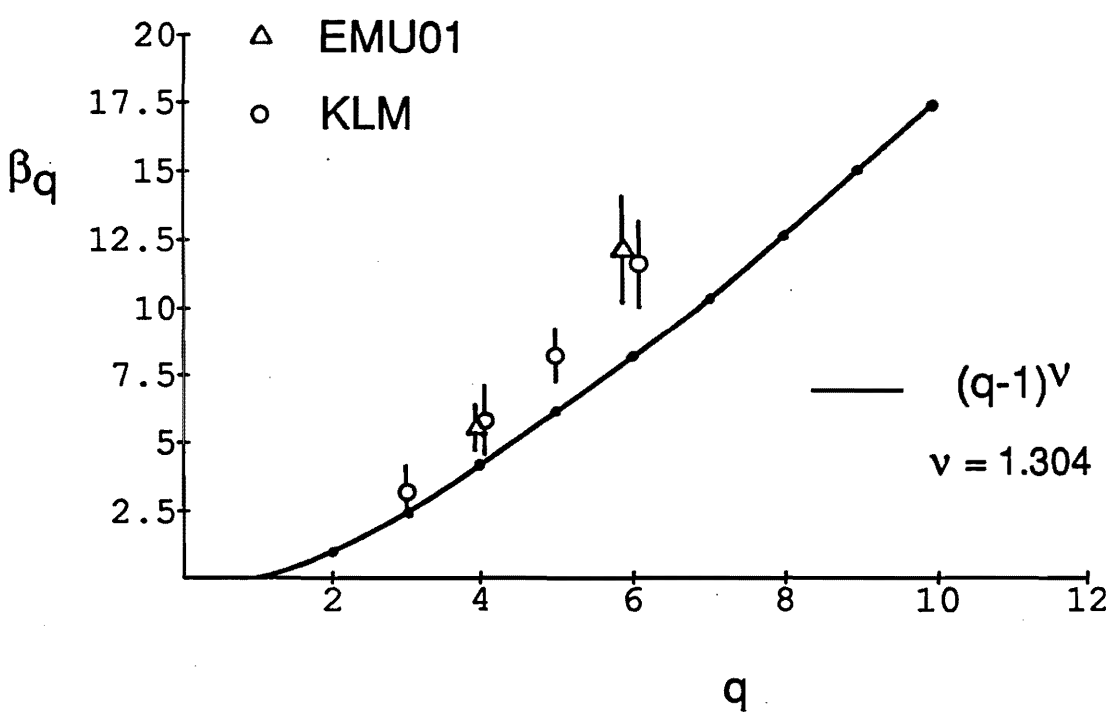
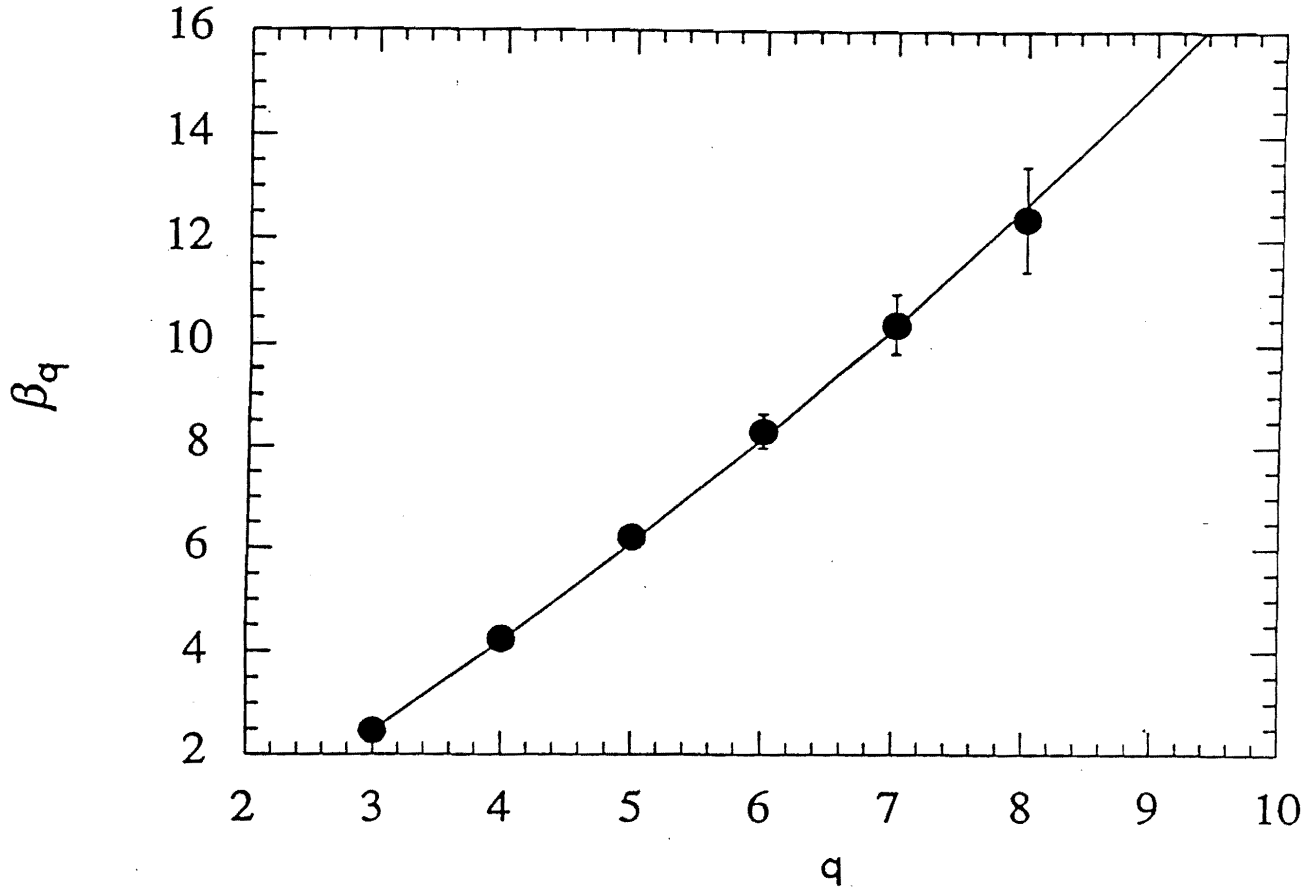

