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LIGHT FRONT TAMM-DANCOFF METHOD HYDROGEN ATOM AND POSITRONIUM

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Abstract

The light front frame and Tamm-Dancoff method is reviewed briefly and the advantages and disadvantages of the light front Tamm-Dancoff method are discussed. The method is applied to the H atom, and positronium.

The light front Tamm-Dancoff (LFTD) method is introduced as an alternative approach to lattice gauge theory to understand relativistic bound states.[1,2]

To alleviate the case the coupling parameter is not small, Tamm and Dancoff[3] (TD) independently considered the possibility of expanding in terms of amplitudes that represent a finite number of particles, and solve the coupled set of integral equations of a small number of these amplitudes. This method was applied to a variety of problems in strong interactions involving the mesons, but it was unsuccessful. Why did the TD method fail? Some of the reasons are (1) non-trivial vacuum structure was an insuperable problem because of the violation of cluster decomposition requirement in TD approximation, and the so called Z diagrams in which the energy in the intermediate state is negative, (2) the non-covariant treatment due to the dropping of amplitudes that represent a larger number than the restricted number of particles, (3) the situations where a small number of particles may not represent the underlying physics very well. The LFTD take care of (1) and (2). To check (3), a large number of model calculations are being performed.

In the light front (LF) or null plane formalism, a physical state is given by the evolution variable that plays the role of time $x^+ = (x^0 + x^3)/\sqrt{2} =$ const, and $x^- = (x^0 - x^3)/\sqrt{2}$, $x_\perp = (x^1, x^2)$ are spatial variables. There are seven generators that leave the LF plane invariant.[4]

The momentum of a particle of mass m is

$$p^\mu = \left(p^+,\ p^- = \left(m^2 + p_\perp^2\right)/2p^+,\ p_\perp = \left(p^1, p^2\right)\right) \qquad \mu = +, -, 1, 2$$

where $p^\pm = (p^0 \pm p^3)/\sqrt{2}$ and $p^2 = m^2 + p_\perp^2 - p_\perp^2 = m^2$, where

$$g_{\mu\nu} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} .$$

The p^+ and $p_\perp$ are conserved at each vertex and particles are on mass shell. We require spectrum of p^μ be contained in the forward cone

$$p^0 > 0, \ \text{ so } \ p^+ = \left(\sqrt{m^2 + p^{3^2} + p_\perp^2} + p^3\right)/\sqrt{2} > 0 \, .$$

The advantages of the LF formalism is that[5] (1) LF subgroup of Poincare group act transitively on the mass shell $p^2 = m^2$, $p^0 > 0$ which means that if the LF wave function of a system is known at rest, it can be obtained at other momentum by boost operators. (2) The property $p^+ > 0$, and its conservation reduces the number of Fock space states needed to generate a covariant result. Consistent treatment of spontaneous symmetry breaking requires the presence of constituents with $p^+ > 0$. (3) The bare vacuum is equal to the physical vacuum. Let us define the bare vacuum as the state with no particles, i.e. $H_0|0>= 0$. If the interaction Hamiltonian H_I involves a certain number of quantized fields with mass, it is impossible to satisfy momentum conservation and $p^+ > 0$ for all fields, hence $H_I|0>= 0$ and thus $H|0>= 0$. But these advantages are accompanied by some disadvantages, (1) It is necessary to require for renormalization, mass and counter terms that depend on the sector of Fock space within which they act. (2) Note if the momentum p^3 is cut off at Λ, $p^+ = (p^0 + p^3)/\sqrt{2} = (2\Lambda + \frac{m^2}{2\Lambda})/\sqrt{2}$. There is an infrared divergence as $p^+ \to$ small. So ultraviolet and infrared divergences have to be treated in a consistent manner. (3) Renormalization 'constants' Z_1 (for vertex) and Z_2 (for wave function renormalization) are momentum dependent.[6] (4) How to handle zero modes and gluons in QCD is not worked out. One particular problem is consistent treatment of spontaneous breaking of chiral symmetry that requires the presence of zero modes.

