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Neutrino Mass Matrix and Double Beta Decay*)

Tsuneyuki KOTANI
Osaka Gakuin University
Suita, Osaka, 564, JAPAN

Abstract

The possible types of light neutrinos are discussed in connection with the recent data on Z^0 decay, which show the number of different neutral elementary particles to be $\mathcal{N}_\nu = 3$. It is shown to be possible in principle to construct models including at most six generations even for $\mathcal{N}_\nu = 3$, if neutrinos are the special (ZKM) Dirac type which means that only one Dirac neutrino plays a role for two generations. The neutrinoless double beta decay accompanied by the emission of a Majoron does not exist, if $\mathcal{N}_\nu = 3$ and the Majoron is the type proposed by Gelmini and Roncadelli. The recent progress of theoretical estimations for nuclear matrix elements are summarized briefly.

1. Introduction

No important and new experimental result on the neutrinoless double beta decay which will be referred to as the $(\beta\beta)_{0\nu}$ mode has been reported for this one year. One of the reasons is that the half life for this decay mode seems to be longer than the one expected until now. Therefore, many new ideas of instruments have been proposed and the next stage of experiments has started. Of course, if neutrinos are the Dirac type, this $(\beta\beta)_{0\nu}$ mode is prohibited, see for example sec. 1.3 of Ref.1. (Hereafter Ref.1 will be referred to as I.)

The possibility has been discussed to observe the neutrinoless double beta decay accompanied by the emission of Majoron, the $(\beta\beta)_{0\nu,B}$ mode, see Eq.(5·2·2) of I. However, this mode seems to be denied by the recent experimental results on the Z^0 decay width, if this Majoron is the type proposed by Gelmini and Roncadelli:[2] The reason is as follows: The number of different light neutral elementary particles which are produced in this Z^0 decay is, for example, $\mathcal{N}_\nu = 2.8\pm0.6$(SLC) and 3.12 ± 0.42(OPAL).[3] If this type of Majoron exists, its contribution is $\mathcal{N}_\nu = 2$ in addition to $\mathcal{N}_\nu = 3$ which corresponds to three massless neutrinos in the context of the standard model.[4),**]

Although $\mathcal{N}_\nu = 3$ simply points out the existence of three generations, we can in principle construct special models including at most six generations consistently with $\mathcal{N}_\nu = 3$ under the following two assumptions:

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*) This review talk was presented on September 30 of 1989. Since new experimental results on the Z^0 decay were announced on October 13, some parts of this written review are different from the oral one.

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(1) Only one Dirac type neutrino plays a common role for two generations. It is known that such type of neutrino is not incompatible with the present experimental data, after Zeldovich and Konopinski and Mahmoud proposed this possibility in 1952.[5] In subsection 3.2 of this review, this type will be defined and referred to as the ZKM Dirac neutrino. (2) All masses of quarks and charged leptons which belong to the fourth and higher generations should be larger than 45 GeV, the half of the Z^0 mass.[6] If all neutrinos are the ZKM Dirac ones, the $(\beta\beta)_{0\nu}$ mode is prohibited.

If the $(\beta\beta)_{0\nu}$ mode is observed, it is necessary to know the magnitude of nuclear matrix elements in order to extract the useful information on the elementary particle physics, for example, such as the mass of neutrino and the right-handed weak interaction. Concerning these nuclear matrix elements, there had been some discrepancies between experimental results and theoretical estimations. One reasonable resolution has been proposed recently.[7),8]

In this short review, the difference between the Dirac type neutrino and the Majorana type one is explained. It is summarized how to distinguish them experimentally. In section 3, various types of neutrinos are classified from the view point of the neutrino mass matrix. In section 4, the recent theoretical and experimental situations on the double beta decay are mentioned briefly.

2. DIRAC, WEYL AND MAJORANA FIELDS

Let us first show that a free Dirac field ϕ consists of two independent Majorana fields. The Lagrangian density for a classical Dirac field is

$$\mathcal{L} = \overline{\phi}\,(i\gamma^0\partial_t + i\gamma\cdot\nabla - m)\,\phi\,. \tag{2.1}$$

Hereafter, the Weyl representation of γ matrices will be used.

If we express the Dirac field ϕ in the ordinary two component form,

$$\phi = \begin{pmatrix}\varphi\\ \chi\end{pmatrix},\quad \phi_L = \frac{(1-\gamma_5)}{2}\,\phi = \begin{pmatrix}0\\ \chi\end{pmatrix},\quad \phi_R = \frac{(1+\gamma_5)}{2}\,\phi = \begin{pmatrix}\varphi\\ 0\end{pmatrix}, \tag{2.2}$$

then the Lagrangian becomes

$$\mathcal{L} = \varphi^+O_+\varphi + \chi^+O_-\chi - \varphi^+m\chi - \chi^+m\varphi\,,\ \text{where}\ O_\pm = i(\partial_t \pm \sigma\cdot\nabla). \tag{2.3}$$

The mass term includes both χ and φ, and it will be referred to as a Dirac type mass term. If $m=0$, then χ and φ represent different Weyl fields. As the quantized field operator, χ with $m=0$ includes both an annihilation operator of particle with the negative helicity and a creation operator of antiparticle with the positive helicity, because of O_- .

Next, let us represent ϕ as a superposition of two independent Majorana fields N_1 and N_2 with the same mass,

$$\phi = (N_1 + i\,N_2)/\sqrt{2}\,, \tag{2.4}$$

where N_j's are defined in the four component form as follows,

$$N_1 = \begin{pmatrix} -i\sigma_2\eta^* \\ \eta \end{pmatrix} \quad \text{and} \quad N_2 = \begin{pmatrix} \xi \\ i\sigma_2\xi^* \end{pmatrix}. \tag{2.5}$$

Note that fields N_j satisfy the self-conjugate condition, $N_j^{\mathrm{c}} = N_j$, where $N^{\mathrm{c}} = C\overline{N}^T$, C being the charge conjugation matrix, see Eq.(2·1·13) of I. Then, by assuming η and ξ to be Grassman numbers, the original Lagrangian density splits completely into two parts such as

$$\mathcal{L} = \mathcal{L}_L(\ \eta\ ) + \mathcal{L}_R(\ \xi\ ), \tag{2.6}$$

where

$$\mathcal{L}_L(\ \eta\ ) = \eta^+ O_- \eta - (m/2)\,(\,\eta^T i\sigma_2\eta - \eta^+ i\sigma_2\eta^*\,),$$

$$\mathcal{L}_R(\ \xi\ ) = \xi^+ O_+ \xi - (m/2)\,(\,\xi^+ i\sigma_2\xi^* - \xi^T i\sigma_2\xi\,). \tag{2.7}$$

These η and ξ will be referred to as the left- and right-handed Majorana fields in the two component form, respectively, because of the $O_{-(+)}$ character. The natural expression for the Majorana field is the two component form like η and ξ, because it has only two freedoms (two spin states). The four component form $N_{1(2)}$ in Eq.(2.5) should be understood as a convention to express the weak charged current compactly. These η and ξ have opposite signs under the CP transformation, because of the factor i in Eq.(2.4), see Eq.(2·5·6) of I.

Mass terms in Eq.(2.7) are the $\eta^T i\sigma_2\eta$ or $\xi^+ i\sigma_2\xi^*$ types (Majorana type mass term), instead of the $\varphi^+\chi$ type (Dirac type mass term) in Eq.(2.3). This Majorana type mass term means that there is no freedom for phase transformation, because of non-existence of the complex conjugate factor of η or ξ. Since the invariance under the phase transformation, namely a global gauge transformation, offers the additive conservations of charge and fermion number within a framework of gauge theory, all charged fermions shoud be treated as the Dirac field. There is, however, a possibility that the neutral fermion like neutrino can be described by the Majorana field, which is more fundamental than the Dirac field. One may have a question why there is a phase freedom for the Dirac field in spite of the fact that it consists of two Majorana fields which have no such phase freedom. The answer for this question is as follows, see Eq.(2·5·8) of I: First let us mix two Majorana fields by an orthogonal transformation,

$$\begin{pmatrix} N_1(\alpha) \\ N_2(\alpha) \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} N_1 \\ N_2 \end{pmatrix}. \tag{2.8}$$

Then the Dirac field ϕ_α is related to ϕ in Eq.(2.4) as

$$\phi_\alpha \equiv [\ N_1(\alpha) + i\,N_2(\alpha)\ ]/\sqrt{2} = e^{i\alpha}\phi. \tag{2.9}$$

The quantized field operator for the Majorana neutrino (η or ξ) is expressed in a four component form as

$$N = \begin{pmatrix} -i\sigma_2\eta^* \\ \eta \end{pmatrix} = N_L + (N_L)^c$$

$$= \int d\Omega_q \sum_s \left[a(q,s)u(q,s)e^{-iqx} + a^+(q,s)v(q,s)e^{iqx} \right] , \tag{2.10}$$

where the four component spinor $u(q,s)$ is nothing but the ordinary Dirac spinor in the Weyl basis of γ matrix, and $v = C\bar{u}^T$. Of course, the spinor parts for η and ξ in the two component form are different from each other and from Eq.(2.10), see Eq.(2·4·6) of I. In the case of the massless N_L (N_R), the operator $a(q,s)$ annihilates a Majorana neutrino with negative (positive) helicity, while $a^+(q,s)$ creates another with positive (negative) helicity. In our point view of Eq.(2.4), annihilation operators (b and c) for the Dirac neutrino and antineutrino are defined as

$$b(q,s) = (a_1 + ia_2)/\sqrt{2}, \quad \text{and} \quad c(q,s) = (a_1 - ia_2)/\sqrt{2}, \tag{2.11}$$

respectively. It is worthwhile to note that the Majorana fields N_j and the Dirac field ϕ are related as follows,

$$N_1 = \frac{1}{\sqrt{2}}(\phi + \phi^c) \qquad \text{and} \qquad N_2 = \frac{-i}{\sqrt{2}}(\phi - \phi^c), \tag{2.12}$$

where the charge conjugation of ϕ is $\phi^c = (N_1 - iN_2)/\sqrt{2}$. The situation where ϕ and ϕ^c are the superposition of N_j is similar to the charged boson case where the field operator is expressed by a superposition of two Hermitian fields.

