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9

Goldberger-Treiman Relation for Technipion

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Abstract

We assume that technipions π_T exist with masses small on the scale of the top-quark mass. An analog of the Goldberger-Treiman relation in the soft-technipion scheme allows us to determine the $tb\pi_T$ Yukawa coupling constant. The widths for the strong decay $t \rightarrow b + \pi_T^+$ and the weak decay $\pi_T^\pm \rightarrow \tau^\pm \pm \nu_\tau$ are calculated.

In technicolor theory it is assumed that techniquarks are coupled to yet another QCD-like gauge field, the technicolor gauge field of QTC, that gives rise to the bindings of the techniquarks and anti-techniquarks to form technicolor-singlet composite states. In the theory of dynamically broken electroweak gauge symmetry [1,2] the massless pseudoscalar composites, technipions ($\pi_T^{\pm}$, π_T^0) and the technieta (η_T), become the longitudinal components of the $W^{\pm}$ and Z^0 vector mesons,[3] playing the role of the elementary Higgs scalars in the spontaneously broken gauge symmetry. The mass of $W^{\pm}$ is given by

$$M_W = \frac{1}{2} g F_{\pi_T} \approx 80 GeV \ , \tag{1}$$

where g is the constant of the SU$(2)_L$ gauge field coupling to the doublet of techniquarks, and F_{π_T} the technipion decay constant. From Eq. (1) it is found

$$F_{\pi_T} \approx 250 GeV \ . \tag{2}$$

This value is to be compared with $f_\pi \approx 93 MeV$ for the ordinary pion.

The mass of the top quark (t) has been determined to be in the range 140 ± 45 GeV from the radiative correction to the decay $Z^0 \to b\bar{b}$ jets, where $b(\bar{b})$ is the bottom (anti-bottom) quark.[4] It is assumed that this reaction was initiated by the transition in which Z^0 decays to a virtual $t\bar{t}$ pair which becomes the $b\bar{b}$ pair by the exchange of $W^{\pm}$.

Such a large t mass can arise only from a large $t\bar{t}$ condensate, indicating that t should be regarded as the techniquark.[5] Assuming that t and b form an isospin doublet of techniquarks, the $t\bar{b}(b\bar{t})$ composites in pseudoscalar states may be identified as the technipions $\pi_T^+(\pi_T^-)$. In the dynamically broken electroweak gauge symmetry, these technipions must be massless so as to be absorbed into the longitudinal components of the $W^{\pm}$ mesons. They are to be regarded as the Nambu-Goldstone bosons of the spontaneously broken chiral symmetry, and as such do not exits in the spectrum of the physical particles.

If, on the other hand, the electroweak gauge symmetry is broken spontaneously by the Higgs mechanism, the technipions, if exist, cannot be absorbed by the $W^{\pm}$ mesons, since the longitudinal components are already occupied by the scalars of the Higgs field that have been absorbed. The technipions are then expected to emerge as pseudo-Goldstone bosons[6] with masses small on the scale of the top-quark mass m_t.

The answer to the question as to whether the electroweak symmetry breaking is spontaneous via the Higgs mechanism or dynamical via the absorption of the Nambu-Goldstone bosons of the spontaneously broken chiral symmetry by the gauge field should be determined by experiment. In the former, technipions are expected to exist as the physical pseudo-Nambu-Goldstone bosons of the broken chiral symmetry, whereas in the latter no such particle should be observed.

The smallness of the mass of the technipion as a pseudo-Goldstone boson on the scale of m_t should make it possible to proceed with the notion of a soft technipion, in analogy with that of the soft pion, and study one of its possible consequences, the Goldberger-Treiman relation.[7,8,9]

The matrix element of the axial vector current for the $t \to b$ transition with emission of π_T^+ may be written as

$$\langle b(k')|A_\mu^1 + iA_\mu^2|t(k)\rangle = \bar{u}_b(k')[\gamma_\mu\gamma_5 g_A(q^2) + q_\mu\gamma_5 h_A(q^2)]\cos\theta_{TC} u_t(k) \ , \qquad (3)$$

where $q_\mu = k'_\mu - k_\mu$ is the momentum transfer, and $g_A(q^2)$ and $h_A(q^2)$ the axial vector and pseudoscalar form factors, respectively. θ_{TC} is the anticipated angle of rotation of the weak interaction eigenstates from the mass eigenstates of the techniquarks t and b. In the chiral $SU(2)_L \times SU(2)_R$ symmetry limit the axial vector is conserved

$$\partial^\mu A_\mu = 0 \ . \tag{4}$$

We then obtain in this limit

$$(m_t + m_b) g_A(q^2) + q^2 h_A(q^2) = 0 \ . \tag{5}$$

In analogy to the case of the axial vector current in the β-decay of the neutron, we assume that $h_A(q^2)$ is dominated by the technipion pole, and proceed to determine its form. The interaction Lagrangian density for the emission of π_T^+ in the $t \to b$ transition and the coupling of π_T^+ to the lepton current is given by