We begin with Einstein's equation, where $P_0^- = H_0$ and $P_I^- = H_I$,

$$\left(M^2 + P_\perp^2 - 2P^+P_0^-\right)\left|\Psi\right>= 2P^+P_I^-\left|\Psi\right> \ . \tag{1}$$

We outline the calculation[7] for the hydrogen atom of state $|H>$.

$$\left|H\right> = C_0\left|Pe\right> + C_1\left|Pe\gamma\right> \tag{2}$$

Project out C_0 amplitude from Eqs.1 and 2

$$FC_0 = e\int L_{01}C_1 + e^2\int L_{00}C_0\,, \tag{3}$$

where L_{00} represents the instantaneous photon exchange and

$$F = \left[M^2 + (p_\perp + k_\perp)^2 - \left(p^+ + k^+\right)\left\{\frac{k_\perp^2 + M_{e0}^2}{k^+} + \frac{p_\perp^2 + M_{P0}^2}{p^+}\right\}\right]\,.$$

Project out C_1 from Eqs.1 and 2 and obtain

$$HC_1 = eL_{10}C_0 + e^2\int L_{11}C_1\,. \tag{4}$$

Eliminate C_1 from Eq.3 with the aid of Eq.4

$$\begin{aligned} FC_0 &= e^2\int L_{01}H^{-1}L_{10}C_0 + e^2\int L_{00}C_0 \\ &= I_1 + I_2 + I_3 + I_4 + I_5\,. \end{aligned} \tag{5}$$

The terms I_1 and I_2 are self energy of the electron and proton and I_3 and I_4 are photon exchange terms. The I_5 is the instantaneous photon exchange term which is split into two terms and added to I_3 and I_4 terms. The self energy terms yield the form

$$I_1 = \{\quad\}\ln\delta + \ldots \qquad I_2 = \{\quad\}\ln\delta + \ldots\,.$$

where δ is a small cut-off to regulate the infrared singularity. If we define M_{P1} and M_{e1} in such a way that the coefficients of $\ln\delta$ vanish. Then,

$$I_1 = \frac{p^+ + q^+}{p^+}\left(M_{P1}^2 - m_{P0}^2\right) \qquad I_2 = \frac{p^+ + q^+}{q^+}\left(M_{e1}^2 - m_{e0}^2\right)\,.$$

Put

$$M_P^2 = M_{P1}^2 + M_{P0}^2 - m_{P0}^2 \qquad M_e^2 = M_{e1}^2 + M_{e0}^2 - m_{e0}^2$$

and let $p_\perp = q_\perp = 0$ in Eq.5, then the relativistic equation for hydrogen atom in LFTD QED is

$$\left[M^2 - \left(p^+ + q^+\right)\left(\frac{M_P^2}{p^+} + \frac{M_e^2}{q^+}\right)\right]C_0 = e^2\int L_{01}H^{-1}L_{10}C_0 + e^2\int L_{00}C_0\,. \tag{6}$$

In the nonrelativistic limit $|p|, |q| < \Lambda \ll M_e$, with

$$M^2 = (M_e + M_P - B)^2 = (M_P + M_e)^2 - 2\,(M_P + M_e)\,\overline{B}$$

$$\left(\overline{B} + \frac{p^2}{2\mu}\right)\Phi\,(p) = \frac{\alpha}{2\pi^2}\int_{-\infty}^{\infty} d^3k\,\frac{\Phi\,(k)}{(p-k)^2} \tag{7}$$

μ is the reduced mass, $\alpha = e^2/4\pi$. This is the Lippman-Schwinger equation and its solution[8] is $\overline{B}_n = \dfrac{\alpha^2\mu}{2n^2}$. A poor person's solution when $p = 0$ can be found by using

the trial function $\Phi(k) = C/(k^2+a^2)^2$, where a is to be determined. We find $\overline{B} = \frac{\alpha}{2}\, a$ where a has dimensions of mass. Put $a = \sqrt{2\overline{B}\mu}$, then

$$\begin{aligned} \overline{B} = \frac{\alpha^2\mu}{2} &= \frac{\alpha^2 m_e}{2} \quad \text{hydrogen} \quad n=1 \\ &= \frac{\alpha^2 m_e}{4} \quad \text{positronium} \quad n=1 \end{aligned} \tag{8}$$

The interesting but difficult terms are the α^4 terms in positronium. There are about 20 T-D amplitudes in α^4. The expected result from magnetic spin interaction[9] is $|\overline{B}| = \alpha^4 m_e[\frac{1}{4}]$.

The diagrams can be arranged into self mass, and vertex correction of the box diagrams in which two photons are exchanged. The renormalization procedure[6] was worked out by Mustaki et al. The renormalized charge is $e_R = e\sqrt{Z_3}$. From the box diagram $\overline{B} = \alpha^4 m_e$ but the factor $\frac{1}{4}$ is elusive.

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