In this review, the leptonic charged currents of the weak interaction with the left- and right-handed weak intermediate bosons (W_L and W_R) are defined respectively as follows,

$$\mathcal{L}_W = \frac{1}{2\sqrt{2}}\left[g_L \sum_l j_{lL}{}^{\rho} W_{L\rho} + g_R \sum_l j_{lR}{}^{\rho} W_{R\rho} \right] + \text{h.c.}, \tag{2.13}$$

where $l = e$, μ and τ and the weak charged currents are

$$j_{lL}{}^{\rho} = \bar{l}\gamma^{\rho}(1 - \gamma_5)\nu_{lL} \qquad \text{the left-handed (V − A) interaction,} \tag{2.14}$$

$$j_{lR}{}^{\rho} = \bar{l}\gamma^{\rho}(1 + \gamma_5)\nu_{lR}, \qquad \text{the right-handed (V + A) interaction.} \tag{2.15}$$

If $m = 0$, it has no physical meaning to distinguish the Weyl neutrino from the Majorana one in the context of the (V − A) and (V + A) weak interactions, because of factors $(1 \pm \gamma_5)$ and Eq.(2.10). However, if $m \neq 0$, it is important to determine whether neutrino is the Dirac or Majorana type. For this purpose, let us consider five cases in the framework of the V − A weak interaction:

(1) Magnetic dipole moment :

A massive Majorana neutrino can not have a magnetic and electric moments, because of the self-conjugate condition characteristic of Majorana neutrinos. Since there is a transition moment, the radiative decay of the heavier Majorana neutrino to the lighter one is allowed, and the spin rotation also occurs, if they pass through a gigantic magnetic field, see Eq.(2·7·1) of I.

On the other hand, a massive Dirac neutrino can have a magnetic moment (and if time reversal invariance is violated, an electric moment, too). This is because the transition moment between two Majorana neutrinos in Eq.(2.4) gives rise to the magnetic moment for the Dirac neutrino, see Eq.(2·7·3) of I. Its magnitude is $\mu_{\nu}^{(V-A)} = 3\times10^{-19}(m_{\nu}/1\ \mathrm{eV})\ \mu_{B}$ in the $SU(2)_{L}\times U(1)$ theories with massive Dirac neutrinos, m_{ν} and μ_{B} being the mass of neutrino in units of 1 eV and the Bohr magneton, respectively. If the V + A weak interaction is taken into account, a little larger value may be derived. The present experimental lower limits are $(1.5\sim12)\times10^{-10}\ \mu_{B}$ from laboratory experiments,[9] and $10^{-12}\sim10^{-13}\ \mu_{B}$ from the SN1987a data.[10]

(2) One neutrino case in the final state :

As an example, let us consider the single β decay, $n \rightarrow p + e^{-} + \overline{\nu_{e}}$, where the emitted neutrino will be referred to as an (electron) antineutrino. The electron (or proton) spetrum shows no difference for both the Dirac and Majorana neutrinos, because the second term in Eq.(2.10) is the same for both cases. The word "antineutrino" will be used even for the Majorana neutrino as the case where it has mainly the positive helicity in the context of V−A weak interaction. Its negative helicity part is proportional to (m_{ν}/ω), typically of order of 10^{-5}, m_{ν} and ω being the mass and energy of neutrino, respectively, see below Eq.(2·4·25) of I.

This emitted antineutrino, for example from a nuclear reactor, can trigger the reaction,

$$\overline{\nu_{e}} + n \rightarrow p + e^{-}, \tag{2.16}$$

for the massive Majorana neutrino case, though it is forbidden for the Dirac neutrino because of the lepton number conservation. Unfortunately, it is almost impossible to observe it experimentally. The reason is the requirement of the helicity matching: That is, in the context of V−A theory, the allowed reaction for either the Dirac or Majorana neutrino is $\nu_{e} + n \rightarrow p + e^{-}$, where the helicity of the incident neutrino is negative mainly. Thus, the reaction rate of Eq.(2.16) is smaller by the order of $(m_{\nu}/\omega)^{2} < 10^{-10}$ in comparison with another normal reaction, $\overline{\nu_{e}} + p \rightarrow n + e^{+}$. Of course, this reaction rate is proportional to the inverse square of the distance of antineutrino propagation, see Eq.(11·1·6) of I.

There may be a chance to observe Eq.(2.16) inside one nucleus, where the normal reaction, $\overline{\nu_e} + p \to n + e^+$, is prohibited by the energy conservation or the competing radiative decay of the excited nuclei. This is the neutrinoless double beta decay, which will be mentioned in the case (5).

Quite similar argument can be applied to the case of neutrino, for example, as solar neutrinos like $2p \to d + e^+ + \nu_e$. The reaction due to the Majorana neutrino corresponding to Eq.(2.16) is

$$\nu_e + p \to n + e^+. \tag{2.17}$$

This case is a little favorable in comparison with Eq.(2.16), because the normal nuclear reaction $\nu_e + n \to p + e^-$ is able to be avoided for low energy neutrino by using the hydrogen or the water as a detector. However, the reducing factor $(m_\nu/\omega)^2 \sim 10^{-10}$ is still obstructive.

There is another problem of the spin-rotation due to the magnetic moment of Dirac neutrino or the transition moment of the Majorana neutrino passing through a gigantic magnetic field B $\sim 10^{13}$ G. We do not discuss it here.[10]

(3) Two neutrinos in the final state :

There are processes like $\mu^+ \to e^+ + \overline{\nu_\mu} + \nu_e$ and $\gamma + \gamma \to \nu_l + \overline{\nu_l}$ in the final stage just before the explosion of supernova. The difference between the Dirac and Majorana neutrinos is terms proportional to (m_ν/ω). It is not easy to measure these terms, see Eq.(11·2·9) of I.

(4) Neutrino oscillation :

In the n massive Majorana neutrino system in the framework of the $SU_L(2)\times U_Y(1)$ model with n left-handed lepton doublets, the number of the CP violating phases is $n(n-1)/2$, which is larger than $(n-1)(n-2)/2$ in the n Dirac neutrino system. The latter corresponds to the Cabibbo-Kobayashi-Maskawa phases in the quark sector. However, this phase difference does not appear in the theoretical expression for the neutrino oscillation, see Eq.(2·3·2) of I.

In the Majorana neutrino case, if the left- and right-handed Majorana type mass matrices corresponding to m_L and M in Eq.(3.3) coexist, then some initial neutrinos (or antineutrinos) transit into sterile neutrinos which do not take part in the V − A interaction, so that the measured flux of neutrinos becomes less than the flux expected for the pure Dirac or the pure left-handed Majorana neutrino case.[11]

In the Majorana neutrino case, it is possible, in principle, that the ν_μ beam from the pion decay, $\pi^{+(-)} \to \mu^{+(-)} + \nu_\mu(\overline{\nu_\mu})$, induces the reaction in Eq.(2.17) (or Eq.(2.16)), if neutrino oscillation occurs. But, it should be noted that there is some contamination of the ν_e beam experimentally.

(5) The Majorana neutrino exchange processes :

The transition of the parent nucleus with $(Z-2)$ protons into the daughter one with Z protons without emitting neutrinos,

$$(A, Z-2) \rightarrow (A, Z) + 2e^- \qquad \textit{the } (\beta\beta)_{0\nu} \textit{ mode}, \tag{2.18}$$

is the realization of Eq.(2.16), as mentioned above, and will be discussed in subsection 4.2 of this review.

There are some other processes to distinguish the Majorana neutrino from the Dirac one like $\mu^- + (A, Z) \rightarrow (A, Z-2) + e^+$ and $K^+ \rightarrow \pi^- + e^+ + \mu^+$. They are listed and discussed in section 11 of I.

