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QUARK MASSES IN DUAL THEORIES *

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ABSTRACT

We consider the $N = 2$, $N_f = 2$ Seiberg-Witten dual theory with two bare quark masses. We find that it is possible to have an arbitrary large mass ratio.

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Recently, a great deal of progress was made in dual theories fostered by Seiberg and Witten who provided the exact vacuum structure and spectrum of four dimensional $N = 2$ Supersymmetric SU(2) QCD with matter [1,2].

We are interested in the hierarchy of quark masses in an $N = 2$, $N_f = 2$ (f= flavor) Seiberg Witten toy model with two bare masses. In order to motivate the equation of family of curves for $N_f = 2$, that parametrize the modular space of the quantum vacau, we briefly outline its derivation.

The low energy effective Lagrangian involves the $N = 2$ vector multiplet A that contains gauge fields A_μ, two Weyl fermions λ, ψ and a scalar ϕ. In terms of $N = 1$ supersymmetry, the fields are vector multiplet $W_\alpha(A_\mu, \lambda)$ and chiral multiplet $\Phi(\phi, \psi)$. The Lagrangian and Kahler potential can be written in terms of a prepotential $\mathcal{F}(A)$ that is an arbitrary holomorphic function of complex variables. The scalar component of the $N = 2$ vector multiplet A is a. The metric on the moduli space parametrized by a is

$$(ds)^2 = Im\tau(a)dad\bar{a}$$

The $\tau(a)$ is a holomorphic function, and $\frac{\partial^2 \mathcal{F}}{\partial a^2} = \tau(a)$. Define $a_D = \frac{\partial \mathcal{F}}{\partial a}$ then $(ds)^2 = Imda_D d\bar{a}$. The tree and one loop contribution to $\mathcal{F}$ is

$$\mathcal{F}_{\text{oneloop}} = i\frac{2}{\pi}A^2 \ln\frac{A}{\Lambda} \qquad \Lambda = scale$$

$\mathcal{F}$ follows from singularity structure and monodromy.

From a_D and $\mathcal{F}_{\text{oneloop}}$ for large a, one has

$$a_D = \frac{4ia}{\pi}\ln(a/\Lambda) + 2ia/\pi$$

that is not a single valued function. Put $u = 2a^2$, and the monodromy is determined by drawing a circle or closed loop around a point u on the u plane at large u,

$$\begin{aligned} u \to e^{2\pi i}u, \quad &\text{or} \quad \ln u \to \ln u + 2\pi i, \\ a \to e^{\pi i}a \to -a \quad &\qquad (\ln a \to \ln a + \pi i). \end{aligned}$$

Then,

$$\begin{aligned} a_D &\to -a_D + 4a, \\ a &\to -a, \end{aligned}$$

or

$$\begin{pmatrix} a_D \\ a \end{pmatrix} \to M_\infty \begin{pmatrix} a_D \\ a \end{pmatrix}, \text{where } M_\infty = \begin{pmatrix} -1 & 4 \\ 0 & -1 \end{pmatrix}.$$

Consider magnetic coupling [3, 4]

$$\beta_D(g_D) = \mu \frac{dg_D}{d\mu} = \frac{g_D^3}{8\pi^2} \quad \text{or} \quad \mu \frac{d\tau_D}{d\mu} = -\frac{i}{\pi},$$

where $\tau_D = 4\pi i/g_D^2(a_D)$ when $\theta_D = 0$. Since the scale μ here is a_D

$$d\tau_D = -\frac{i}{\pi}\frac{da_D}{a_D},$$

or

$$\tau_D = -\frac{i}{\pi}\ln a_D.$$

Define $h_D(A) = \frac{\partial \mathcal{F}}{\partial A_D}$, $A_D =$ dual vector multiplet, and scalar component of A_D is a_D, then

$$\frac{dh_D(a)}{da_D} = \frac{\partial^2 \mathcal{F}}{\partial a_D \partial a_D} = \tau_D$$

hence

$$h_D(a) = \int \tau_D da_D = -\frac{i}{\pi}\int \ln a_D da_D$$

In the $N = 2$ gauge theory, the central charge Z is

$$Z = a(n_e + \tau_{cl} n_m) = (an_e + a_D n_m),$$

where $\tau_{cl} \sim \frac{\theta}{2\pi} + \frac{4\pi i}{g^2}$, n_e and n_m are integers. The BPS bound as the mass of the dyon is $M \geq \sqrt{2}|Z|$.