$$L' = i\sqrt{2}\ g_{tb\pi_T} \bar{b}\gamma_5 t \pi_T^+ + \sqrt{2}\ F_{\pi_T} \cos\theta_{TC} \partial_\alpha \pi_T^+\ l^\alpha, \tag{6}$$

where $g_{tb\pi_T}$ is the Yukawa coupling constant at the $tb\pi_T$ vertex, F_{π_T} the technipion decay constant given by Eq. (1), and l^α the lepton current. The pseudoscalar form factor $h_A(q^2)$ dominated by π_T is then obtained from Eq. (6). It is given by

$$h_A(q^2) = -\sqrt{2}\ g_{tb\pi_T} F_{\pi_T}\ \frac{1}{q^2 - m_{\pi_T}^2} \ . \tag{7}$$

In the soft π_T limit ($m_{\pi_T}^2 \to 0$) we obtain from Eqs. (5) and (7)

$$(m_t + m_b) g_A(0) = 2 g_{tb\pi_T}(0) F_{\pi_T} \ . \tag{8}$$

Further, assuming $q_{tb\pi_T} = q_{tb\pi_T}(m_{\pi_T}^2)$, we obtain the Goldberger-Treiman relation for our case

$$(m_t + m_b) g_A(0) = 2 g_{tb\pi_T} F_{\pi_T} \ . \tag{9}$$

For $m_t = 140GeV$ and F_{π_T} given by Eq. (2) we obtain

$$g^2_{tb\pi_T}/4\pi = 0.00660(g_A(0))^2 \ . \tag{10}$$

Assuming $g_A(0)^2 \approx 1$, we find $g^2_{tb\pi_T}/4\pi$ to be of the order of magnitude of $\alpha = e^2/4\pi$. Such a small Yukawa coupling constant at the $tb\pi_T$ vertex may be a reflection of the reduction in the QTC gauge coupling strength responsible for the binding of $\bar{t}b(\bar{b}t)$ to form the technipion $\pi^-_T(\pi^+_T)$ at the energy scale set by $m_t \approx 140GeV$.

For several values assumed for m_t and m_{π_T} we have calculated the widths for the decay transition $t \to b + \pi^+_T$ with the Yukawa coupling constant $g_{tb\pi_T}$ determined by Eq. (9). The results are compared with the decay widths for $t \to b + W^+$ obtained in the standard model for the electroweak interactions.

In the rest system of t, we get from the Yukawa coupling term in Eq. (6)

$$\Gamma(t \to b + \pi^+_T) = \frac{g^2_{tb\pi_T}}{4\pi}\frac{(m_t - m_b)^2}{m_t^2}k \ , \tag{11}$$

where

$$k = [\{\frac{m_t^2 - m_b^2 + m_{\pi_T}{}^2}{2m_t}\}^2 - m^2_{\pi_T}]^{1/2} \ . \tag{12}$$

The decay width for $t \to b + W^+$ is obtained from the interaction Lagrangian density

$$L' \ = -\frac{g}{\sqrt{2}}\ (\bar{b}_L\gamma^\alpha t_L \cos\theta_{TC} + \bar{\tau}_L\gamma^\alpha\nu_\tau)W^-_\alpha + h.c. \ , \tag{13}$$

where $g = e/\sin\theta_W$, with θ_W the Weinberg angle. In the rest system of t we obtain for the decay width

$$\Gamma(t \to b + W^+) = \frac{1}{2}g^2\cos^2\theta_{TC}\frac{1}{16\pi m_t^2}[m_t^2 + m_b^2 - 2m_W^2 + (m_t^2 - m_b^2)^2/m_W^2]k \ , \tag{14}$$

where

$$k = [\{\frac{m_t^2 - m_b^2 + m_W^2}{2m_t}\}^2 - m_W^2]^{1/2} \ . \tag{15}$$

The results are given in Table 1 for unspecified values for $(g_A(0))^2$ and $cos^2\theta_{TC}$.

The weak leptonic and quark-antiquark decays of $\pi_T^{\pm}$ mediated by $W^{\pm}$ can be obtained immediately from the interaction Lagrangian

$$L' \ = \frac{g}{2} F_{\pi_T} cos\theta_{TC} \partial^{\alpha} \pi_T^{+} W_{\alpha}^{-} + \frac{g}{\sqrt{2}} \ [\bar{\nu}_{\tau} \gamma^{\beta} \tau_L^{-} + \bar{c}_L \alpha^{\beta} s_L cos\theta_C] W_{\beta}^{-} + h.c. \ , \tag{16}$$

where τ^- and ν_τ are the τ^--lepton and τ-like neutrino, respectively, and c and s the charm and strange quarks, respectively, and θ_C the Cabibbo angle.