3. Neutrino mass matrix

As the simplest extension of the standard theory, we shall assume that in each generation, there is only one massless left-handed Majorana neutrino before the spontaneous symmetry breaking and it will get some mass by the Higgs mechanism. Furthermore, it is assumed to be able to add another massless right-handed Majorana neutrino which appears in the left-right symmetric models like $SU(2)_L \times SU(2)_R \times U(1)_Y$ gauge theory. Let us first examine one generation case.

3.1 One generation case

The Lagrangian density of the neutrino mass part after the symmtery breaking is

$$\mathscr{L}_m = -\frac{1}{2}\left(\overline{(\nu_L^0)^c},\ \overline{\nu_R^{0\prime}}\right) \mathcal{M} \begin{pmatrix} \nu_L^0 \\ (\nu_R^{0\prime})^c \end{pmatrix} + \text{h.c.}, \tag{3.1}$$

where ν_L^0 and $\nu_R^{0\prime}$ do not mean to take the left- and right-handed parts of ν^0, but stand for seeds which are characterized respectively as the left- and right-handed Majorana massless neutrinos by their kinetic terms like the first terms in Eq.(2.7) before the spontaneous symmetry breaking. In the four component form, they are

$$\nu_L^0 = \begin{pmatrix} 0 \\ \eta^0 \end{pmatrix} \quad \text{and} \quad (\nu_R^{0\prime})^c = \begin{pmatrix} 0 \\ i\sigma_2 \xi^{0*} \end{pmatrix}, \tag{3.2}$$

and belong to the same representation of the homogeneous Lorentz group.

The mass matrix $\mathcal{M}$ in Eq.(3.1) is represented as

$$\mathcal{M} = \begin{pmatrix} m_L & m_D^T \\ m_D & M \end{pmatrix}, \tag{3.3}$$

where m_L, m_D and M are the vacuum expectation values of neutral Higgs bosons multiplied by the Yukawa coupling constants of interactions among Higgs bosons, ν_L^0 and $\nu_R^{0\prime}$. These m_L, M and m_D will be referred to as the left-, right-handed Majorana type and Dirac type mass terms (mass matrices for many generation case), respectively. For simplicity, they are assumed to be real (the CP conservation). The transposed notation T of

m_D has no meaning in this one generation case, though it comes from an identity

$$\overline{\nu_R^{o'}}\, m_D\, \nu_L^o = \overline{(\nu_L^o)^c}\, m_D^T\, (\nu_R^{o'})^c . \tag{3.4}$$

Since the mass matrix $\mathcal{M}$ is a real symmetric matrix, it can be diagonalized by the following transformation,

$$U_\nu^T\, \mathcal{M}\, U_\nu = \begin{pmatrix} m_{\mathrm{I}} & 0 \\ 0 & m_{\mathrm{II}} \end{pmatrix} . \tag{3.5}$$

Although it is enough mathematically to use an orthogonal matrix for the transformation matrix U_ν, a unitary matrix is chosen so that both eigen values (m_{I} and m_{II}) become real positive and give the masses of the Majorana neutrinos, see Eq.(2·3·16) of I. The corresponding eigen vectors which will be referred to as (Majorana) mass eigenstates, are represented by N_{I} and N_{II}, respectively, where N_j is defined as

$$N_j = N_{jL} + (N_{jL})^c = N_{jL} + N_{jR} . \tag{3.6}$$

Here sufices L and R mean to take the left- and right-handed projection of the mass eigenstate field operator N_j as in Eq.(2.2). If all elements of one column of U_ν are pure imaginary and those of another column are real, then N_{I} and N_{II} have opposite CP values.

According to their assumed kinetic terms, the full Lagrangian densities for the left- and right-handed massive Majorana neutrinos are expressed in the four component form as

$$\mathcal{L}_L = \overline{N_{\mathrm{I}L}}\, i\gamma^\rho \partial_\rho N_{\mathrm{I}L} - \frac{1}{2}\left(\overline{(N_{\mathrm{I}L})^c}\, m_{\mathrm{I}} N_{\mathrm{I}L} + \overline{N_{\mathrm{I}L}}\, m_{\mathrm{I}} (N_{\mathrm{I}L})^c\right) ,$$

$$\mathcal{L}_R = \overline{N_{\mathrm{II}R}}\, i\gamma^\rho \partial_\rho N_{\mathrm{II}R} - \frac{1}{2}\left(\overline{(N_{\mathrm{II}R})^c}\, m_{\mathrm{II}} N_{\mathrm{II}R} + \overline{N_{\mathrm{II}R}}\, m_{\mathrm{II}} (N_{\mathrm{II}R})^c\right) , \tag{3.7}$$

corresponding to Eq.(2.7) in the two component form. If the transformation matrix U_ν and the mass eigenstate neutrinos N_j are expressed in the column matrix form as

$$U_\nu = \begin{pmatrix} U_1 \\ V_1^* \end{pmatrix} = \begin{pmatrix} U_{\mathrm{I}} & U_{\mathrm{II}} \\ V_{\mathrm{I}}^* & V_{\mathrm{II}}^* \end{pmatrix} \quad \text{and} \quad N = \begin{pmatrix} N_{\mathrm{I}} \\ N_{\mathrm{II}} \end{pmatrix} , \tag{3.8}$$

then the original ν_L^o and $\nu_R^{o'}$ are defined as superpositions of the mass eigenstate neutrinos,

$$\nu_L^o = U_1 N_L \qquad \text{and} \qquad \nu_R^{o'} = V_1 N_R , \tag{3.9}$$

respectively. Note that in the one generation case, ν_L^o and $\nu_R^{o'}$ are equal to the weak eigenstate ν_{lL} of the V − A interaction in Eq.(2.14) and ν_{lR}'

of V + A in Eq.(2.15), respectively.

It is worthwhile to mention about the sterile neutrino here again. Though U_ν itself is a unitary matrix, its part U_1 in Eq.(3.9) is not unitary. Therefore, for example, the antineutrino $\overline{\nu_l}$ produced by the V − A weak interaction in Eq.(2.14) can go to some other state (the sterile neutrino ν_l'') coming from $(\nu_R^{0\prime})^c$, which corresponds to the weak eigenstate ν_{Rl}' in the V + A interaction of Eq.(2.15) and does not play a role in the standard V − A theory. In general, conditions to make the neutrino oscillation are; (1) the difference between $m_{\rm I}$ and $m_{\rm II}$ is so small that a coherent superposition of the state vectors derived from the field operator $N_{\rm I}$ and $N_{\rm II}$ is formed by the weak interaction, and (2) each mass of them is negligible in comparison with their momenta.[11] Each transition amplitude is expressed as

$$a(\overline{\nu_l} \rightarrow \overline{\nu_k}) = \textstyle\sum_{j={\rm I},{\rm II}} (U_1^*)_{kj}\, e^{-iE_jt} (U_1)_{lj} ,$$

$$a(\overline{\nu_l} \rightarrow \nu_k'') = \textstyle\sum_{j={\rm I},{\rm II}} (V_1)_{kj}\, e^{-iE_jt} (U_1)_{lj} , \tag{3.10}$$

where $k = l$ for one generation. The sum of these two transition probabilities shoud be equal to unity. The concept of such transition to the sterile neutrino does not exist for the pure Dirac case (only $m_D \neq 0$) or for the pure left-handed Majorana case (only $m_L \neq 0$).

Let us examine three special cases for m_L, m_D and M.

(3.1.A) The $M = -m_L$ case (the degenerate mass case):

The transformation matrix U_ν and the degenerate mass are

$$U_\nu = \frac{1}{\sqrt{2}} \begin{pmatrix} ({\rm c}+{\rm s}) & i\,({\rm c}-{\rm s}) \\ ({\rm c}-{\rm s}) & -i\,({\rm c}+{\rm s}) \end{pmatrix} \quad \text{and} \quad m_{\rm I} = m_{\rm II} = \sqrt{m_L^2 + m_D^2} , \tag{3.11}$$

where ${\rm c} = \cos\theta$, ${\rm s} = \sin\theta$, $\tan 2\theta = (m_L/m_D)$ and $\sin 2\theta = (m_L/m_{\rm I})$. Thus, $N_{\rm I}$ and $N_{\rm II}$ have the degenerate mass but opposite *CP* values, because of the i factor in U_ν. If we define one Dirac type field as

$$\phi = ({\rm N_I} + i\,{\rm N_{II}})/\sqrt{2} \qquad \text{and} \qquad \phi^c = ({\rm N_I} - i\,{\rm N_{II}})/\sqrt{2}, \tag{3.12}$$

then we have

$$\nu_L^0 = \cos\theta\ \phi_L + \sin\theta\ (\phi^c)_L ,$$

$$\nu_R^{0\prime} = -\sin\theta\ (\phi^c)_R + \cos\theta\ \phi_R . \tag{3.13}$$

In the limit $\theta \rightarrow 0$, namely $m_L \rightarrow 0$, we obtain the usual result for the (ordinary) Dirac neutrino with the mass $m_{\rm D}$,

$$\nu_L^0 = \phi_L \qquad \text{and} \qquad \nu_R^{0\prime} = \phi_R , \tag{3.14}$$

corresponding to ϕ in Eq.(2.2).

