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GLOBALLY CORRECT QUANTIZATION OF A PURE YANG-MILLS THEORY : PERTURBATIVE EXPANSION

Stefano Fachin

Physics Department,
New York University
4 Washington Place,
New York, NY 10003, USA

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Abstract

We consider a new Lorentz-covariant quantization of gauge fields which overcomes the problem of Gribov copies and verify, by explicit one- and two-loop perturbative expansion, that the two-point function in the new gauge (maximal Landau gauge) agrees with the conventional perturbative correlation function in the Landau gauge. It is shown that this result can be extended, at least up to one-loop level, to all correlation functions .

1 Introduction

One of the main difficulties presented by QCD is that the quanta of the fundamental gauge-coupled fields are not particles observed in nature, but are instead believed to be in a confined phase that is not described by conventional renormalized perturbation theory.

In searching for non-perturbative results in continuum gauge theory a new gauge (maximal Landau gauge) has been introduced [1, 2]. This replaces the Faddeev-Popov quantization, which is not correct because of the existence of Gribov copies [3, 4], and arises quite naturally from the necessity of quantizing the gauge fields by integrating over the correct classical configuration space.

We will review this aforementioned Lorentz-covariant quantization procedure in the following section. Then, to ensure consistency of the new formulation, we revert to perturbation theory: first we derive the Feynman rules in the new gauge, stressing the relationship enforced by gauge invariance among the new vertices (secs. 3, 4) and then we explicitly calculate the one- (sec. 5) and two- (sec. 7) loop corrections to the two-point correlation function. These will be found to agree, in the limit in which the gauge-fixing parameter is taken to infinity, with the conventional perturbative correlation functions that one finds in the Landau gauge. We will also show that this result holds, at one-loop level, for all correlation functions (sec. 6) .

2 The new gauge-fixing procedure

The expectation value of a gauge invariant observable $O[A]$ is given by the formal Euclidean functional integral

$$< O >= \int \mathcal{D}A\, e^{-S_{cl}[A]}\, O[A]\,, \tag{1}$$

where $S_{cl}[A] = \frac{1}{2}\beta|F|^2 = \frac{1}{4}\beta \int d^Dx(F^a_{\mu\nu}F^a_{\mu\nu})$ is the usual kinetic action for the gauge fields and the proper normalization factor is understood in front of the integral; A is the unrenormalized classical connection and β is related to the unrenormalized coupling constant g by $\beta = 1/g^2$. Since the action $S_{cl}[A]$ is gauge invariant, the functional integration in Eq.(1) has to be restricted to the physical configuration space; integrating blindly over all field configurations

would result in a divergence due to redundant infinite integrations, since the gauge group is not compact in the continuum case. This is usually achieved by gauge-fixing, i.e. by choosing a representative field on each gauge orbit. The set of all these representatives constitute what is known as fundamental modular region, and is the correct region over which one should integrate.

All gauge-fixing procedures have to face the problem that there is no global section of the fiber bundle of connections[1] (on a torus), so that they are locally correct, but globally they may contain gauge-equivalent copies (Gribov copies) of the representative connection. A globally correct gauge-fixing procedure that overcomes the problem of Gribov copies can be introduced [1, 2]. The correctness of the procedure we are about to review rests on some recent results that we are going to summarize first.

Let Ω be the Gribov region, that is the region characterized by

$$\Omega = \{A \in \mathcal{A} : \partial \cdot A = 0, K[A] = -\partial \cdot D[A] \geq 0\} \tag{2}$$

where $\mathcal{A}$ is the set of all connections for a fixed principal bundle $P(M, G)$ with gauge group G over a base-space M (M is to be identified with the D-dimensional, usually compactified, space-time; in our case the D-dimensional torus T^D). Let Λ be the fundamental modular region. It has been shown that:

1. Every gauge orbit $A^g (g \in G)$ does intersect Ω, so that $\Lambda \subseteq \Omega$ [7].

2. There are Gribov copies inside the Gribov region Ω, i.e. some of the orbits intersect Ω more than once, so that the inclusion $\Lambda \subset \Omega$ is proper [3, 4].

3. The region Ω is the region where the minima of the functional

$$S_A[g] \equiv |A^g|^2 = -tr \int d^D x A^g_\mu(x) A^g_\mu(x) \tag{3}$$

lie [8]. For fixed A , $S_A[g]$ is a functional of g whose domain is the orbit through A. Its most important feature is that it reaches its absolute minimum on each gauge orbit [7].

[1]This result was first proved by Singer for the case of a principal bundle over a four-sphere with compact (non-Abelian) Lie group [5] and , more recently, by Killingback for a principal bundle over a four-torus with any compact non-Abelian semi-simple Lie group G, and even for the Abelian $G = U(1)$ case [6] .

In absence of degeneracy of the absolute minimum (outside of a set of measure zero), the feature described in (3) makes it possible to choose the region of absolute minima of $S_A[g]$ as the fundamental modular region, i.e. to choose, for every fixed A, the absolute minimum of $S_A[g]$ as representative on the orbit that passes through A. This in turn is achieved, given that the region of absolute minima of $S_A[g]$ is not known explicitly, by adding a gauge-fixing term $M^2|A^g|^2$ to the classical action (where M is a gauge parameter with the dimension of mass) so that, as $M \longrightarrow \infty$, the Euclidean probability gets concentrated at the absolute minimum of $|A^g|^2$ on each gauge orbit, and the path-integration is thus automatically confined to the interior of the physical configuration space Λ.

Following the standard Faddeev-Popov procedure one defines the gauge invariant function

$$I[A] \equiv \int dg e^{-\frac{1}{2}\beta M^2|A^g|^2} \tag{4}$$

and rewrites Eq.(1) in the equivalent form

$$< O >= \int \mathcal{D}A\, e^{-S_{cl}[A]}\, O[A]\, I^{-1}[A] \int dg e^{-\frac{1}{2}\beta M^2|A^g|^2} \tag{5}$$

By exploiting the gauge-invariance of the measure, of $S_{cl}[A]$ and $O[A]$, we can factorize out the (infinite) volume of the gauge orbits and include it in the normalization constant so that Eq.(5) reduces to

$$< O >= \int \mathcal{D}A\, O[A] e^{-S_{cl}[A]} e^{-S_{gf}[A]} (I[A])^{-1} \tag{6}$$

where $S_{gf}[A] \equiv \frac{1}{2}\beta M^2|A|^2$. The (gauge-variant) gauge-fixing term $e^{-S_{gf}[A]}$ gives a different weight to each point on a gauge orbit in such a way that the points having the maximum weight are the ones inside the fundamental modular region Λ: by taking the limit $M \longrightarrow \infty$ at the end of the calculation the probability gets concentrated at the absolute minima of $S_A[g]$, i.e. inside Λ.

To evaluate $I[A]$ one takes the limit $M \longrightarrow \infty$, so the Gaussian approximation becomes exact and one obtains

$$I[A] = e^{-\frac{1}{2}\beta M^2|B|^2} det^{-\frac{1}{2}}(-\partial \cdot D[B]) \tag{7}$$

where B is the configuration on the gauge orbit on which the absolute minimum is attained.