For later convenience,

$$Z = (n_m, n_e)\begin{pmatrix} a_D \\ a \end{pmatrix} \equiv qa^\alpha.$$

Note $\sqrt{2}a_D$ is the mass of the monopole and duality symmetry is $a \leftrightarrow a_D$, $n_e \leftrightarrow n_m$, $\tau \leftrightarrow -\frac{1}{\tau} = \tau_D$. The contribution of monopole to BPS bound is $\sqrt{2}a_D n_m$. The full duality group is SL(2, Z) under which a, a_D transform as a doublet.

Take a massless monopole at u_0

$$u_D(u_0) = 0.$$

Near $u_0(= \Lambda^2)$,

$$a_D(u) = c_0(u - u_0),$$

$$a(u) = -h_D = \frac{i}{2\pi} c_0(u - u_0) \ln(u - u_0) + a_0,$$

where

$$a_0 = a(u_0), \qquad dh_D = \tau_D da_D.$$

Let u circle around u_0 in the u plane,

$$\ln(u - u_0) \to \ln(u - u_0) + 2\pi i$$

or

$$a(u) \to a - a_D,$$

$$a_D(u) \to a_D,$$

hence

$$M_{u_0} = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$$

When the three monodromies are taken in counter clockwise direction where from a base point a large loop that includes those that include u_0 and $-u_0$, the monodromies obey

$$M_{u_0} M_{-u_0} = M_\infty,$$

so

$$M_{-u_0} = \begin{pmatrix} -1 & 4 \\ -1 & 3 \end{pmatrix}.$$

The pair of singularies $\pm u_0$ is required by the Z_2 symmetry $u \to -u$. The transformation of functions around a complex singularity are monodromies and the matrices are called monodromy matrices. Note

$$h(A) = \frac{\partial \mathcal{F}}{\partial A} \quad \tau(A) = \frac{\partial^2 \mathcal{F}}{\partial A^2} = \frac{\partial h(A)}{\partial A} = h',$$

$$A_D = \frac{\partial \mathcal{F}}{\partial A} = h(A) \text{ or } A = -h_D(A_D).$$

Duality transformation has freedom to shift A_D by a constant, and

$$-\frac{1}{\tau(A)} = -\frac{1}{h'(A)} = \tau_D(A_D) = h'_D(A_D),$$

$$S\begin{pmatrix} h_D \\ h \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} h_D \\ h \end{pmatrix} = \begin{pmatrix} h \\ -h_D \end{pmatrix},$$

duality implements SL(2, Z) generator S.

A natural physical interpretation of singularities in u plane is that some additional massless particles are appearing at a particular value of u [5]. What particle becomes massless to generate the singularity? The massless particle that produces a monodromy M satisfies,

$$\begin{array}{lll} qM = q, & q = (n_m, n_e), & \\ q = (1,0), & qM_{u_0} = (1,0)\begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} = (1,0), & \text{monopole.} \\ q = (1,-1), & qM_{u_0} = (1,-2)\begin{pmatrix} -1 & 4 \\ -1 & 3 \end{pmatrix} = (1,-2) & \text{dyon.} \\ q = (n_m, n_e) & qM_\infty = (n_m, n_e)\begin{pmatrix} -1 & 4 \\ 0 & -1 \end{pmatrix} = (-n_m, 4n_m - n_e) & \end{array}$$

There is no simple solution.

The moduli space M of quantum vacua is the u plane with singularities. We described the u plane punctured at the singularities $-\Lambda^2$, Λ^2 and ∞ with their monodromy matrices.

The family of curves of the moduli space can be parametrized by the cubic equation

$$y^2 = (x + \Lambda^2)(x - \Lambda^2)(x - u).$$

For every u, there is a genus one Riemann surface-torus determined by the equation above with discriminant

$$\Delta = \prod_{i<j}(e_i - e_j)^2 = (e_1 - e_2)^2(e_1 - e_3)^2(e_2 - e_3)^2 = 4\Lambda^4(\Lambda^4 - u^2)^2.$$

The previously found $M_{-\Lambda}, M_\Lambda, M_0$ are obtained in Refs. [1] and [2]. The x plane has 4 branch points at $x = -\Lambda^2, \Lambda^2, u, \infty$. The branch points are joined pair wise by two cuts along $-\Lambda^2$ to Λ^2 and u to ∞ in x plane.

Consider the toy model massive $N_f = 2$, with masses m_1 and m_2. The most general structure of the curve is

$$y^2 = (x^2 - t\Lambda_2^4)(x-u) + \mu^2\Lambda_2^3(ax+bu) + 2cm^2\Lambda_2^4,$$

where

$$\mu^2 = m_1 m_2, \qquad m^2 = \frac{1}{2}(m_1^2 + m_2^2).$$

and t, a, b, c are constants to be determined.