On the other hand, if $\theta \neq 0\,(m_L \neq 0)$, the mass degeneracy are broken slightly by the higher order effects of the weak V − A interaction obtained by substituting ν_L^0 of Eq.(3.13) into ν_{lL} of Eq.(2.14). Thus we have two Majorana neutrinos which have the tiny mass difference and opposite *CP* values. These two neutrinos induce the decay of the $(\beta\beta)_{0\nu}$ mode in Eq.(2.18), where the effective neutrino mass $\langle m_\nu \rangle$ is proportional to $2m_{\mathrm{I}} \sin 2\theta$. A pair of these neutrinos is called as the *pseudo Dirac neutrino*, according to Wolfenstein.

In the opposite limit of $\theta \to \pi/4$ $(m_D \to 0)$, N_{I} and N_{II} contribute to the V − A and V + A interactions separately.

(3.1.B) The $m_{\mathrm{D}} \gg m_L \sim M$ case:

Let us choose the transformation matrix U_ν and two masses as follows,

$$U_\nu = \begin{pmatrix} \cos\theta & i\sin\theta \\ \sin\theta & -i\cos\theta \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} m_{\mathrm{I}} \\ m_{\mathrm{II}} \end{pmatrix} = D \pm \frac{1}{2}(M + m_L), \tag{3.15}$$

where $\tan 2\theta = m_D/[(m_L - M)/2]$, $\sin 2\theta = m_D/D$ and $D = \{[(M - m_L)/2]^2 + m_D^2\}^{1/2}$. We have another type of pseudo Dirac neutrino, if $m_D \gg |M - m_L|/2$. These two Majorana neutrinos have opposite *CP* values and their mass difference can be chosen as the larger value than the case (3.1.A), because m_L and M are free parameters. In the limit where $m_L = M = 0$, of course, we have one Dirac field.

If the number of light neutrinos is $\mathcal{N}_\nu = 3$ from the Z^0 decay width, the existence of these pseudo Dirac neutrinos in subsections 3.1A and 3.1B seems to be unlikely, because it gives $\mathcal{N}_\nu = 2$ even for one generation.

(3.1.C) The $M \gg m_D \gg m_L \geqq 0$ case (the seesaw mechanism case):

The transformation matrix U_ν and two masses are chosen as

$$U_\nu = \begin{pmatrix} i\cos\theta & \sin\theta \\ -i\sin\theta & \cos\theta \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} m_{\mathrm{I}} \\ m_{\mathrm{II}} \end{pmatrix} = D \mp \frac{1}{2}(M + m_L)\,, \tag{3.16}$$

where $\tan 2\theta = m_D/[(M - m_L)/2]$ and $\sin 2\theta = m_D/D$. Then, two Majorana neutrinos with opposite helicities appear. If we assume $m_D = m_e = 0.5$ MeV and $M = 100$ GeV, then two masses become

$$m_{\mathrm{I}} \sim (m_D M^{-1} m_D) - m_L \sim 2.5\times10^{-6}\ \mathrm{eV} - m_L,$$

$$m_{\mathrm{II}} \sim M + (m_D M^{-1} m_D) \quad \sim 100\ \mathrm{GeV}. \tag{3.17}$$

This is the so-called seesaw mechanism to explain the smallness of the neutrino mass in comparison with masses of the corresponding charged leptons. Parameters M and m_L are free in the grand unified theories like the SO(10) gauge theory, though the value of m_D are restricted by the known quantities like quark mixing, masses of quarks and charged leptons.

While, if we choose parameters like $m_L \gg (m_D^2/M)$, then we have two

Majorana neutrinos with the same helicity. In this case, the transformation matrix U_ν is obtained by taking out the i factor of the first column in Eq.(3.16), and m_{I} becomes $m_{\mathrm{I}} = [(M + m_L)/2] - D \sim m_L$ and $m_{\mathrm{II}} \sim M$.

This (3.1C) case is one of the realistic models for neutrinos. In this case, it is easy to see the following relation from Eq.(3.8) and Eq.(3.16),

$$|U_{\mathrm{I}}| = |V_{\mathrm{II}}| \gg |U_{\mathrm{II}}| = |V_{\mathrm{I}}|. \tag{3.18}$$

This inequality is general for the seesaw mechanism models of three generations with $|M| \neq 0$, though the equalities should be read as the same order of magnitude, because all of them are complicated 3×3 matrices.

3.2 Three generation case

In our simplest extension of the standard theory, ν_L^0 and $\nu_R^{0\prime}$ in Eq.(3.1) are considered to represent columns like

$$\nu_L^0 = \begin{pmatrix} \nu_{eL}^0 \\ \nu_{\mu L}^0 \\ \nu_{\tau L}^0 \end{pmatrix} \quad \text{and} \quad \nu_R^{0\prime} = \begin{pmatrix} \nu_{eR}^{0\prime} \\ \nu_{\mu R}^{0\prime} \\ \nu_{\tau R}^{0\prime} \end{pmatrix}. \tag{3.19}$$

Accordingly, three mass matrix elements, m_L, m_D and M in Eq.(3.3), become 3×3 matrices, so that the whole matrix $\mathcal{M}$ is a real symmetric 6×6 one. The symmetric character of m_L is proved by the identity,

$$\overline{(\nu_{lL})^c}(m_L)_{lk}\,\nu_{kL} = \overline{(\nu_{kL})^c}(m_L)_{lk}\,\nu_{lL}, \tag{3.20}$$

where definitions $\overline{(\nu_{lL})^c} = -(\nu_{lL})^T C^{-1}$ and $C^T = -C$ have been used. The symmetric features of M can be proved similarly.

Therefore, the symmetric mass matrix $\mathcal{M}$ can be diagonalized by using a 6×6 unitary matrix U_ν as in Eq.(3.5). The mass eigenstate field N_j with mass m_j is classified by extending the notation in Eqs. (3.8) and (3.5) as follows;

$$N_{\mathrm{I}} = \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix},\ m_{\mathrm{I}} = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix},\ \text{and}\ N_{\mathrm{II}} = \begin{pmatrix} N_4 \\ N_5 \\ N_6 \end{pmatrix},\ m_{\mathrm{II}} = \begin{pmatrix} m_4 & 0 & 0 \\ 0 & m_5 & 0 \\ 0 & 0 & m_6 \end{pmatrix}, \tag{3.21}$$

where all m_j are real and positive.

The weak eigenstate neutrino fields [ν_{lL} of the V − A interaction in Eq.(2.14) and ν_{lR}' of V + A in Eq.(2.15)] are defined as superpositions of the mass eigenstate neutrinos N_j with mass m_j multiplied by mixing matrix elements U_{lj} and V_{lj}, respectively;

$$\nu_{lL} = \textstyle\sum_j U_{lj} N_{jL} \quad \text{with} \quad U = U_{cl}^{+} U_1 = U_{cl}^{+}(U_{\mathrm{I}} + U_{\mathrm{II}}),$$

and

$$\nu_{lR}' = \textstyle\sum_j V_{lj} N_{jR} \quad \text{with} \quad V = V_{cl}^{+} V_1 = V_{cl}^{+}(V_{\mathrm{I}} + V_{\mathrm{II}}), \tag{3.22}$$

where 3×3 unitary matrices U_{cl} and V_{cl} are transformation matrices for the left- and right-handed charged leptons, respectively. Note that the Dirac type mass matrix for charged leptons ($\mathcal{M}_D$) is diagonalized as $V^{+}_{cl}\mathcal{M}_D U_{cl}$. The 3×6 transformation matrices for neutrinos (U_1 and V_1) are defined from U_ν similarly to Eq.(3.8).

Thus, all theoretical expressions for three generation case are derived from the corresponding quantities for one generation case by replacing them with matrix forms, except the neutrino mixing matrices U and V in Eq.(3.22), *cf.* Eq.(3.9). Hereafter, we shall assume transformation matrices for charged leptons U_{cl} and V_{cl} to be a unit matrix in order to simplify the description. Also eigen values m_1, m_2 and m_3 are assummed to correspond to different masses of ν_e, ν_μ and ν_τ.

Various types of neutrinos introduced for the one generation case can be defined also in the three generation case similarly. For example, we have three light left-handed Majorana neutrinos and three heavy right-handed ones under the seesaw mechanism. They are consistent with the Z^0 decay data, say, $\mathcal{N}_\nu = 3$ and three kinds of charged leptons.[6)]

However, there are special types of neutrino characteristic of many generations. Before discussing them, it is convenient to explain how to obtain the *ordinary Dirac neutrinos* from our view point of Eq.(2.4) for ϕ. The word "ordinary Dirac" means to guarantee the lepton number conservations for each generations and/or $L_e + L_\mu + L_\tau = \text{const}$. Three following relations are required;

(1) $m_{\mathrm{I}} = m_{\mathrm{II}}$ in Eq.(3.21):

Two masses in Eq.(3.21) should degenelate such as $m_j = m_{3+j}$. These eigen values are obtained from the following diagonalization by using U_ν in E.(3.8),

$$V^{+}_{\mathrm{I}} m_D U_{\mathrm{I}} = m_{\mathrm{I}} \; (= m_{\mathrm{II}} = V^{+}_{\mathrm{II}} m_D U_{\mathrm{II}}). \tag{3.23}$$

(2) Two Majorana type mass matrices should be zero, $m_L = M = 0$:

This is necessary to avoid the pseudo Dirac neutrinos due to the mass splitting which come from the radiative correction like Eq.(3.13).