3 The magic of gauge-invariance: Ward-Takahashi Identities

Consider a gauge invariant functional $F[A]$ of the gauge-field A and let us assume that F can be expanded in powers of A :

$$F[A] = \sum_{n=o}^{\infty} \frac{1}{n!} \int d^D x_1 \dots d^D x_n F^{(n)}(x_1, \dots, x_n)^{a_1 \dots a_n}_{\mu_1 \dots \mu_n} A^{a_1}_{\mu_1}(x_1) \dots A^{a_n}_{\mu_n}(x_n). \quad (8)$$

What one can show is that the gauge-invariance of F entails some relationships among the coefficients $F^{(n)}$ of its power expansion. Starting from the condition which expresses the gauge-invariance of F

$$\int d^D x D^{ab}_{\mu}[A(x)] \frac{\delta F}{\delta A^b_{\mu}(x)} = 0 \quad (9)$$

and substituting for F its power expansion of Eq.(8) one finds, after a little algebra (and Fourier-transforming to momentum space)

$$-ik_{\mu} F^{(n+1)}(k, k_1, \dots, k_n)^{a a_1 \dots a_n}_{\mu \mu_1 \dots \mu_n} + \sum_{r=1}^{n} f^{a a_r c} F^{(n)}(k_1, \dots, k_r + k, \dots, k_n)^{a_1 \dots c \dots a_n}_{\mu_1 \dots \mu_r \dots \mu_n} = 0, \quad (10)$$

which provides a recursion relation for the longitudinal part of these coefficients. At $k = 0$ this reduces to a condition among the coefficients of a given order n:

$$\sum_{r=1}^{n} f^{a a_r c} F^{(n)}(k_1, \dots, k_r, \dots, k_n)^{a_1 \dots c \dots a_n}_{\mu_1 \dots \mu_r \dots \mu_n} = 0, \quad (11)$$

which expresses global gauge invariance.

One can now apply this general technique to our case and find relationships among the vertices. Following the standard procedure, we rescale the field A by g, the coupling constant, before deriving the Feynman rules. The action $S[A] \equiv S_{cl}[A] + S_{gf}[A]$ that appears in the functional integral in Eq.(6) contains a term that breaks gauge-invariance: $S[A] = S_{inv}[A] + \frac{1}{2} M^2 |A|^2$. Nevertheless, since

$$(D \cdot \frac{\delta}{\delta A}) S[A] = M^2 (\partial \cdot A) \quad (12)$$

is linear in A, if one expands $S[A]$ in powers of A and follows the procedure used above, he will find for the coefficients $S^{(n)a_1\ldots a_n}_{\mu_1\ldots\mu_n}(x_1,\ldots,x_n) = \delta^n S[A]/(\delta A^{a_1}_{\mu_1}(x_1)\ldots A^{a_n}_{\mu_n}(x_n))\ |_{A=0}$ of the expansion the same relationship we stated in Eq.(10); only the one pertaining to the linear term has to be modified. We will find convenient, in the calculations that follow, to split the action into two pieces

$$S[A] = S^{loc}[A] + S^{nl}[A] \tag{13}$$

where the local part is defined by

$$S^{loc}[A] \equiv \frac{1}{2}|F|^2 + \frac{1}{2}M^2|A|^2 \tag{14}$$

and the remaining part is

$$S^{nl}[A] \equiv\ -\frac{1}{2}M^2\ |B[A]|^2\ -\ tr\ln(-\partial\cdot D[B[A]]) \equiv S^{nl}_{gl} + S^{nl}_{gh}; \tag{15}$$

Eq. (10) will hold separately for each piece.

For example one finds that (for $n=2$), in lowest order, the S^{nl}_{gl} contribution to the inverse gluon propagator[2]

$$\mathcal{F}\left[\left.\frac{\delta^2 S^{nl}_{gl}}{\delta A^{a_1}_{\mu_1}(x_1)\delta A^{a_2}_{\mu_2}(x_2)}\right|_{A=0}\right] =\ -M^2 P^T(k)_{\mu_1\mu_2}\delta^{a_1a_2} \tag{16}$$

and the non-local three-point vertex function

$$\mathcal{F}\left[\left.\frac{\delta^3 S^{nl}_{gl}}{\delta A^{a_1}_{\mu_1}(x_1)\delta A^{a_2}_{\mu_2}(x_2)\delta A^{a_3}_{\mu_3}(x_3)}\right|_{A=0}\right] \ =\ V^3_{nl}(k_1,k_2,k_3)^{a_1,a_2,a_3}_{\mu_1,\mu_2,\mu_3} \tag{17}$$

are related by

$$ik_\mu V^3(k,k_1,k_2)^{aa_1a_2}_{\mu\mu_1\mu_2} - M^2P^T(k_2)_{\mu_1\mu_2}f^{aa_1a_2} - M^2P^T(k_1)_{\mu_1\mu_2}f^{aa_2a_1} = 0 \tag{18}$$

One can therefore extract the color dependence by defining[3]

[2] $\mathcal{F}[.]$ indicates the Fourier transform of the object in brackets.

[3] Clearly Eq. (18) gives a constraint only on the longitudinal part of V^3_{nl} and one should allow in Eq. (19) for a dependence on the totally symmetric tensor $d^{a_1a_2a_3}$ as well. But, as we will show in the next section, all non-local vertices must have at least two longitudinal legs, so that the only dependence on the color indices that is actually allowed is the one shown in Eq. (19).

$$V^3_{nl}(k_1,k_2,k_3)^{a_1,a_2,a_3}_{\mu_1,\mu_2,\mu_3} \equiv i f^{a_1a_2a_3} V^3_{nl}(k_1,k_2,k_3)_{\mu_1,\mu_2,\mu_3} \tag{19}$$

and Eq.(18) gives

$$k_\mu V^3_{nl}(k_1,k_2,k_3)_{\mu_1,\mu_2,\mu_3} = M^2\ [P^T(k_1) - P^T(k_2)]_{\mu_1\mu_2}. \tag{20}$$

With the notation for the (non-local) n-point function

$$V^n_{nl}(x_1,\ldots,x_n)^{a_1\ldots a_n}_{\mu_1\ldots\mu_n} \equiv \left.\frac{\delta^n S^{nl}_{gl}}{\delta A^{a_1}_{\mu_1}(x_1)\ldots\delta A^{a_n}_{\mu_n}(x_n)}\right|_{A=0} \tag{21}$$

Eq.(10) gives

$$ik_\mu V^{n+1}_{nl}(k,k_1,\ldots,k_n)^{aa_1\ldots a_n}_{\mu\mu_1\ldots\mu_n} = \sum_{r=1}^{n} f^{aa_rb} V^n_{nl}(k_1,\ldots,k_r+k,\ldots,k_n)^{a_1\ldots b\ldots a_n}_{\mu_1\ldots\mu_r\ldots\mu_n}\ (n\geq 2) \tag{22}$$

4 Perturbative Expansion

The Feynman rules for the perturbative expansion can be read off, as usual, from the generating functional Z,

$$Z[J] = \int \mathcal{D}A\, e^{-S_{cl}[A]-S_M[A]+(J,A)+\frac{1}{2}tr\log(-\partial\cdot D[B[A]])} \tag{23}$$

where

$$S_M[A] \equiv S_{gf}[A] - \frac{1}{2}M^2|B|^2 = \frac{1}{2}M^2(|A|^2 - |B[A]|^2). \tag{24}$$

Since gauge-invariance enforces relationships among the vertices, as we saw in the previous section, one need not calculate explicitly the expansion of $B[A]$ in powers of A; all one has to do is to find the lowest order term of the expansion and make use of the recursion relationship given in Eq.(22).