The $U(1)_R$ charge (global phase rotation) is conserved, where the assignment of charges for the parameters and x and y are

u	x	Λ	m	y
4	4	2	2	6.

Define λ_2 such that when m_2 is large low energy $N_f = 1$ is finite $\Lambda_1^3 = m_2\lambda_2^2$ $m_2 \to \infty, \lambda_2^2 \to 0$.

Then, we obtain $a = \frac{1}{4}, b = 0$ and $c = -\frac{1}{64}$ and

$$y^2 = (x^2 - t\Lambda_2^4)(x-u) + \frac{1}{4}m_1 m_2 x - \frac{1}{64}(m_1^2 + m_2^2)\Lambda_2^4.$$

To find t, put $m_2 = 0$, $u = m_1^2$ in discriminant Δ, put coefficients of the leading term in m_1^8 in Δ equal to zero, and get $t = \frac{1}{64}$. Then the equation with $s = \Lambda^2/8$ is

$$y^2 = (x^2 - s^2)(x-u) + 2\mu^2 sx - 2m^2 s^2.$$

We are interested in examining the possible mass heirarchy in this $N_f = 2$ model. Define $m_1 = M + D$, $m_2 = M - D$ so $\mu^2 = m_1 m_2 = M^2 - D^2$. We determined the Δ with masses μ^2 and m^2. Then, m^2 is replaced by $m^2 = \mu^2 + (m^2 - \mu^2)$. The μ^2 part leads to the previous equal mass case and the coefficient of $(m^2 - \mu^2) = 2D^2$ leads to the constraint. We find

$$\Delta = 4s^2[(u+s)^2 - 8\mu^2 s](u - s - \mu^2)^2$$

with a constraint

$$D^2[(3s^2 - su)M^2 + suD^2 - us^2 + \frac{u^3}{9}] = 0.$$

If $D = 0$, $\mu^2 = m^2$ and the equal mass result is obtained [2, 6].

If $D \neq 0$, we get

$$(3s^2 - su)M^2 + suD^2 - us^2 + \frac{u^3}{9} = 0.$$

Consider u near the double zero of $\Delta = 0$,

$$\mu^2 = u - s = M^2 - D^2,$$

then we find

$$M^2 = \frac{u^2}{3s} - \frac{u^3}{27s^2} = u\left(\frac{u}{3s} - \frac{u^2}{27s^2}\right)$$

$$D^2 = u\left[\frac{s}{u} - 1 + \left(\frac{u}{3s} - \frac{u^2}{27s^2}\right)\right]$$

When $m_1 > 0, m_2 > 0$, we require

$$M^2 > D^2 \quad M^2 > 0 \quad D^2 > 0$$

and note

$$\frac{m_1}{m_2} = \frac{M+D}{M-D} = \frac{(M+D)^2}{M^2 - D^2}.$$

For $\frac{u}{s} = 1 + \epsilon, \quad \epsilon > 0, \quad s = \Lambda^2/8$

$$\frac{m_1}{m_2} = \frac{32}{27}\frac{1}{\epsilon}.$$

Large mass ratio is possible. For $\frac{u}{s} = 2$

$$\frac{m_1}{m_2} = \frac{29}{27} + \frac{4}{27}\sqrt{7} \approx 1.47.$$

Consider next the two other zeros of Δ

$$\mu = \pm(u+s)/\Lambda,$$

then

$$M^2 = u\left(3 + \frac{16}{3}\frac{u}{s} + \frac{71}{27}\frac{u^2}{s^2}\right),$$

$$D^2 = u\left(-13 - \frac{8}{3}\frac{u}{s} - 8\frac{s}{u} + \frac{71}{27}\frac{u^2}{s^2}\right).$$

For $\frac{u}{s} = 3 \quad M^2 = 42.6 \quad D^2 = 0 \quad m_1 = m_2 = \Lambda/2$ and a large mass ratio is not possible. For $\frac{u}{s} = 4, \quad m_1/m_2 \approx 3$.

We verify that the relation u/s is in the strong coupling regime for all of the preceding values [7]. The weak coupling regime satisfies $\Lambda^2 \ll u$. For the case of double zeros, $u/s = 1 + \epsilon$ or $u/\Lambda^2 \approx 1/8$ or $\Lambda^2 \sim 8u$ so $\Lambda^2 \ll u$ is not satisfied. The case of $\mu = 1/2(u+s)/\Lambda, \quad \Lambda^2 = 8u/3$ so again $\Lambda^2 \ll u$ is not satisfied.

The problem with $N_f = 3$, 3 masses m_a, m_b, and m_c is being studied with G. Cleaver.

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