(3) Two Majorana neutrinos should have opposite helicities:

If they are the j-th neutrinos of N_{I} and N_{II} with the degenerate mass m_j, this condition is expresssed as

$$(U_{\mathrm{II}})_{lj} = i(U_{\mathrm{I}})_{lj} \quad \text{and} \quad (V_{\mathrm{II}})_{lj} = i(V_{\mathrm{I}})_{lj}\,. \tag{3.24}$$

Note that if $\mathcal{N}_\nu = 3$ from the Z^0 data is established, three Dirac neutrinos are consistent with it, but this type of pseudo Dirac neutrinos due to $|m_L| \neq 0$ and/or $|M| \neq 0$ corresponding to Eqs.(3.13) and (3.15) is prohibited, because $\mathcal{N}_\nu > 3$.

Now we shall return and see the new features characteristic of many generations. Let us first consider a new type of Dirac neutrino which consists of two left-handed Majorana neutrinos. As an example, the case of two left-handed electron and muon neutrinos will be examined, see

Eq.(2·5·9) of I. Then the Lagrangian density for the mass part is,

$$\mathcal{L}_{mL2} = -\frac{1}{2}\left(\overline{(\nu^{o}_{eL})^{c}}, \overline{(\nu^{o}_{\mu L})^{c}} \right) \begin{pmatrix} m_{ee} & m_{e\mu} \\ m_{e\mu} & m_{\mu\mu} \end{pmatrix} \begin{pmatrix} \nu^{o}_{eL} \\ \nu^{o}_{\mu L} \end{pmatrix} + \text{h.c.}, \tag{3.25}$$

where Eq.(3.20) has been used. This has mathematically the same structure as Eq.(3.3) for the one generation case, except the seesaw mechanism in subsection 3.1.C. Thus, if $m_{\mu\mu} = -m_{ee}$, then analogously to Eq.(3.13), we have

$$\begin{aligned} \nu_{eL} &= \cos\theta\ \phi_L + \sin\theta\ (\phi^{c})_L, \\ \nu_{\mu L} &= -\sin\theta\ \phi_L + \cos\theta\ (\phi^{c})_L, \end{aligned} \tag{3.26}$$

where $\tan 2\theta = (m_{ee}/m_{e\mu})$, $\sin 2\theta = (m_{ee}/m_{\mathrm{I}})$ and $m_{\mathrm{I}} = [m_{ee}^2 + m_{e\mu}^2]^{1/2}$.

In the limit of $\theta \to 0$ ($m_{ee} \to 0$), we have one Dirac type neutrino. This is the *ZKM Dirac neutrino*, which is a superposition of two left-handed Majorana neutrinos with the degenerate mass $m_{e\mu}$ and opposite helicities, *cf.* Eq.(2·1·7) of I. In this example, we have $m_1 = m_2$ and $(U_{\mathrm{I}})_{l1} = \pm i(U_{\mathrm{I}})_{l2}$ for $l = e$ and μ in order. Note that the difference of lepton numbers $(L_e - L_\mu)$ is conserved, because the mass term $m_{\mathrm{I}}\overline{\phi}\phi$ and the weak charged current in Eq.(2.14) is invariant under the phase transformation,

$$\nu_{eL} \to e^{i\alpha}\nu_{eL} \quad \text{and} \quad \nu_{\mu L} \to e^{-i\alpha}\nu_{\mu L}. \tag{3.27}$$

Only one ZKM Dirac neutrino appears for two generations. Therefore, if all neutrinos are the ZKM Dirac neutrino, six generations are compatible with $\mathcal{N}_\nu = 3$. The intermediate case of four or five generations is allowed, depending on the structure of the mass matrix. Strictly speaking, since the transformation matrix U_{cl} for charged leptons is not the unit matrix in general, the mixing angle θ in Eq.(3.26) should be chosen as having $\nu_{eL} = \phi_L$ and $\nu_{\mu L} = (\phi^{c})_L$.

If $\theta \neq 0$ in Eq.(3.26), we have the *pseudo ZKM Dirac neutrino* which is equivalent with two left-handed Majorana neutrinos. In this case, $\mathcal{N}_\nu = 3$ indicates three generations with one pseudo ZKM Dirac neutrino and one Majorana neutrino. Of course, the $(\beta\beta)_{0\nu}$ mode is allowed in this case, though it is prohibited for the pure ZKM Dirac neutrino.

If we take into account the right-handed Majorana neutrinos, say ν^{o}_{eR}' and $\nu^{o}_{\mu R}$', then we can have two different ZKM Dirac neutrinos by applying the seesaw mechanism. One is light and another is heavy (>100GeV), and the difference of lepton numbers $(L_e - L_\mu)$ is conserved. Such case is realized by assuming that both symmetric left- and right-handed Majorana mass matrices (m_L and M) have only off-diagonal elements and the Dirac mass matrix (m_D) has only diagonal elements. Again, if all six neutrinos are these types of the ZKM Dirac ones, then six generations are compatible with $\mathcal{N}_\nu = 3$. The situation is similar to the case of m_L only. If requirements on m_L, M and m_D are not satisfied, then only three generations are allowed from $\mathcal{N}_\nu = 3$.

Furthermore, it is possible to construct another type of the ZKM Dirac neutrino, which consists of one left-handed Majorana neutrino and aother right-handed one with the degenerate mass $m_j = m_{3+k}$ for $k \neq j$. The case of three generations is only compatible with $\mathcal{N}_\nu = 3$. The seesaw mechanism does not work, so that this case seems not to be realistic.

4. Double beta decay

Two following decay modes compete with the $(\beta\beta)_{0\nu}$ mode in Eq.(2.18);

$$(A, Z-2) \rightarrow (A, Z) + 2e^- + 2\overline{\nu_e} \qquad \text{the } (\beta\beta)_{2\nu} \text{ mode,}$$

$$(A, Z-2) \rightarrow (A, Z) + 2e^- + \chi^0 \qquad \text{the } (\beta\beta)_{0\nu,B} \text{ mode,} \tag{4.1}$$

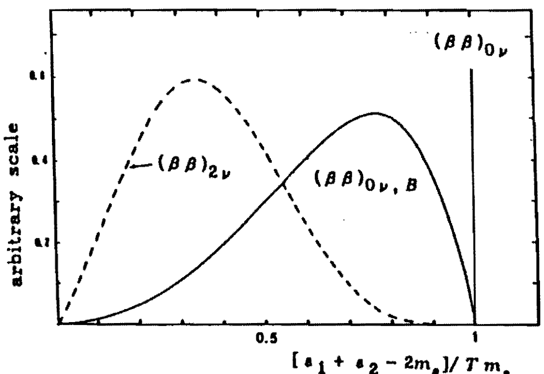

Fig. 1.

where χ^0 is the Majoron. These three decay modes can be distinguished experimentally by measuring the sum-energy spectrum of two electrons, as shown in Fig. 1. The total kinetic energy released in the decay is defined as,

$$T = (M_i - M_f - 2m_e)/m_e,$$

where m_e, M_i and M_f are masses of electron, parent and daughter nuclei, respectively.

4.1 The $(\beta\beta)_{2\nu}$ mode

This decay mode is allowed for either Dirac or Majorana neutrino. Since this decay rate can be calculated unambiguously by the standard V − A model, it is used to check the reliability on the theoretical estimations of nuclear matrix elements.