To find this lowest order term let us denote, for a fixed A, by $g = e^\omega$ the gauge-transformation that brings A to the point $B[A]$ where the functional $S_A[g]$ reaches its absolute minimum: $B[A] = A^g$. The condition that B be a stationary point of $S_A[g]$ is, for an infinitesimal gauge-transformation,

$$\partial\cdot B = 0 = \partial\cdot A^g = \partial\cdot A + \partial\cdot D[A]\omega + O(\omega^2) \tag{25}$$

Eq.(25) shows that ω is of order A^L (A^L denotes the longitudinal component of the gauge field A). Expanding ω in powers of A, $\omega = \sum_{n=1}^{\infty} \omega^{(n)}$, where $\omega^{(n)}$ is of order $O(A^n)$ in the gauge field, and substituting this expression in Eq.(25) one finds

$$\partial \cdot A^L + \partial^2 \omega^{(1)} = 0. \tag{26}$$

Solving for $\omega^{(1)}$

$$\omega^{(1)} = -\Delta^{-1} \partial \cdot A^L \tag{27}$$

which gives the lowest order term of the expansion of B

$$\begin{aligned} B[A] &= A + D[A]\omega + O(\omega^2) = A + \partial\omega^{(1)} + O(A^2) = \\ &= A - \partial\Delta^{-1}\partial \cdot A^L = A - A^L = A^T \end{aligned} \tag{28}$$

so that

$$|B[A]|^2 = |A^T|^2 + O(A^3) \tag{29}$$

and

$$S_M[A] = \frac{1}{2} M^2 |A^L|^2 + \dots \ . \tag{30}$$

From (30) one concludes that:

a) The quadratic piece of S_M gives a purely longitudinal contribution to the usual Landau-gauge gluon propagator which vanishes as $M \longrightarrow \infty$.

b) All vertices that one finds by expanding S_M in powers of A [and that we will sometimes refer to as non-local, since they originate from the non-local part of the action $S^{nl}[A]$ of Eq. (15)], have the following remarkable features:

1. They have at least two legs which are longitudinal.
2. They are related to each other in virtue of Eq.(22).

Inverting the quadratic part of the action $S_{cl} + S_M$ one finds the zeroth-order gluon propagator in momentum space

$$\mathcal{D}^{ab}_{\mu\nu}(k) = \delta^{ab}[\frac{1}{k^2} P^T_{\mu\nu}(k) + \frac{1}{M^2} P^L_{\mu\nu}(k)] \tag{31}$$

where

$$P^T_{\mu\nu}(k) = \delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \ \text{ and } \ P^L_{\mu\nu}(k) = \frac{k_\mu k_\nu}{k^2} \tag{32}$$

are the projectors on the transverse and the longitudinal subspaces, respectively. The transverse part of $\mathcal{D}$, which is the only one that survives in the $M \longrightarrow \infty$ limit, is the usual Landau-gauge gluon propagator. However the longitudinal part will contribute in diagrams containing non-local vertices, which are proportional to M^2.

Summarizing: the cubic and quartic part of $S_{cl}[A]$ give rise to the usual three- and four-leg vertices that represent the self-interaction of the gluons, while the non-local part

$$S^{nl}[A] \equiv -\frac{1}{2}\beta M^2 |B[A]|^2 - \frac{1}{2} tr \ln(-\partial \cdot D[B[A]]) \tag{33}$$

is made up by two pieces: the first one, proportional to $|B|^2$, gives rise to the new (non-local) vertices for the self-interaction among the gluons, while the second one is just one-half of the usual ghost contribution, but with the field $B[A]$ taking the place of A. One apparently needs once again to expand $B[A]$ in powers of A up to the required order in each perturbative calculation. This expansion would produce new ghost-ghost-gluons vertices; from the fact that $B[A^L = 0, A^T] = A^T$ and $B[A] = A^T + O(A^2)$ it follows that

1. The lowest order vertex is the same as the usual Landau-gauge ghost-ghost-gluon vertex.

2. All higher order vertices (i.e. vertices with two ghost lines and two or more gluon lines) have at least one gluon line which is longitudinal.

Therefore, if one considers ghost contributions to the totally-transverse n-point function, he will soon realize that, in any order in perturbation theory, the diagrams containing higher order ghost vertices are $O(\frac{1}{M^2})$, and will not survive the $M \longrightarrow \infty$ limit.

One can therefore conclude that *the ghost contribution to the totally-transverse n-point function that comes from the S^{nl}_{gh} part of the action is, in the limit $M \longrightarrow \infty$, exactly one-half of the standard Landau-gauge ghost contribution, and this is true in any order in perturbation theory.*

5 One-loop correction to the vacuum polarization

To understand the mechanism that makes the correlation functions in the maximal Landau gauge coincident with the ones relative to the standard Landau-gauge, let us consider first the simplest case: the first-order contribution to the two-point function. The (totally-transverse) one-loop contribution to the vacuum polarization has the diagrammatic representation shown in Fig. 1, where the "L" over a line indicates the longitudinal part of the gluonic propagator and the dot on a vertex indicates its non-local origin.

We will label with $\Pi^T(p)^{ab}_{\mu\nu}(i)(i = 1, \ldots, 6)$ the contributions to $\Pi^T(p)^{ab}_{\mu\nu}$ coming from the diagrams of Fig. 1, and with $\Pi^{(n)T}_{gh}(p)^{ab}_{\mu\nu}$ the one-loop Landau-gauge ghost contribution to the n-point function.

We are interested in the totally transverse part of it since the longitudinal part does not survive the $M \longrightarrow \infty$ limit once the (full) propagator is put on the external legs; therefore all external lines in the Fig. 1 are to be thought of as transverse.

Since the three- and four-leg local vertices are the same as the ones one finds in the Landau-gauge and the gluon propagator differs from the usual one only by a longitudinal part that vanishes as $M \longrightarrow \infty$, the contributions coming from diagrams containing only local vertices (e.g. $\Pi^T(1)$ and $\Pi^T(2)$) are the same that one finds in the Landau-gauge. As already explained the "tr ln" contribution is just one-half of the (totally-transverse) Landau-gauge ghost contribution, $\Pi^T(p)^{ab}_{\mu\nu}(\text{“}tr\ ln\text{”}) = \frac{1}{2}\,\Pi^{(2)}_{gh}(p)^{ab}_{\mu\nu}$, so that our claim is equivalent to say that all contributions coming from diagrams containing non-local vertices add up to give, in the limit $M \longrightarrow \infty$, one-half of the standard Landau-gauge ghost contribution.

Let us consider the contributions to the two-point function given by the diagrams containing non-local vertices. Let us start with the contribution of the fourth diagram, $\Pi^T(p)^{ab}_{\mu\nu}(4)$. The fact that at least two of the three-legs of the non-local vertex have to be longitudinal forces the internal propagators to be longitudinal; but the longitudinal part of the gluon propagator is proportional to $\frac{1}{M^2}$ (Eq.(31)), while the non-local vertices are proportional to M^2, since they arise from the expansion of $S_M[A] = \frac{1}{2}M^2(|A|^2A - |B[A]|^2)$. Therefore $\Pi^T(p)^{ab}_{\mu\nu}(4)$ is proportional to $\frac{1}{M^2}$ and goes to zero as M goes to infinity.