The leptonic decay width is given by[10]

$$\Gamma(\pi_T^{\pm} \to \tau^{\pm} \pm \nu_\tau) = \frac{1}{128\pi} F_{\pi_T}^2 \ \cos^2\theta_{TC} \ g^4 \ (m_W)^{-4} \ m_{\pi_T} (1 - m_\tau^2/m_{\pi_T}^2)^2 m_\tau^2 \ , \tag{17}$$

where $g = e/\sin\theta_W$. For the quark-antiquark decay width $\Gamma(\pi_\tau^{\pm} \to \pm c \mp s)$ the last factor in Eq. (17) should be replaced by $(1 \ - \ m_c^2/m_{\pi_T}^2)^2 m_c^2 \cos^2\theta_c$. We have kept only the most massive lepton and the quark. The results are given in Table 2.

In Table 1 $\Gamma(t \to b + \pi_T^{+})/g_A(0)^2$ and $\Gamma(t \to b + W^{+})/\cos^2\theta_{TC}$ are given at several values of m_t and m_{π_T}. Depending upon the values of the unkown parameters $g_A(0)^2$ and $\cos^2\theta_{TC}$, $\Gamma(t \to b + \pi_T^{+})$ may not be negligibly small in comparison to $\Gamma(t \to b + W^{+})$.

In Table 2 $\Gamma(\pi_T^{\pm} \to \tau^{\pm} \pm \nu_t)/\cos^2\theta_{TC}$ and $\Gamma(\pi_T^{\pm} \to \pm c \mp s)/\cos^2\theta_{TC}$ are listed at several values of m_{π_T}. These decay width are small by $0(10^{-3})$ in comparison to the decay widths of the hadronic resonances.

The ordinary pions are pseudo-Nambu-Goldstone bosons of the spontaneously broken chiral symmetry in QCD. If the technipions are similar objects in QTC, relatively light spin-zero pseudoscalar $t\bar{b}(b\bar{t})$ composite states are expected to exist in nature. As was emphasized, the existence of such states, if observed, would lead to the view that it is the Higgs scalars that constitute the longitudinal components of the $W^{\pm}$ mesons, in exclusion of the technipions, thus favoring the spontaneous breaking of the elctroweak symmetry over the dynamical breaking. On the other hand, the consistent non-observation of technipions would suggest that the electroweak symmetry breaking is dynamical, in that the technipions are true Nambu-Goldstone bosons constituting the longitudinal components of the $W^{\pm}$ mesons.

The technipions, if exist with masses less than 45 GeV, are likely to have been observed already at LEP. For larger masses they could possibly be looked for at the existing and future proton-antiproton colliders and the future electron-positron colliders.

Finally, another possible effect of interest in which the technipions could play a role is in the radiative correction to the observed phenomenon $Z^0 \rightarrow b\bar{b}$ in which the virtual $t\bar{t}$ from Z^0 is subsequently converted to $b\bar{b}$ via the exchanges of $\pi_T^{\pm}$, in addition to those of $W^{\pm}$.[4]

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Table 1*

$m_t = 100 GeV$ $g^2_{tb\pi_T}/4\pi = 0.00344(g_A(0))^2$

m_{π_T}	$k(\pi_T)$	$\Gamma(t \rightarrow b + \pi^+_T)/g_A(0)^2$	$k(W^+)$	$\Gamma(t - b + W^+)/cos^2\theta_{TC}$
80	17.6	0.0543	17.6	0.0865
60	31.7	0.0984		
40	41.9	0.130		
20	47.9	0.149		
10	49.4	0.153		

$m_t = 140 GeV$ $g^2_{tb\pi_T}/4\pi = 0.00660(g_A(0))^2$

m_{π_T}	$k(\pi_T)$	$\Gamma(t \rightarrow b + \pi^+_T)/g_A(0)^2$	$k(W^+)$	$\Gamma(t - b + W^+)/\cos^2\theta_{TC}$
120	18.2	0.112	47.0	0.627
100	34.1	0.209		
60	57.1	0.350		
20	68.5	0.420		
10	69.6	0.427		

$m_t = 200 GeV$ $g^2_{tb\pi_T}/4\pi = 0.0132(g_A(0))^2$

m_{π_T}	$k(\pi_T)$	$\Gamma(t \rightarrow b + \pi^+_T)/g_A(0)^2$	$k(W^+)$	$\Gamma(t - b + W^+)/cos^2\theta_{TC}$
150	43.6	0.546	83.9	2.08
100	74.9	0.940		
50	93.7	1.18		
20	98.9	1.24		
10	99.7	1.25		

The top-quark decay rates for different values assumed for m_t andm_{π_T}

* All masses, momenta, and the decay widths are in units of GeV

Table 2

$\underline{m_{\pi_T}}$	$\Gamma(\pi_T^+ \to \tau^+ + \nu_\tau)/\cos^2\theta_{TC}$	$\Gamma(\pi_T^+ \to c + +\bar{s})/\cos^2\theta_{TC}$
10 GeV	19.1 keV	12.6 keV
20	46.6	26.3
40	74.0	31.1
60	112.0	79.3
100	186.0	132.0

The leptronic and hadronic decay rates of π_T at diffeent masses assumed for π_T.

Notes and References

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