The half-life of the $0^+ \rightarrow 0^+$ transition in the $(\beta\beta)_{2\nu}$ mode is given as

$$[T_{2\nu}(0^+ \rightarrow 0^+)]^{-1} = \left| M_{GT}^{(2\nu)}/\mu_0 \right|^2 G_{GT}, \tag{4.2}$$

where the integrated kinematical factor G_{GT} is the known numerical quantities.[12] The factor $|M_{GT}^{(2\nu)}/\mu_0|$ is related to the reduced nuclear matrix elements of the double Gamow-Teller (nuclear spin flip) transitions [$M_{GTa}^{(2\nu)}$ defined in Eq.(3·2·4b) of I] by the following definition:

$$\left[\frac{M_{GT}^{(2\nu)}}{\mu_0}\right] = \Sigma_a \left[\frac{M_{GTa}^{(2\nu)}}{\mu_a}\right], \tag{4.3}$$

where Σ_a means the sum over the intermediate nucleus (N_a) at the energy state E_a and by neglecting the lepton energy part, the denominator becomes

$$\mu_a m_e = E_a - (M_i + M_f)/2. \tag{4.4}$$

As an example, let us consider the following decay,

$$ {}^{82}_{34}\mathrm{Se} \rightarrow {}^{82}_{36}\mathrm{Kr} + 2e^{-} + 2\overline{\nu_e} \quad \text{with} \quad T = 5.861 = 2.995\ \mathrm{MeV}/m_e. \tag{4.5} $$

Experimental values of the half-life are in units of 10^{20} years

(1.30 ± 0.05)	(Heidelberg, 1986),[13]
(1.2 ± 0.1)	(Missouri, 1988),[14]
(1.1 + 0.8 − 0.3)	(Irvine, 1987),[15]

where the first two are obtained by the geochemical method and the last one by the time proportional chamber at laboratory. As the integrated kinematical factor is $(G_{GT})^{-1} = 2.276\times10^{17}$ yr for this ^{82}Se decay,[12] we have from the Irvine data

$$ \left| M_{GT}^{(2\nu)}/\mu_0 \right| = 0.046. \tag{4.6} $$

Theoretical estimates of it was (0.083 ~ 0.123) before 1986. In 1986, the Caltech and Tübingen groups proposed the theoretical nuclear models to reproduce the strong suppression, the proton-neutron quasiparticle random phase approximation (*pnQRPA*) with particle-particle interaction (g_{pp}).[7] The Heidelberg group performed the laborious calculation and found the reasonable agreements with present experimental limits for various nuclei.[8] However, there remains some problem on the ${}^{128}_{52}$Te and ${}^{130}_{52}$Te nuclei. We shall return it in next subsection.

Before closing this subsection, we would like to add one comment. Strictly speaking, the denominator of Eq.(4.3) includes the lepton energy difference originally, because of the second order perturbation, *i.e.*, $\mu_a \pm K_D$ (or L_D), where

$$ K_D = (\varepsilon_1 + \omega_1 - \varepsilon_2 - \omega_2)/2m_e \quad \text{and} \quad L_D = (\varepsilon_1 + \omega_2 - \varepsilon_2 - \omega_1)/2m_e, \tag{4.7} $$

ε_j being the energy of the j-th electron. The integrated kinematical factor G_{GT} in Eq.(4.2) takes into account this lepton energy dependence as factors like $[1-(K_D/\langle\mu_a\rangle)^2]$, where $\langle\mu_a\rangle$ is defined from Eq.(4.4) by using some appropriate average of E_a.[12] Since $\mu_a > 10$ and $\langle K_D\rangle \ll T/4$, the error due to this replacement is small, say less than 5%. The value in Eq.(4.6) has been obtained from the experimental data by using Eq.(4.2), so that it is a little diffferent from the direct theoretical evaluation of Eq.(4.3), though this deviation is supposed not to be so serious.

4.2 The $(\beta\beta)_{0\nu}$ mode

Let us consider this decay mode as the transitions of two neutron into two protons inside nucleus, the $2n$ mechanism, as shown in Fig.2. In the minimum standard model, an antineutrino ($\overline{\nu}_e$) with positive helicity is emitted from the n_1 vertex, while a neutrino (ν_e) with negative helicity is absorbed at the n_2 vertex, as shown in Fig.2(a) where the main helicity states of leptons with large momenta are shown by short arrows. These two neutrino lines can not be connected in the standard model. In order

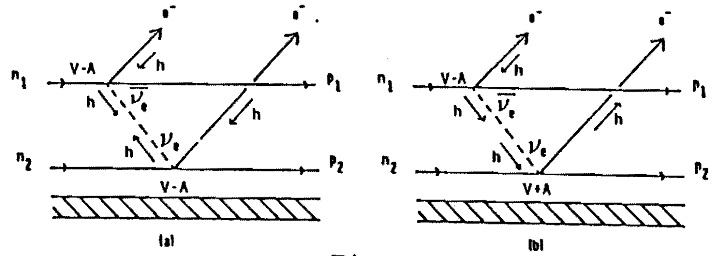

Fig.2.

for the $(\beta\beta)_{0\nu}$ mode to occur, two conditions are required: (1) *The lepton number nonconservation, i.e.*, this virtual neutrino should be the Majorana type. (2) *The helicity matching, i.e.*, both neutrinos should have the same helicity component to connect them.

The second condition is satisfied within the V − A interaction, if neutrinos are massive (the m_ν part), as explained below Eq.(2.16). In addition, there is another possible case (the V + A part), as shown in Fig.2(b). Transition amplitude due to the j-th virtual neutrino is proportinal to $m_j U_{ej}^2$ for the m_ν part, while $\lambda q_j U_{ej} V_{ej}$ for the V + A part, where U and V are the neutrino mixing matrices defined in Eq.(3.22), λ stands for the ratio of the V + A to V − A interactions and q_j is the 4-momentum (ω_j, q) of neutrino, see Eq.(3·3·1) of I. Thus, the m_ν part and the virtual neutrino energy (ω) term of the V + A part contribute to only the $0^+ \to 0^+$ nuclear transition, because both final electrons are in the S wave and there is no daughter nuclei with unit spin in nature. On the other hand, the q term of the V + A part triggers both the $0^+ \to 0^+$ and $0^+ \to 2^+$ transitions, because q requires one more parity odd term like an electron in the $P_{1/2(3/2)}$ wave. There is another parity odd term, the nucleon recoil term. Its contribution is not small, because both final electrons can be in the S wave. It is worthwhile to mention that if the $0^+ \to 2^+$ transition in the $(\beta\beta)_{0\nu}$ mode is observed, then it means unambiguously that at least one neutrino is the massive Majorana and there is the V + A interaction. Its mass may be a tiny radiative correction due to the weak interaction.

The effective interaction Hamiltonian in the simplest extension of the standard model is expressed as

$$H_W = (G/\sqrt{2})[j_{L\rho}(J_L^{\rho +} + \kappa J_R^{\rho +}) + j_{R\rho}(\eta J_L^{\rho +} + \lambda J_R^{\rho +})] + \text{h.c.}\,, \tag{4.8}$$

where $j_{L(R)\rho}$ is the leptonic current defined in Eqs.(2.14 − 15). The ratio of the V + A to V − A interactions is expressed by λ, η and κ, which are written approximately as, for example in the $SU(2)_L \times SU(2)_R \times U(1)_Y$ models,

$$\lambda \sim (M_{WL}/M_{WR})^2, \qquad \eta = \kappa \sim -\tan\zeta, \tag{4.9}$$

where ζ is the mixing angle between the left-handed gauge boson W_L with the mass M_{WL} and the right-handed gauge boson W_R with M_{WR}, *cf.* Eq.(A·2·5) of I. In the $(\beta\beta)_{0\nu}$ decay, the κ term is neglected, because it appears always as $(1 \pm \kappa)$ and $|\kappa| \ll 1$ is expected, see Eq.(C·1·5) of I.

The hadronic V - A (V + A) current $J_{L(R)\rho}$ is within the non-relativistc impulse approximation,

$$J_L^{\rho+}(x) = \sum_n \tau_n^+[(g_V - g_A C_n)g^{\rho 0} + (g_A \sigma_n^k - g_V D_n^k)g^{\rho k}]\delta(x - r_n),$$

$$J_R^{\rho+}(x) = \sum_n \tau_n^+[(g'_V + g'_A C_n)g^{\rho 0} + (-g'_A \sigma_n^k - g'_V D_n^k)g^{\rho k}]\delta(x - r_n), \qquad (4.10)$$

where τ_n^+, σ_n and r_n are the isospin raising, spin and position operators acting on the n-th nucleon, respectively, and $g^{\rho\mu} = (1, -1, -1, -1)$ is the metric tensor. The operators C_n and D_n represent the nucleon recoil terms defined in Eq.(3·1·17) of I, where only the first and second terms are kept in the expansion of the inverse of nucleon mass (m_N). If $m_j > m_N$, the non-relativistic approximation of Eq.(4.10) should be carefully examined, because the third terms contribute to the m_ν part. If $m_j \gg m_N$, then the quark structure of nucleons should be taken into account. In this review, these heavier neutrino case will not be considered.

The quark mixing and the renormalization effect due to the strong interaction are included as follows;

$$g_V = \cos\theta_c = 0.9729, \quad g'_V = e^{i\alpha}\cos\theta'_c \quad \text{and} \quad (g_A/g_V) = (g'_A/g'_V) = 1.254, \qquad (4.11)$$

where θ_c and θ'_c are the Cabibbo-Kobayashi-Maskawa mixing angle for the left- and right-handed d and s quarks, respectively, and α is the CP violating phase, see Eq.(3·1·11) of I.