The third diagram gives[4]

$$\Pi^T(p)^{ab}_{\mu\nu}(3) = \frac{1}{2}\int \frac{d^Dk}{(2\pi)^D}\, V_3^{nl}(p,k,m)^{acd}_{\mu\alpha\beta}\, D^L(-k)^{cs}_{\alpha\tau}\, D^L(-m)^{dr}_{\beta\sigma}\, V_3^{nl}(-p,-k,-m)^{bsr}_{\nu\tau\sigma}$$
$$(p+k+m=0). \quad (34)$$

Substituting for the longitudinal part of the propagator D^L its expression (Eq.(31)) and making use of

$$k_\mu V_3^{nl}(k,l,m)^{abc}_{\mu\nu\rho} = iM^2 f^{abc}[P^T(l) - P^T(m)]_{\nu\rho} \qquad (k+l+m=0), \quad (35)$$

one finds

$$\Pi^T(p)^{ab}_{\mu\nu}(3) = -\frac{1}{2} tr(T^aT^b) P^T(p)_{\mu\beta} R(p)_{\beta\tau} P^T(p)_{\tau\nu} \quad (36)$$

where

$$R(p)_{\beta\tau} \equiv \int \frac{d^Dk}{(2\pi)^D}\, \frac{k_\beta k_\tau}{k^2(k+p)^2}. \quad (37)$$

The fifth diagram gives

$$\Pi(p)^{ab}_{\mu\nu}(5) = \frac{1}{2}\int \frac{d^Dk}{(2\pi)^D}\, V_4^{nl}(p,k,-k,-p)^{abcd}_{\mu\nu\rho\sigma} D^L(k)^{bc}_{\nu\rho} \Pi^T(p)^{ab}_{\mu\nu}(5). \quad (38)$$

Making use of Eq.(22)

$$k_\nu V_4^{nl}(k,-k,-p,p)^{bcda}_{\nu\rho\sigma\mu}\delta^{bc} = -tr(T^aT^d)[V_3^{nl}(-k,-p,p+k) - V_3^{nl}(-k,k-p,p)]_{\rho\sigma\mu} \quad (39)$$

and one finds

$$P^L(k)_{\nu\rho} V_4^{nl}(k,-k,-p,p)^{bbda}_{\nu\rho\sigma\mu} = -2\frac{1}{k^2} tr(T^aT^d)\, M^2[P^L(p) - P^L(p+k)]_{\mu\sigma}. \quad (40)$$

Therefore the totally transverse part of $\Pi(p)^{ab}_{\mu\nu}(5)$ is [changing the integration variable k into $-(k+p)$]

$$\Pi^T(p)^{ab}_{\mu\nu}(5) = tr(T^aT^b) P^T(p)_{\mu\alpha} R(p)_{\alpha\beta} P^T(p)_{\beta\nu}. \quad (41)$$

[4]In the following we will always consider all divergent integrals regularized by dimensional regularization. Actually we will never need to evaluate them and all cancellation that we will find are true in any other regularization scheme that does not break gauge invariance, so that the recursion relation (10) holds.

Adding all the contributions coming from diagrams containing non-local vertices together

$$\lim_{M \longrightarrow \infty} \left[\Pi^T(p)(3) + \Pi^T(p)(5)\right]^{ab}_{\mu\nu} = \frac{1}{2} tr(T^a T^b) P^T(p)_{\mu\alpha} R(p)_{\alpha\beta} P^T(p)_{\beta\nu}. \tag{42}$$

Now we have to compare (42) with $\Pi^{(2)}_{gh}(p)^{ab}_{\mu\nu}$. From

$$\begin{aligned} \det(-\partial \cdot D[A]) &= e^{tr \ln(-\partial \cdot D[A])} = \\ &= \int \mathcal{D}\bar{\eta}\, \mathcal{D}\eta \ \exp\left\{ - \int dx\, dy \bar{\eta}^a(x)(-\partial \cdot D[A])^{ab} \delta^D(x-y) \eta^b(y) \right\} \end{aligned} \tag{43}$$

one derives the usual Landau-gauge Feynman rules for the ghosts

$$\begin{array}{ll} \textit{ghost propagator} & D^{gh}(k)^{ab} = \frac{1}{k^2}\, \delta^{ab} \\ \textit{ghost-ghost-gluon vertex} & V^{gh}(p,q,r,)^{abc}_{\rho} = -i f^{abc} q_\rho \equiv i T^c_{ab} q_\rho \end{array} \tag{44}$$

Hence the usual ghost contribution to the transverse part of the two-point function is

$$\begin{aligned} \Pi^{(2)}_{gh}(p)^{ab}_{\mu\nu} &\equiv -2\frac{1}{2} tr(T^a T^b) P^T(p)_{\mu\mu_1} \int \frac{d^D k}{(2\pi)^D} \frac{(i)^2}{k^2 q^2} q_{\mu_1} k_{n_1} P^T(p)_{n_1\nu} \\ &= tr(T^a T^b) P^T(p)_{\mu\mu_1} R(p)_{\mu_1 n_1} P^T(p)_{n_1\nu} \qquad (p+q=k). \end{aligned} \tag{45}$$

so that, indeed

$$\lim_{M \longrightarrow \infty} \Pi^T(p)_{nl} = \frac{1}{2} \Pi^{(2)}_{gh}(p). \tag{46}$$

We can thus conclude that *the first-loop correction to the two-point function calculated in the maximal Landau gauge coincides with the usual results one finds in the Landau gauge.*

6 One-loop correction to the totally transverse part of the n-point function

Let us now consider how it is possible to generalize the previous result to Green's functions of every order n. A generic one-loop contribution to the n-point function has the form of a loop with a certain number V of vertices

and n external lines. If we let ν_k be the number of vertices with $(k+2)$ legs appearing in the loop, then $V = \sum_{k=1}^{n} \nu_k$ and $n = \sum_{k=1}^{n} k\nu_k$.

Once again one has to consider only the totally-transverse part of the n-point function, since parts containing one or more longitudinal components can be written as linear combinations of totally-transverse parts of lower order in virtue of a recursive use of Eq.(10). As we are going to see, these transverse parts have a finite $M \longrightarrow \infty$ limit, hence the parts containing longitudinal components will not survive the limit when the propagator is put on the external legs.

Since we are considering the totally transverse part of the n-point function, all external legs are transverse; therefore both propagators originating from a non-local vertex have to be longitudinal. Since the longitudinal part of the gluon propagator is proportional to $\frac{1}{M^2}$ while all non-local vertices are proportional to M^2, the only contributions that do not vanish in the $M \longrightarrow \infty$ limit are those in which all the vertices in the loop are either local or non-local.

In the case in which all vertices are local, all internal propagators can be replaced by their transverse part (the only one that survives in the $M \longrightarrow \infty$ limit), so that they all add up to give the usual (Landau-gauge) gluonic contribution to the (totally transverse part of the) n-point function.

To show that the totally transverse part of the n-point function is the same as the one that it is found in the Landau-gauge it is once again sufficient to prove that all the contributions coming from loops containing only non-local vertices add up to give one-half of the totally transverse part of the of the Landau-gauge ghost contribution, the other one-half necessary to reconstruct the standard ghost contribution coming from the "$tr\ln$" part of the action.

One can construct all possible diagrams with n external legs by considering all possible partitions [5] $\boldsymbol{\nu} = [\nu_1 \ldots \nu_n]$ of n. Clearly diagrams associated to different partitions will be topologically inequivalent.