EMU01
KLM
β_q
20
17.5
15
12.5
10
7.5
5
2.5
$(q-1)^\nu$
$\nu = 1.304$
2
4
6
8
10
12
q

Fig. 2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14
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β_q
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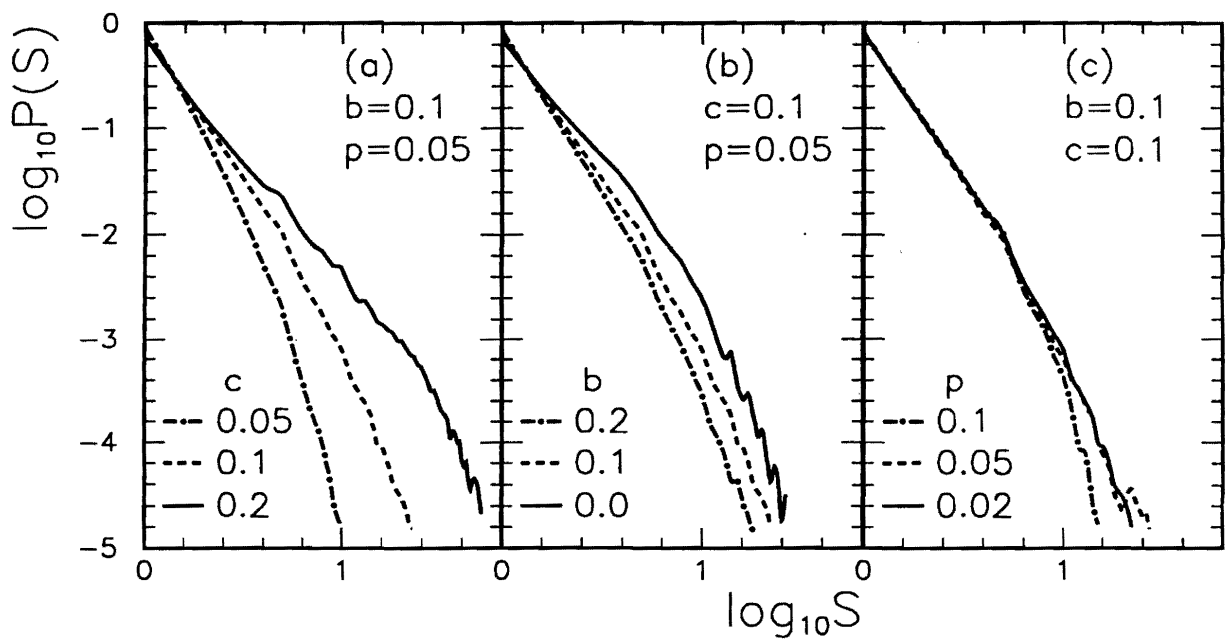
5
6
7
8
9
10
q

Fig. 3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−3
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−5
$\log_{10}P(S)$
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b=0.1
p=0.05
(b)
c=0.1
p=0.05
(c)
b=0.1
c=0.1
c
0.05
0.1
0.2
b
0.2
0.1
0.0
p
0.1
0.05
0.02
0
1
0
1
0
1
$\log_{10}S$

Fig. 4