Thus, we have three unknown effective parameters for masses of virtual neutrinos and the V + A part,

$$\langle m_\nu \rangle = |\sum'_j m_j U_{ej}^2|, \quad \langle\lambda\rangle = \lambda\,|\sum'_j U_{ej}V_{ej}(g'_V/g_V)|, \quad \langle\eta\rangle = \eta\,|\sum'_j U_{ej}V_{ej}|, \qquad (4.12)$$

where the primed sum extends over only the light neutrinos (m_j<10 MeV). The reason for this restriction on the sum is that if m_j is smaller than the average value of the intermediate nuclear energy level μ_a in Eq.(4.4) which is of order of 20, then the neutrino potential due to the virtual neutrino exchange is a simple Coulomb type (ϕ/r) independent of m_j, see Fig.3.4 of I and Fig.8 of the second paper of Ref. 8. If $m_j > m_N$, the neutrino potential becomes a Yukawa type depending on m_j, so that the expressions of these effective parameters should include the ratio of nuclear matrix elements like $[\langle\phi' e^{-m_j r}/r\rangle/\langle\phi/r\rangle]$. However, as mentioned already, the non-relativistic approximation of Eq.(4.10) and the quark structure of nucleons should be carefully examined for the heavier neutrino case, especially for the m_ν part. In addition, the mixing matrix element U_{ej} becomes smaller in general. Therefore, we do not write this case explicitly. However, it is worthwhile to note that the absolute value of the contribution from some heavier neutrino should be equal to the one from the lighter neutrino, because $\langle m_\nu\rangle$ is proportional to m_j.

Now let us examine the kinematical characters of the m_ν and V + A parts. From the dimensional analysis, the effective mass $\langle m_\nu \rangle$ is normalized by some typical energy scale of the decay process, say m_e. Then, we have a big suppression factor, $(\langle m_\nu \rangle / m_e) \sim 10^{-5}$. While, the corresponding quantity of the V + A part is the average energy of virtual neutrino which is of order of $\langle \omega \rangle \sim \langle q \rangle \sim \langle 1/R \rangle \sim 80\ m_e$, where R is the nuclear radius. Therefore, it may be expected that the V + A part gives the larger contribution than the $\langle m_\nu \rangle$ part, but it is not so in reality. If all neutrinos are lighter than 10 MeV, then the unitality property of U_ν in Eq.(3.8) requires the relation,

$$\Sigma_j U_{ej} V_{ej} = 0. \tag{4.13}$$

Thus, the non-zero values of $\langle \lambda \rangle$ and $\langle \eta \rangle$ in Eq.(4.12) mean to measure the small deviations from zero which are based on the contributions from the virtual heavy neutrinos, in addition to the smallness of λ and η themselves.

Contributions from the m_ν and V + A parts can be distinguished in principle by measuring the angular correlation between two final electrons, because the former shows the $(1-\cos\theta)$ type but the latter is the mixing of $(1+\cos\theta)$ and isotropic types, see Eq.(6·2·1) of I.

The half-life of the $0^+ \rightarrow 0^+$ transition in the $(\beta\beta)_{0\nu}$ mode is given as

$$[T_{0\nu}(0^+ \rightarrow 0^+)]^{-1} = \left| M_{GT}^{(0\nu)} \right|^2 \Big[C_1(\langle m_\nu \rangle/m_e)^2 + C_2 \langle \lambda \rangle (\langle m_\nu \rangle/m_e) \cos\phi_1$$
$$+ C_3 \langle \eta \rangle (\langle m_\nu \rangle/m_e) \cos\phi_2 + C_4 \langle \lambda \rangle^2 + C_5 \langle \eta \rangle^2 + C_6 \langle \lambda \rangle \langle \eta \rangle \cos(\phi_1 - \phi_2) \Big], \tag{4.14}$$

where ϕ_1 and ϕ_2 take 0 or π, if CP is conserved. The double Gamow-Teller nuclear matrix element is defined as

$$M_{GT}^{(0\nu)} \equiv \Sigma_a \langle 0_f^+ \| \ h_+(r_{nm}, E_a) \sigma_n \cdot \sigma_m \ \| 0_i^+ \rangle, \tag{4.15}$$

where h_+ is the neutrino potential, r_{nm} is the distance between the n-th and m-th decaying neutrons and the abbreviation for the reduced nuclear matrix element had been defined in Eq.(B·1·5) of I. Coefficients C_j are some combinations of 8 integrated kinematical factors and other 8 nuclear matrix elements, see Eq.(3·5·10) of I.

We shall consider experimental data on two nuclei as examples.

For the decay of ${}^{76}_{32}\mathrm{Ge} \rightarrow {}^{76}_{34}\mathrm{Se} + 2e^-$, the largest upper limit of the half-life for the $0^+ \rightarrow 0^+$ transition with T = 3.991 (= 2.0396 MeV) in the $(\beta\beta)_{0\nu}$ mode is,

$$T_{0\nu}(0^+ \rightarrow 0^+; {}^{76}\mathrm{Ge}) > 8\times 10^{23} \mathrm{yr} \qquad (\text{UCSB-LBL, 1988}). \tag{4.16}$$

(UCSB-LBL, 1988).[16]

Using estimations obtained by the Heidelberg group,[8] we find

$\langle m_\nu \rangle < 1.7$ eV, *i.e.* $(\langle m_\nu \rangle / m_e) < 3.3\times 10^{-6}$,
$\langle \lambda \rangle < 3.1\times 10^{-6}$,

$$\langle \eta \rangle < 1.7\times10^{-8}. \tag{4.17}$$

Each limit is obtained by assuming two other parameters to be zero. The smaller upper limit of $\langle \eta \rangle$ is due to the large contributuion from the nucleon recoil term accompanied with the induced weak magnetism (g_W).

Next, let us consider the ratio of total half-lives of ^{130}Te to ^{128}Te :

$$R_T^{-1} = \left[\frac{T_{2\nu}(^{130}\mathrm{Te})}{T_{2\nu}(^{128}\mathrm{Te})}\right]\left[\frac{\{1+(T_{2\nu}/T_{0\nu})\}(^{128}\mathrm{Te})}{\{1+(T_{2\nu}/T_{0\nu})\}(^{130}\mathrm{Te})}\right]. \tag{4.18}$$

The interest in this ratio is that the maximum kinetic energy release of ^{128}Te ($T = 1.700$) is much smaller than that of ^{130}Te ($T = 4.957$). Thus, if the $(\beta\beta)_{0\nu}$ mode exists, its yield in the ^{128}Te decay may be comparable with the $(\beta\beta)_{2\nu}$ mode, while in the ^{130}Te decay the yield of the $(\beta\beta)_{0\nu}$ mode is much less than the $(\beta\beta)_{2\nu}$ mode, because the half-lives of the three body decay (the $(\beta\beta)_{0\nu}$ mode) and the five body decay (the $(\beta\beta)_{2\nu}$ mode) depend roughly on T as T^{-5} and T^{-11}, respectively. In fact, theoretical estimates by the Heidelberg group[8] confirm the tendency of this rough idea;

$$\begin{aligned} &T_{2\nu}(^{130}\mathrm{Te}) = 1.84\times10^{21}\mathrm{yr}, \qquad T_{0\nu}(0^+\to0^+;\ ^{130}\mathrm{Te}) > 1.68\times10^{23}\mathrm{yr},\\ &T_{2\nu}(^{128}\mathrm{Te}) = 2.63\times10^{24}\mathrm{yr}, \qquad T_{0\nu}(0^+\to0^+;\ ^{128}\mathrm{Te}) > 2.49\times10^{24}\mathrm{yr}, \end{aligned} \tag{4.19}$$

where $T_{0\nu}(0^+\to0^+)$ is calculated by assuming $\langle m_\nu\rangle < 1.7$ eV and $\langle\lambda\rangle = \langle\eta\rangle = 0$.

While, the small T value means the long life time and the difficulty to measure electrons precisely. This ratio has been measured by the geochemical method, which can not discriminate two decay modes and each ($0^+\to0^+$ or 2^+) transition. The ($0^+\to2^+$) transition is known to be small in the $(\beta\beta)_{2\nu}$ mode, but is the open question in the $(\beta\beta)_{0\nu}$ mode. But as it is irrelevant to the present discussion, we do not consider it.

It is clear from Eq.(4.19) that $(T_{2\nu}/T_{0\nu}) \ll 1$ for the ^{130}Te decay. Thus, we have the following inequality;

$$R_T^{-1} \geqq (R_{2\nu})^{-1} \equiv \left[\frac{T_{2\nu}(^{130}\mathrm{Te})}{T_{2\nu}(^{128}\mathrm{Te})}\right] = (1.78\pm0.08)\times10^{-4}\left|\frac{[M_{GT}^{(2\nu)}/\mu_0](^{128}\mathrm{Te})}{[M_{GT}^{(2\nu)}/\mu_0](^{130}\mathrm{Te})}\right|^2, \tag{4.20}$$

where the equality means no $(\beta\beta)_{0\nu}$ mode and Eq.(4.2) has been used. It was pointed out by Pontecorvo that the similar values are expected for nuclear matrix elements of ^{128}Te and ^{130}Te.

Three recent experimental results are

$$\begin{aligned} R_T^{-1} = &(1.01 \pm 1.13)\times10^{-4} && \text{(Heidelberg, 1983),[13]}\\ &(3.9\ {+1.5}\ {-0.8})\times10^{-4} && \text{(Missouri, 1988),[14]}\\ &(3.2 \pm 1.3)\times10^{-4} && \text{(Yamagata, 1989).[17]} \end{aligned} \tag{4.21}$$

The theoretical estimates in Eq.(4.19) gives $(R_{2\nu})^{-1} = 7.0\times10^{-4}$. This value is larger than experimental values and inconsistent with the inequality in Eq.(4.20). The calculated value for the ratio of nuclear matrix elements is 1.98. If the uncertainty of 2σ is allowed for experimental data, then we may say that this theoretical value of $(R_{2\nu})^{-1}$ mean the very small limit on $\langle m_\nu \rangle$ or no $(\beta\beta)_{0\nu}$ mode. It is necesssary to recheck theoretical estimates on the nuclear matrix elements for the $(\beta\beta)_{2\nu}$ mode.

4.3 The $(\beta\beta)_{0\nu,B}$ mode

The recent experimental data on the invisible decay width of the Z^0 boson exclude the Majoron which has hypercharge larger than or equal to 1,[18] as already mentioned in section 1. At present, we do not know the model of Majoron which make the significant contribution to the $(\beta\beta)_{0\nu,\mathrm{B}}$ mode, but does not couple with the neutral gauge boson. Therefore, we present only experimental upper limits for this mode;

$$T_{0\nu,B}(0^+\rightarrow 0^+;\ ^{76}\mathrm{Ge}) > 1.4\times10^{21}\mathrm{yr} \qquad (\text{UCSB-LBL, 1988}).^{16)}$$
$$T_{0\nu,B}(0^+\rightarrow 0^+;\ ^{82}\mathrm{Se}) > 1.6\times10^{21}\mathrm{yr} \qquad (\text{Irvine, 1988}).^{15)}$$
$$T_{0\nu,B}(0^+\rightarrow 0^+;\ ^{100}\mathrm{Mo}) > 3.3\times10^{20}\mathrm{yr} \qquad (\text{LBL-Hol.-NM, 1988}).^{19)} \qquad (4.22)$$

5. Summary

The number of light neutrinos is assumed to be restricted to $\mathcal{N}_\nu = 3$ from the invisible decay width of the Z^0 decay.[3] There are three possible types of light neutrinos, *i.e.*, ordinary Dirac, left-handed Majorana and ZKM Dirac defined below Eq.(3.26). The numbers of the former two types are supposed to be m and n, respectively. Then the number of generations and these possible types are related as follows:

No. of generations	$m + n$	No. of ZKM Dirac	
3	3	0	
4	2	1	
5	1	2	
6	0	3	(5.1)

Thus total 10 combinations are consistent with $\mathcal{N}_\nu = 3$. Concerning the ZKM Dirac neutrino, we considered only the case where it consists of two left-handed Majorana neutrinos. There are other possibilities that it is a superposition of one left-handed Majorana neutrino and another right-handed one but not the ordinary Dirac; for example, only three ZKM Dirac neutrinos for three generations are compatible with $\mathcal{N}_\nu = 3$. We did not include these possibilities in this list. Various models can be constructed from these 10 cases. For example, a pair of two left-handed Majorana neutrinos may be treated as one pseudo ZKM Dirac type. For the left-handed Majorana and ZKM Dirac types, we can add the corresponding heavier neutrinos by applying the seesaw mechanism, by which masses of light neutrinos are of order of a few eV or less. Of course, all masses of quarks and charged leptons of the fourth and higher generations should be greater than 50 GeV, the half of the Z^0 mass.

The $(\beta\beta)_{0\nu}$ mode has not yet been observed. It is a still open question whether neutrinos are Dirac or Majorana types. If this mode is observed, then at least one of neutrinos is a massive Majorana one unambiguously, see section A.1 of I. The precise knowledge of nuclear matrix elements is required in order to take out the useful information on the effective neutrino mass parameter $\langle m_\nu \rangle$ and the effective V + A interaction parameters $\langle \lambda \rangle$ and $\langle \eta \rangle$.

If the $0^+ \to 0^+$ transition is observed and $\langle m_\nu \rangle$ is obtained, then at least one of neutrino masses m_j satisfies $m_j \geqq \langle m_\nu \rangle$, where the equality stands for the special case with only one light left-handed massive Majorana neutrino. If the finite values of $\langle \lambda \rangle$ and $\langle \eta \rangle$ are determined, they give the most severe restrictions on the V + A interaction and also mean the finite deviation from the unitality condition in Eq.(4.13), that is, the existence of heavy neutrino, say $m_j > 10$ MeV. The present upper limits are $\langle m_\nu \rangle < 1.7$ eV, $\langle \lambda \rangle < 3.1\times10^{-6}$ and $\langle \eta \rangle < 1.7\times10^{-8}$.

If the $0^+ \to 2^+$ transition is observed, the existence of the V + A interaction is established uniquely. However, if all masses of neutrinos are less than 10 MeV, the transition due to the V + A interaction in the $(\beta\beta)_{0\nu}$ mode is forbidden. In this case, experimental data from the $\pi^+ \to \mu^+ \to e^+$ decay[20] give the restrictions on parameters of the V + A interaction defined in Eq.(4.9), *i.e.*, $\lambda \leqq 0.0263$ ($m_{WR} \geqq 432$ GeV) and $|\eta| \leqq 0.037$, *cf.*, Table 10.1 of I.

References

1) M. Doi, T. Kotani and E. Takasugi, Prog. Theor. Phys. Supp. 83 (1985) 1. All articles cited in this reference 1 are not repeated in this short review except some special case.
2) G.B. Gelmini and M. Roncadelli, Phys. Lett. B99 (1981), 411.
3) G.S. Abrams, *et al.*, Phys. Rev. Lett. 63 (1989), 2181.
L3 Collaboration, Phys. Lett. B231 (1989), 509.
ALEPH Collaboration, Phys. Lett. B231 (1989), 519.
OPAL Collaboration, Phys. Lett. B231 (1989), 530.
DELPHI Collaboration, Phys. Lett. B231 (1989), 539.
4) V. Barger, *et al.*, Phys. Lett. B192 (1987), 212.
V. Barger, *et al.*, Phys. Rev. D26 (1982), 218.
H.M. Georgi, S.L. Glshow and S. Nussinov, Nucl. Phys. B193 (1981), 297.
5) Ya.B. Zeldovich, Dok. Akad. Nauk. USSR 86 (1952), 505.
E.J. Konopinski and H. Mahmoud, Phys. Rev. 92 (1953), 1045.
6) OPAL Collaboration, CERN preprint, CERN-EP/89-147.
7) P. Vogel and M.R. Zirnbauer, Phys. Rev. Lett. 57 (1986), 3148.
O. Civitarese, A. Faessler and T. Tomoda, Phys. Lett. B194 (1987), 11.
T. Tomoda and A. Faessler, Phys. Lett. B199 (1987), 475.
J. Engel, P. Vogel and M.R. Zirnbauer, Phys. Rev. C37 (1988), 731.
8) K. Muto, E. Bender and H.V. Klapdor, Z. Phys. A 334 (1989), 177 and 187 and references therein.
9) A.V. Kyuldjiev, Nucl. Phys. B243 (1987), 387.

10) R. Barbieri and R.N. Mohapatra, Phys. Rev. Lett. 61 (1988), 27.
I. Goldman, *et al.*, Phys. Rev. Lett. 60 (1988), 1789.
J.M. Lattimer and J. Cooperstein, Phys. Rev. Lett. 61 (1988), 23 and 2633 (Err).

11) B. Pontecorvo, Zh. Eksz. Teor. Fiz. 33 (1957), 549 [Sov. Phys. - JETP 6 (1958), 429].
V. Barger, *et al.*, Phys. Rev. Lett. 45 (1980), 692.
J. Schechter and J.W.F. Valle, Phys. Rev. D22 (1980), 2227.
S.M. Bilenky and S.T. Petcov, Rev. Mod. Phys. 59 (1987), 671.

12) M. Doi, T. Kotani and E. Takasugi, Phys. Rev. C37 (1988), 2104.

13) T. Kirsten, *et al.*, Proc. of International Symposium on Nuclear Beta Decays and Neutrinos, ed. by Kotani, Ejiri and Takasugi (World Scientific Pub., Singapore, 1986), p.81 and references therein.

14) W.J. Lin, *et al.*, Nucl. Phys. A481 (1988), 477 and 484.

15) S.R. Elliott, A.A. Hahn and M.K. Moe, Phys. Rev. Lettt. 59 (1987), 2020 and 1649.

16) D.O. Caldwell, *et al.*, Phys. Rev. Lett. 59 (1987), 419.
D.O. Caldwell, Univ. of California at Santa Barbara preprint (1989), UCSB-HEP-88-8.

17) N. Takaoka, private communication.

18) M.C. Gonzakez-Garcia and Y. Nir, SLAC preprint (1989), SLAC-PUB-5090.

19) M. Alston-Garnjost, *et al.*, Phys. Rev. Lett. 60 (1988), 1928.

20) D.P. Stoker, *et al.*, Phys. Rev. Lett. 54 (1985), 1887.
J. Carr, *et al.*, Phys. Rev. Lett. 51 (1983), 627.