To a given partition $\boldsymbol{\nu} = [\nu_1 \ldots \nu_n]$ of n one can associate a certain number $N[\boldsymbol{\nu}]$ of topologically independent diagrams obtained by placing the V vertices on the loop in different ways. $N[\boldsymbol{\nu}]$ is found by considering that, of the possible $V!$ permutations of the V vertices, the ones involving only

[5] The standard way of writing a partition of n as a set of non-increasing integers $l_1 \geq l_2 \geq \ldots \geq l_n$ such that $\sum_{r=1}^{n} l_r = n$ is obtained by defining $l_k \equiv \sum_{r=k}^{n} \nu_r$.

vertices of the same kind do not give a distinct diagram, so that

$$N[\boldsymbol{\nu}] = \frac{V!}{\nu_1! \dots \nu_n!} \tag{47}$$

Finally each of these $N[\boldsymbol{\nu}]$ diagrams can be attached to the n external points in

$$C[\boldsymbol{\nu}] = \frac{n!}{\prod_{r=1}^{n} (r!)^{\nu_r}} \tag{48}$$

different ways. Of course we are overcounting both the number of topologically inequivalent diagrams and the number of different ways one can attach them to the external points: diagrams obtained by a rigid rotation or by reflection of a given diagram are not inequivalent. To compensate for this overcounting each diagram should carry a symmetry factor $1/(2V)$, where V stands for the rotations and 2 for the reflection.

We will first show that each of these $N[\boldsymbol{\nu}] \cdot C[\boldsymbol{\nu}]$ diagrams gives a contribution that is proportional to the contribution

$$\begin{aligned} \tilde{\Gamma}_{gh}^{T\dots T}(p_1, \dots, p_n)^{a_1 \dots a_n}_{\mu_1 \dots \mu_n} \;&\equiv\; \frac{1}{n}\{\Gamma_{gh}^{T\dots T}(p_1, \dots, p_n)^{a_1 \dots a_n}_{\mu_1 \dots \mu_n} + \\ &+ (n! \text{ permutations of the n triplets } (a_1 p_1 \mu_1) \dots (a_n p_n \mu_n)\} \end{aligned} \tag{49}$$

coming from the ghosts, so that the proof of our main claim is reduced to a combinatorial problem.

Let us consider one of the ν_r non-local vertices with $r+2$ legs that appear in the loop. Let us label with $p_i (i = 1, \dots, r)$ the external (transverse) incoming momenta and with k_n and $-k_r$ the (longitudinal) internal ones

$$\begin{aligned} &V_{r+2}^{nl}(k_n, p_1, \dots, p_r, -k_r)^{b_n a_1 \dots a_r b_r}_{\beta_n \mu_1 \dots \mu_r \alpha_r} = \\ &= P^T(p_1)_{\mu_1 \nu_1} \dots P^T(p_r)_{\mu_r \nu_r} P^L(k_n)_{\beta_n \rho} V_{r+2}^{nl}(k_n, p_1, \dots, p_r, -k_r)^{b_n a_1 \dots a_r b_r}_{\rho \nu_1 \dots \nu_r \alpha_r}. \end{aligned} \tag{50}$$

By applying the recursion relation (22) and making use of the fact that a non-local vertex must have at least two longitudinal legs (so that, for example, $V_{r+1}^{nl}(p_1, \dots, p_r, k_n - k_r)^{a_1 \dots a_r b_r}_{\mu_1 \dots \mu_r \alpha_r} = 0$) one finds

$$\begin{aligned} &V_{r+2}^{nl}(k_n, p_1, \dots, p_r, -k_r)^{b_n a_1 \dots a_r b_r}_{\beta_n \mu_1 \dots \mu_r \alpha_r} = \\ &= (-i)\frac{(k_n)_{\beta_n}}{k_n^2} \sum_{s=1}^{r} P^T(p_1)_{\mu_1 \nu_1} \dots P^T(p_r)_{\mu_r \nu_r} T^{a_s}_{b_n v} V_{r+1}^{nl}(p_1, \dots, p_s + k_n, \dots, p_r, -k_r)^{a_1 \dots v \dots a_r b_r}_{\nu_1 \dots \nu_s \dots \nu_r \alpha_r} \end{aligned} \tag{51}$$

Repeating this procedure of pulling out one longitudinal projector and of using the relationship (22) r times one finds

$$
\begin{aligned}
& V^{nl}_{r+2}(k_n, p_1, \ldots, p_r, -k_r)^{b_n a_1 \ldots a_r b_r}_{\beta_n \mu_1 \ldots \mu_r \alpha_r} = \\
= \; & -(-i)^r M^2 \frac{(k_n)_{\beta_n}}{k_n^2} \frac{(k_r)_{\alpha_r}}{k_r^2} (T^{a_1} T^{a_2} \ldots T^{a_r})_{b_n b_r} \\
& \left[P^T(p_1) \cdot (k_n + p_1)\right]_{\mu_1} \ldots \left[P^T(p_r) \cdot (k_n + p_1 + p_2 + \ldots + p_r)\right]_{\mu_r} \\
& \frac{1}{(k_n + p_1)^2} \frac{1}{(k_n + p_1 + p_2)^2} \cdots \frac{1}{(k_n + p_1 + p_2 + \ldots + p_{r-1})^2} + \\
+ \; & r! \text{ permutations of the } r \text{ triplets } \{(a_1 \mu_1 p_1) \ldots (a_r \mu_r p_r)\}. \qquad (52)
\end{aligned}
$$

The link of the non-local vertex V^{nl}_{r+2} with the ghosts becomes apparent by considering the Feynman rules of Eq.(44) for the ghost-ghost-gluon vertex and the ghost propagator. Comparing the analytic expression of the diagram shown in Fig. 2 with Eq.(52) one obtains the useful identity that is expressed pictorially in Fig. 3, and which can be used to reduce each of the $N[\boldsymbol{\nu}]C[\boldsymbol{\nu}]$ diagrams that contribute to the totally transverse part of the n-point function to the form of a ghost contribution. Let us consider the contribution to the n-point function coming from a given diagram (i.e. given $\boldsymbol{\nu}$) attached to the external points in a definite way (i.e. fixed i, j with $(i = 1, \ldots, N[\boldsymbol{\nu}]; j = 1, \ldots, C[\boldsymbol{\nu}])$). By use of the identity of Fig. 3 one finds

$$
\Gamma^{T \ldots T}_{ij}[\boldsymbol{\nu}](p_1, \ldots, p_n)^{a_1 \ldots a_n}_{\mu_1 \ldots \mu_n} = (-)^{V+1} \left\{ \Gamma^{T \ldots T}_{gh}(p_1, \ldots, p_n)^{a_1 \ldots a_n}_{\mu_1 \ldots \mu_n} + \prod_{r=1}^{n} (r!)^{\nu_r} \text{permutations} \right\}. \qquad (53)
$$

This is the contribution to the n-point function coming from a given diagram attached to the external points in a definite way. Summing over the $C[\boldsymbol{\nu}]$ different ways of attaching a given diagram to the external points gives

$$
\begin{aligned}
\Gamma_j[\boldsymbol{\nu}](p_1, \ldots, p_n)^{a_1 \ldots a_n}_{\mu_1 \ldots \mu_n} \; & \equiv \; \sum_{i=1}^{C[\boldsymbol{\nu}]} \Gamma_{ij}[\boldsymbol{\nu}](p_1, \ldots, p_n)^{a_1 \ldots a_n}_{\mu_1 \ldots \mu_n} = \qquad (54) \\
& = \; (-1)^{V-1} [\Gamma_{gh}(p_1, \ldots, p_n)^{a_1 \ldots a_n}_{\mu_1 \ldots \mu_n} + \\
& + \; n! \text{ permutations of the triplets } \{(a_1 \mu_1 p_1), \ldots, (a_r \mu_r p_r)\}] \\
& = \; n(-1)^{V-1} \tilde{\Gamma}^{T \ldots T}_{gh}(p_1, \ldots, p_n)^{a_1 \ldots a_n}_{\mu_1 \ldots \mu_n}.
\end{aligned}
$$

The sum of all the contributions pertaining to the $N[\boldsymbol{\nu}]$ topologically

inequivalent diagrams (with the appropriate symmetry factor) gives simply

$$\Gamma[\boldsymbol{\nu}](p_1,\ldots,p_n)^{a_1\ldots a_n}_{\mu_1\ldots\mu_n} \equiv \sum_{j=1}^{j=N[\boldsymbol{\nu}]} \Gamma_j[\boldsymbol{\nu}](p_1,\ldots,p_n)^{a_1\ldots a_n}_{\mu_1\ldots\mu_n} = \tag{55}$$
$$= \frac{1}{2V}\frac{V!}{\prod_{r=1}^{n}(\nu_r!)}(-1)^{V-1}n\tilde{\Gamma}^{T\ldots T}_{gh}(p_1,\ldots,p_n)^{a_1\ldots a_n}_{\mu_1\ldots\mu_n}.$$

Finally one has to sum [6] over all possible partitions of n :

$$\sum_{\boldsymbol{\nu}}\Gamma[\boldsymbol{\nu}](p_1,\ldots,p_n)^{a_1\ldots a_n}_{\mu_1\ldots\mu_n} \equiv \tag{57}$$
$$\equiv \sum_{\substack{\nu_1\ldots\nu_n \\ \sum_{r=1}^{n} r\nu_r=n}} \frac{1}{2V}\frac{V!}{\prod_{r=1}^{n}(\nu_r!)}(-1)^{V-1}n\left\{\tilde{\Gamma}^{T\ldots T}_{gh}(p_1,\ldots,p_n)^{a_1\ldots a_n}_{\mu_1\ldots\mu_n}\right\} =$$
$$= \frac{1}{2}\tilde{\Gamma}^{T\ldots T}_{gh}(p_1,\ldots,p_n)^{a_1\ldots a_n}_{\mu_1\ldots\mu_n} \tag{58}$$

which proves our claim

$$\lim_{M\longrightarrow\infty} \Pi^{(n)}_{nl}(p_1,\ldots,p_n) = \Pi^{(n)}_{gh}(p_1,\ldots,p_n). \tag{59}$$

7 Two-loop correction to the vacuum polarization

One would like to see if what was shown to hold at one-loop level can be extended to higher orders in perturbation theory: is it still true that the contributions coming from the "new" diagrams, i.e. the one containing non - local vertices, add up to give, once the limit $M \longrightarrow \infty$ is taken, one-half of the usual (Landau-gauge) ghost contribution ? We will consider the two-loop correction to the two-point function and show that this is still the case.

[6] Using the identity

$$\sum_{\substack{\nu_1\ldots\nu_n \\ \sum_{r=1}^{n} r\nu_r=n}} \frac{1}{2V}\frac{V!}{\prod_{r=1}^{n}(\nu_r!)}(-1)^{V-1} = \frac{1}{2n} \tag{56}$$

which is proved in the appendix.

By using the Feynman rules derived in Sec. (4) one can write down the diagrams that contribute to the (totally transverse part of the) two-loop vacuum polarization, as shown in Figure 4. In that picture explicit dependence on Lorentz and color indices has been suppressed for simplicity while the appropriate symmetry factors pertaining to each diagram have been explicitly shown. The vertices with three or four legs in each diagram can be either local or non-local. Making use once again of the fact that each non-local vertex has at least two longitudinal legs (and is proportional to M^2), and of the form of the propagator (31), one can extract from the list of Figure 4 those diagrams which contain at least one non-local vertex and which survive in the $M \longrightarrow \infty$ limit [whose sum we will indicate with $\lim_{M \longrightarrow \infty} \Lambda_{nl}(p)$]. These diagrams are shown in Figure 6; the additional numbers in front represent a symmetry factor that arises when we split the propagator into its transverse (T) and longitudinal (L) part. For example the factor two in the first diagram takes into account that one has also to consider another diagram where the L and T lines that join the same two vertices are exchanged. Which lines are longitudinal and which are transverse is explicitly indicated in the first three diagrams.

By using the fundamental identity shown in Fig. 3, one can reduce every non-local vertex to a gluon-ghost-ghost vertex and every non-local diagram to a sum of ghost diagrams i.e. of diagrams that contain ghost propagators. The result of this operation on the non-local diagrams that contribute to the two-loop vacuum polarization is shown in Fig. 6, where the value of every non-local diagram is given in terms of ghost diagrams, as defined in Fig. 5.

Adding up all these diagrams one finally finds

$$\lim_{M \longrightarrow \infty} \Lambda_{nl}(p) = \frac{1}{4} \{4\Lambda_1 + 2\Lambda_2 + 2\Lambda_3 + 4\Lambda_4 + \Lambda_5\} \tag{60}$$

This has to be compared with the usual Landau-gauge ghost contribution to the two-loop vacuum polarization:

$$\Lambda_{gh}(p) = \frac{1}{2} \{4\Lambda_1 + 2\Lambda_2 + 2\Lambda_3 + 4\Lambda_4 + \Lambda_5\} \tag{61}$$

so that, indeed, one finds once again

$$\lim_{M \longrightarrow \infty} \Lambda_{nl}(p) = \frac{1}{2} \Lambda_{gh}(p) \tag{62}$$

and the same mechanism that was shown to work at one-loop level appears to be still valid at two-loop level: the extra diagrams coming from the presence of the non-local vertices add up to give a contribution that is one-half of the usual Landau gauge ghost contribution; this together with the fact that the ghost contribution coming for the "tr log" part of the action in the new theory gives exactly the same result as in the usual Landau-gauge, but is reduced by a factor one-half by the presence of a different exponent in the Faddeev-Popov determinant, makes the two perturbative expansions coincident order by order in perturbation theory.

8 Conclusion

It has been verified by explicit one- and two-loop perturbative expansion that the two-point function in the maximal Landau gauge coincides with the one in the ordinary Landau gauge. This result has been extended, up to one-loop order, to all correlation functions. As a byproduct of the calculation it has been seen that the gauge symmetry-breaking term $S_{gf} = \frac{1}{2}\beta M^2|A|^2$ is not affected by radiative corrections.

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A Combinatorics

We claim that the following identity holds

$$W_n \equiv \sum_{\substack{\nu_1 \dots \nu_n \\ \sum_{r=1}^n r\nu_r = n}} \frac{1}{V} \frac{V!}{\prod_{r=1}^n (\nu_r!)} (-1)^{V-1} = \frac{1}{n} \tag{63}$$

where $V = \sum_{r=1}^{n} \nu_r$. To prove it let us consider the generating function for the W_n

$$F(x) \equiv \sum_{n=1}^{\infty} x^n W_n \tag{64}$$

so that

$$W_n = \frac{1}{n!} \frac{d^n}{dx^n} F(x) \bigg|_{x=0} \tag{65}$$

Summing over all $[\nu_1 \ldots \nu_n]$ with the restriction $\sum_{r=1}^{n} r\nu_r = n$ and over all $n \geq 1$ is equivalent to summing over all possible ν's without any restriction (only the case in which all ν's are simultaneously zero is excluded), which is in turn equivalent to summing over all possible ν's with the restriction $\sum_{r=1}^{\infty} \nu_r = V$ and then over all $V \geq 1$; therefore

$$\begin{aligned}
F(x) &= \sum_{\nu_1 \ldots \nu_n \ldots} (-1)^{\nu_1+\nu_2+\ldots+\nu_n+\ldots-1} (\nu_1 + \nu_2 + \ldots + \nu_n + \ldots - 1)! \prod_{r=1}^{\infty} \left(\frac{x^{r\nu_r}}{\nu_r!} \right) \\
&= \sum_{V=1}^{\infty} (-1)^{V-1} \sum_{\substack{\nu_1 \ldots \nu_n \ldots \\ \sum_{r=1}^{\infty} \nu_r = V}} \frac{(V-1)!}{\prod_{r=1}^{\infty} (\nu_r!)} \prod_{r=1}^{\infty} x^{r\nu_r} \\
&= (-) \sum_{V=1}^{\infty} \frac{(-1)^V}{V} \sum_{\substack{\nu_1 \ldots \nu_n \ldots \\ \sum_{r=1}^{\infty} \nu_r = V}} \binom{V}{\nu_1 \ldots \nu_n \ldots} x^{\nu_1} x^{2\nu_2} \ldots x^{n\nu_n} \ldots \\
&= (-) \sum_{V=1}^{\infty} \frac{(-1)^V}{V} \left(\sum_{k=1}^{\infty} x^k \right)^V \\
&= -\log \left(1 + \sum_{k=1}^{\infty} x^k \right) \\
&= \log(1-x) \\
&= \sum_{n=1}^{\infty} \frac{x^n}{n}
\end{aligned} \tag{66}$$

By equating coefficients with the same power of x it follows

$$W_n = \frac{1}{n}. \tag{67}$$

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0 Figure Captions

Fig. 1: One-loop contributions to the vacuum polarization.

Fig. 2: A useful diagram and its analytic expression; it is understood that there are no propagators on the external ghost lines.

Fig. 3: Relation between non-local vertices and ghosts.

Fig. 4: Diagrams that contribute to the two-loop vacuum polarization and do not contain ghost propagators.

Fig. 5: Ghost contributions to the two-loop vacuum polarization.

Fig. 6: Non-local contributions to the two-loop vacuum polarization and their value, in term of ghost diagrams.

$\Pi^T(p)^{ab}_{\mu\nu} \equiv$

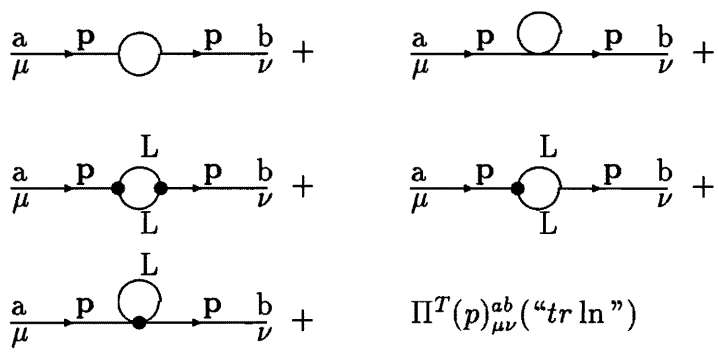
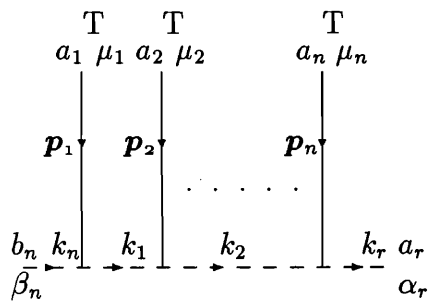


Figure 1:

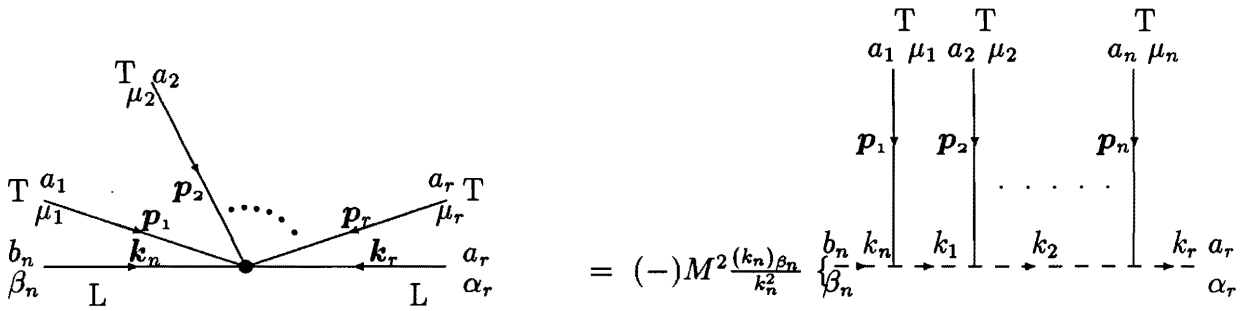


$$= (-i)^r \left(T^{a_1} T^{a_2} \ldots T^{a_r}\right)_{b_n b_r}$$

$$\left[P^T(p_1) \cdot (k_n + p_1)\right]_{\mu_1} \ldots \left[P^T(p_r) \cdot (k_n + p_1 + p_2 + \ldots + p_r)\right]_{\mu_r}$$

$$(k_n + p_1)^{-2} \; (k_n + p_1 + p_2)^{-2} \quad \ldots \; (k_n + p_1 + p_2 + \ldots + p_{r-1})^{-2}$$

Figure 2:



$+(r!)$ permutations of the triplets $(a_1, \mu_1, p_1) \ldots (a_r, \mu_r, p_r)\} \frac{(k_r)_{\alpha_r}}{k_r^2}$

Figure 3:

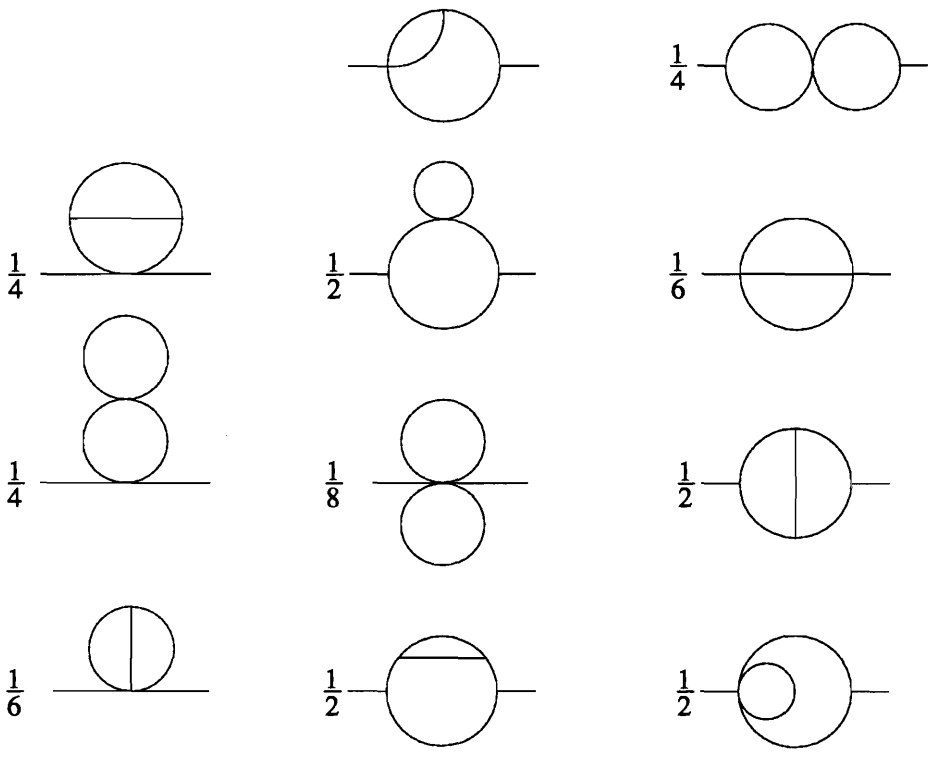


Figure 4:

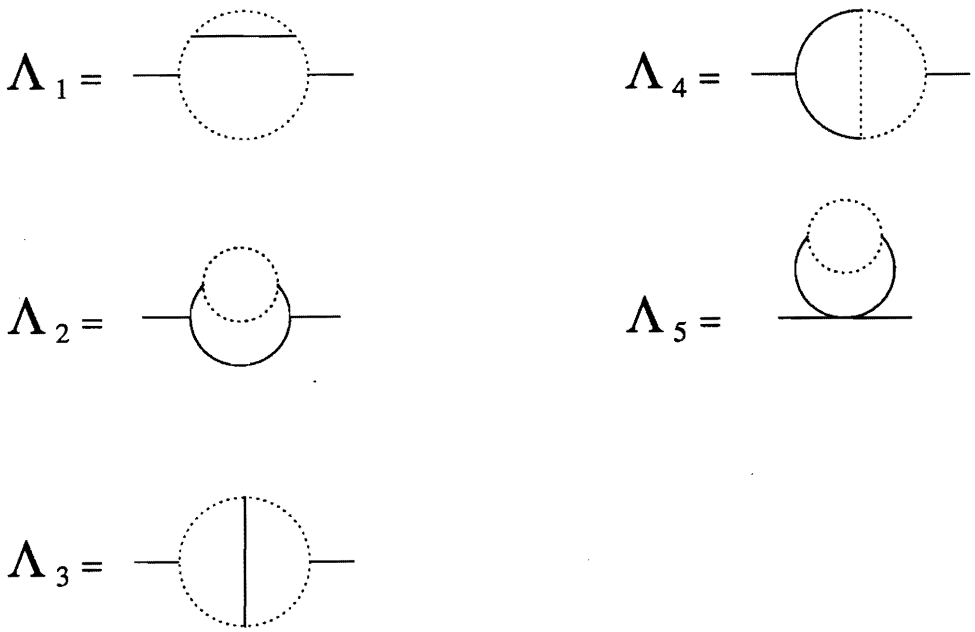


Figure 5:

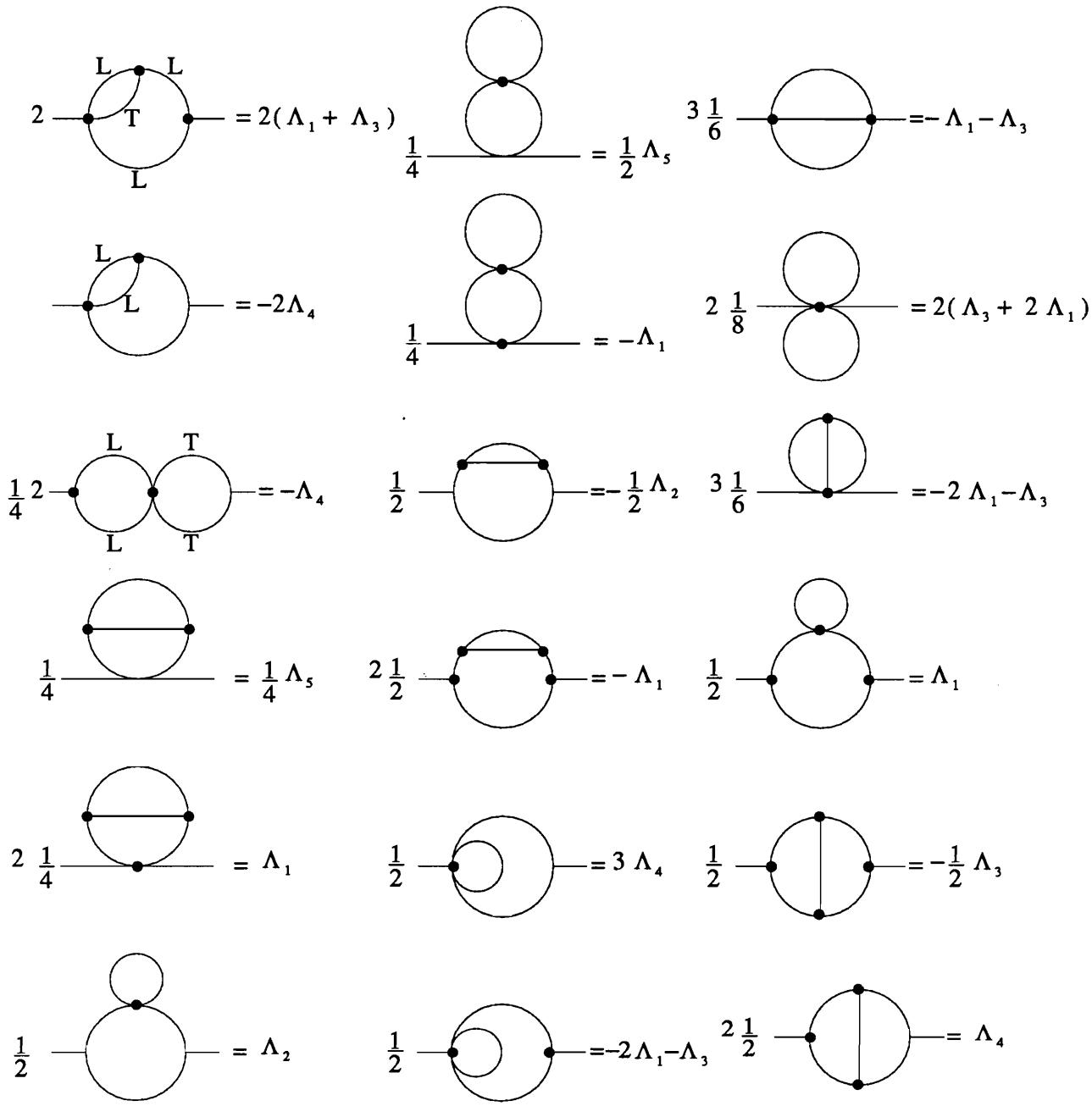

2 L L T L = 2(Λ1 + Λ3)
1/4 = 1/2 Λ5
3 1/6 = − Λ1 − Λ3
L L = −2Λ4
1/4 = −Λ1
2 1/8 = 2(Λ3 + 2 Λ1)
1/4 2 L T L T = −Λ4
1/2 = − 1/2 Λ2
3 1/6 = −2 Λ1 − Λ3
1/4 = 1/4 Λ5
2 1/2 = − Λ1
1/2 = Λ1
2 1/4 = Λ1
1/2 = 3 Λ4
1/2 = − 1/2 Λ3
1/2 = Λ2
1/2 = −2Λ1 − Λ3
2 1/2 = Λ4

Figure 6: