NYU-TH-91-02-04

NYU-TH-91-02-04

N.Y.U. February 1990

# Higher Order Contributions to $\Delta r$

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**ABSTRACT**

A simple expression for $\Delta r$ is proposed, within the on-shell scheme, which incorporates leading higher order contributions as well as subleading logarithms of $O(\alpha^2)$. The results obtained for $\Delta r$, $\sin^2\theta_W$ and $m_W$ are found to be very close to those derived in the $\overline{MS}$ approach, over a large range of $m_t$ and $m_H$ values.

*In Memoriam* M. A. B. Bég

To be Published: Bég Memorial Volume

The radiative correction $\Delta r$ plays a significant role in current discussions of electroweak physics. For example, the relation[1]

$$\sin^2\theta_W = \frac{A^2}{m_W^2(1-\Delta r)}\,, \tag{1}$$

where

$$\sin^2\theta_W = 1 - \frac{m_W^2}{m_Z^2} \tag{2}$$

and $A = \left(\pi\alpha/\sqrt{2}G_\mu\right)^{1/2} = (37.2803 \pm 0.0003)\mathrm{GeV}$ , can be employed to compute $\sin^2\theta_W$ as a function of $m_t$, $m_H$ and $m_Z$. In turn, this parameter leads immediately to a prediction for $m_W$, can be compared with other determinations of $\sin^2\theta_W$ to constrain $m_t$ and can also be used, in conjunction with other corrections, to calculate asymmetries and other observables at the $Z^0$ peak. As many of these observables depend sensitively on $\sin^2\theta_W$, considerable effort has been devoted to its accurate computation.

The one loop contributions to $\Delta r$ were given several years ago[1,2] and since that time they have been confirmed by several groups. Some time later[3] it was shown that the insertion of the one-loop result in the denominator of Eq.(1), augmented by the inclusion of the two-loop irreducible contributions to the vacuum polarization function, incorporates not only the leading logarithms of $O((\alpha\ln(m_Z/m_f))^n)$ (here $m_f$ is a generic fermion mass) but also the subleading logarithms of $O(\alpha^2\ln(m_Z/m_f))$. The theoretical underpinning of this result is the theorem on cancellation of mass singularities for total decay rates.[4,5] It is worth noting that the leading logarithms derived in this method coincide with those obtained by the renormalization group analysis.[6]

Motivated by increasing indications that the top quark is heavy, the corrections of $O((\alpha m_t^2/m_W^2)^2)$ to $\Delta r$ have been recently studied.[7–10] The analysis of Refs.7–9 has been carried out within the on-shell scheme where the basic renormalized parameters are identified with the conventionally defined $\alpha$, $\sin^2\theta_W$ given by Eq.(2), and $m_Z$, while the investigations of Ref.10 employ the $\overline{MS}$ framework and establish relationships between the two schemes. On the basis of these studies, the authors of Refs.7–9 have proposed the following expression for $\Delta r$ :

$$\frac{1}{1-\Delta r} = \frac{1}{(1-\Delta\alpha)(1+\frac{c^2}{s^2}\Delta\bar{\rho})} + (\Delta r)_{rem}\,, \tag{3}$$

where henceforth $s^2 = 1 - c^2$ is an abbreviation for $\sin^2\theta_W$, $\Delta\alpha$ represents the contribution of leptons and the first five hadronic flavors to the vacuum polarization function $\Pi_{\gamma\gamma}(0) - Re\Pi_{\gamma\gamma}(m_Z^2)$, $\Delta\bar{\rho}$ is the leading correction to the $\rho$ parameter for

large $m_t$ and $(\Delta r)_{rem}$ stands for additional, non-dominant contributions to $\Delta r$ (the subscript *rem* means "remainder"). Current analyses lead to $\Delta\alpha = 0.0602 \pm 0.0009$[11] and

$$\Delta\bar{\rho} = x_t[1 - \frac{(2\pi^2 - 19)}{3} x_t] , \tag{4}$$

$$x_t = \frac{3G_\mu m_t^2}{8\pi^2\sqrt{2}} , \tag{5}$$

where $x_t$ is the well-known Veltman correction[12] and the second term in Eq.(4) is the leading two-loop irreducible contribution to the $\rho$ parameter.[13] The first term in the r.h.s. of Eq.(3) resums leading logarithms to all orders and contains also the leading corrections of $O((\alpha m_t^2/m_W^2)^2, \alpha^2(m_t^2/m_W^2)\ln(m_Z/m_f))$. The remainder is given by

$$(\Delta r)_{rem} = \Delta r^{(1)} + \frac{c^2}{s^2} x_t - \Delta\alpha , \tag{6}$$

where $\Delta r^{(1)}$ stands for the one-loop contributions to $\Delta r$,[1,2] augmented by QCD and electromagnetic corrections of $O(\alpha\alpha_s)$ and $O(\alpha^2)$ associated with self-energy contributions involving leptons and light quark isodoublets (these additional corrections are quite small but they are usually included in current analyses). The last two terms in Eq.(6) simply subtract from $\Delta r^{(1)}$ the $O(\alpha)$ corrections that have been resummed in the first term of Eq.(3). It should be pointed out that there are also QCD corrections to $\Delta\bar{\rho}$, $\Delta\alpha$ and $(\Delta r)_{rem}$ associated with the $t$-$b$ isodoublet. The more recent investigations are reported in Refs.14–16. Their discussion lies, however, beyond the scope of this paper.

Regarding the treatment of $(\Delta r)_{rem}$, there is a certain ambiguity in the literature. Whereas certain contributions to $(\Delta r)_{rem}$ are written using $\alpha/s^2$ as the coupling constant, in others $\alpha/s^2$ is replaced by its tree level equivalent $\sqrt{2}G_\mu m_W^2/\pi$. However, as implied by the analysis of Ref.3 and discussed at the end of this paper, neither of these two choices incorporate the subleading contributions of $O(\alpha^2\ln(m_Z/m_f))$. In this paper we propose a simple modification of Eq.(3) that automatically includes such terms, to wit

$$1 - \Delta r = (1 - \Delta\alpha)(1 + \frac{c^2}{s^2}\Delta\bar{\rho}) - (\Delta r)_{rem} . \tag{7}$$

The analysis of Ref.3 implies that if $(\Delta r)_{rem}$ is evaluated via Eq.(6) employing $\alpha$ and $\alpha/s^2$ as expansion parameters, as in the original papers of Refs.1,2, then Eq.(7) incorporates the subleading logarithms of $O(\alpha^2\ln(m_Z/m_f))$. At this point, we introduce a modification that preserves this desirable property but at the same time improves the treatment of the subleading terms of $O((\alpha/2\pi s^2)^2 m_t^2/m_Z^2)$. Calling $A_{ZZ}(m_Z^2)$ and $A_{WW}(m_W^2)$ the Z and W self-energies, we separate in $\Delta r^{(1)}$ (Cf.

Eq.(6) and Refs.1,2) the gauge invariant contribution

$$-\left(\frac{c^2}{s^2}\right) X \equiv \left(\frac{c^2}{s^2}\right) Re \left[\frac{A_{ZZ}(m_Z^2)}{m_Z^2} - \frac{A_{WW}(m_W^2)}{m_W^2}\right]_{\text{fin.}} ,$$

which represents the finite part of the counterterm $\delta s^2/s^2$ ( fin. means that we subtract the "divergent parts" proportional to $\delta = (n-4)^{-1} + C + \ln(m_Z/\mu)$ where $\mu$ is the 't Hooft mass scale). Next we combine this quantity with $(c^2/s^2)x_t$ (Cf. Eq.(6)) which cancels the leading $m_t$ contribution in $-(c^2/s^2)X$, and evaluate $(c^2/s^2)(x_t - X)$ employing the effective coupling constant $\sqrt{2}G_\mu m_W^2(1-\Delta\alpha)/\pi$, i.e. in the formal expansion in terms of $\alpha/s^2$ we replace $\alpha/s^2 \to \sqrt{2}G_\mu m_W^2(1-\Delta\alpha)/\pi$. After substracting the terms proportional to $\delta$, the remaining contributions in Eq.(6) are evaluated using $\alpha$ and $\alpha/s^2$ as expansion parameters, as in the original papers of Refs.1,2. The justification for this special treatment of the terms involving $(c^2/s^2)x_t$ and $(c^2/s^2)X$ can be seen on the basis of the arguments of Ref.7. Indeed, in this paper it was pointed out that

$$G_\mu/\sqrt{2} = [e_0^2/8m_W^2 s_0^2]/[1 + (e_0^2/s_0^2)f] ,$$

where $e_0^2$ and $s_0^2$ are the bare parameters and $(e_0^2/s_0^2)f$ represents non-leading $O(\alpha)$ corrections. Disregarding for the purposes of this argument the two-loop irreducible contributions to $\Delta\bar{\rho}$ (second term in Eq.(4)), one finds that the finite part of $s_0^2$ is given by $(s_0^2)_{\text{fin}} = s^2[1+(c^2/s^2)X]$ where $X$ is evaluated in terms of $\sqrt{2}G_\mu m_W^2/\pi$ and we have neglected a small $O(\alpha^2)$ correction involving $(c^2/s^2)X(e^2/s^2)f$. This relation between $s_0^2$ and $s^2$ is precisely the origin of the first term in Eq.(3) and Eq.(7), where only the dominant contributions involving $x_t$ have been retained and the factor $(1-\Delta\alpha)$ arises from the renormalization of $e_0^2$. The above argument indicates that the relevant quantity occurring in $\mu$ decay is $X$ rather than its dominant part $x_t$ and it is therefore natural to treat all the contributions to $X$ on the same footing. This is indeed the case when $(c^2/s^2)(x_t - X)$ in $(\Delta r)_{rem}$ is evaluated as explained after Eq.(7). We mention parenthetically that the difference between this evaluation and the corresponding one in terms of $\alpha/s^2$ is of $O((c^2/s^2)^2 X(x_t - X))$. Although it is a subleading $O(\alpha^2)$ contribution, the presence of the $(c^2/s^2)^2$ enhancement factor makes it relatively large, reaching values of $\approx 3 \cdot 10^{-3}$ for $m_t = 300$GeV.

In the $\overline{MS}$ framework it is convenient to introduce two basic radiative corrections, $\Delta\hat{r}_W$ and $\Delta\hat{r}$, by means of the relations[10,17,18]

$$\hat{s}^2 = \frac{A^2}{m_W^2(1-\Delta\hat{r}_W)} , \tag{8}$$

$$\hat{s}^2\hat{c}^2 = \frac{A^2}{m_Z^2(1-\Delta\hat{r})} , \tag{9}$$

where $\hat{s}^2 \equiv 1 - \hat{c}^2 \equiv \sin^2\hat{\theta}_W$ is the weak interaction parameter defined by modified minimal subtraction at mass scale $m_Z$. Thus, $\Delta\hat{r}_W$ and $\Delta\hat{r}$ represent the quantum corrections when $\hat{s}^2$ is expressed in terms of ($G_\mu$, $\alpha$, $m_W$) and ($G_\mu$, $\alpha$, $m_Z$), respectively. Defining $\hat{\rho} \equiv c^2/\hat{c}^2 = (1-\Delta\hat{r})/(1-\Delta\hat{r}_W)$ and $\Delta\hat{\rho} = 1 - \hat{\rho}^{-1}$ one finds[10] an alternative expression for $1 - \Delta r$, namely

$$1 - \Delta r = [1 + (c^2/s^2)\Delta\hat{\rho}][1 - \Delta\hat{r}_W]\,. \tag{10}$$

Detailed expressions and tables for $\Delta\hat{r}_W$ and $\Delta\hat{r}$, as well as the values of $\Delta r$ obtained from Eq.(10), are given in Ref.10. As the dominant terms in $\Delta\hat{\rho}$ and $\Delta\hat{r}_W$ coincide with those in $\Delta\bar{\rho}$ and $\Delta\alpha$, respectively, Eq.(10) has a structure reminiscent of the leading term in Eq.(7). Comparison of Eqs.(7) and (10) shows that they contain the same $O(\alpha)$ terms, leading logarithms, terms of $O\Big((\alpha\frac{m_t^2}{m_W^2})^2, \alpha^2(\frac{m_t^2}{m_W^2})\ln(\frac{m_Z}{m_f})\Big)$ and subleading logarithms of $O(\alpha^2\ln(m_Z/m_f))$. They differ, however, by subleading terms of $O((\alpha/2\pi s^2)(c^2/s^2)\Delta\bar{\rho})$. As the irreducible contributions of this order have not been evaluated, it seems impossible, at this stage, to rigorously prove which of the two expressions (Eq.(7) or Eq.(10)) is more accurate. We also recall that the parameters $s^2$ and $m_W$ can be obtained from the expression

$$s^2 = \frac{1}{2}\left\{1 - \left[1 - \frac{4A^2}{m_Z^2(1-\Delta r)}\right]^{1/2}\right\}. \tag{11}$$

Tables 1-4 compare the values of $\Delta r$ extracted iteratively by combining Eq.(11) with either Eq.(3), Eq.(7) or Eq.(10), as a function of $m_t$, for several $m_H$ entries and $m_Z = 91.177$GeV. Here it is understood that $(\Delta r)_{rem}$ has been evaluated in the manner explained after Eq.(7). Tables 5-8 display the corresponding values of $s^2$ and $m_W$. For brevity, quantities evaluated on the basis of Eq.(10) ($\overline{MS}$ approach) are denoted in the Tables with a subscript I, while those based on the on-shell expressions of Eq.(7) and Eq.(3) are identified with the subscripts II and III, respectively. In order to facilitate the discussion, in tables 1-4 we include the value of $(\Delta r)_{rem}$ (evaluated with $s^2$ extracted from Eqs.(7) and (11); corresponding values of $(\Delta r)_{rem}$ obtained from Eqs.(3) and (11) differ only slightly).

It is apparent that $(\Delta r)_I$ and $(\Delta r)_{II}$ are very close in the full range 80GeV $\leq m_t \leq$ 300GeV, 50GeV $\leq m_H \leq$ 1TeV, their difference reaching a maximum of only $3.8 \cdot 10^{-4}$ at $m_t = 300$GeV. On the other hand, both differ from $(\Delta r)_{III}$ at small $m_t$. For example, for $m_H = 1$TeV and $m_t \leq 150$GeV this difference is $> 10^{-3}$, which is larger than the current uncertainty of $9 \cdot 10^{-4}$ associated with the hadronic contributions to $\Delta\alpha$. The difference between $(\Delta r)_{II}$ and $(\Delta r)_{III}$ is easily understood. If we neglect terms of $O(((\Delta r)_{rem})^2)$, Eq.(3) is equivalent to a modified Eq.(7) in which $(\Delta r)_{rem}$ has been transformed into $(\Delta r)_{rem}(1-\Delta\alpha)^2(1+$

$(c^2/s^2)\Delta\bar{\rho})^2$. For small $m_t$, the contribution of $(\Delta r)_{rem}$ is thus reduced by a factor $\approx 1 - 2\Delta\alpha$, i.e. 12%. As an example, Table 3 shows that, for $m_t = 100$GeV and $m_H = 250$GeV, $(\Delta r)_{rem} \approx 0.01$ and we expect $(\Delta r)_{III}$ to be smaller than $(\Delta r)_{II}$ by $\approx 1.2 \cdot 10^{-3}$, which is roughly the observed difference. As $m_t$ increases $(\Delta r)_{II} - (\Delta r)_{III}$ first decreases for two reasons: *i)* the fortuitous circumstance that the logarithmic terms of $O(\alpha \ln(m_t/m_Z))$ lead to a decrease of $|(\Delta r)_{rem}|$ and *ii)* the fact that $(1 - \Delta\alpha)^2(1 + (c^2/s^2)\Delta\bar{\rho})^2$ becomes increasingly close to unity. At sufficiently large values of $m_t$, these observations are no longer true and $|(\Delta r)_{II} - (\Delta r)_{III}|$ increases again, reaching a value of $7.4 \cdot 10^{-4}$ for $m_t = 300$GeV and $m_H = 50$GeV. The differences at smaller $m_t$ have some effect in the prediction of $m_W$ and the asymmetries. For example, in Table 8 we see that for $m_H = 1$TeV and $m_t = 90$GeV, $(m_W)_{III} - (m_W)_{II} = 38$MeV, while $(m_W)_{II}$ and $(m_W)_I$ coincide. Similarly, for the same entries $s^2_{III} - s^2_{II} = -7.4 \cdot 10^{-4}$. This leads to a difference $(A_{LR})_{III} - (A_{LR})_{II} \approx -8(s^2_{III} - s^2_{II}) = 5.9 \cdot 10^{-3}$ in the predicted value of the on-resonance left-right asymmetry, which is considerably larger than the expected experimental error of $3 \cdot 10^{-3}$. We note that, unlike $\Delta\rho$ effects, these shifts in $s^2$ at small $m_t$ are not partially compensated by changes in the electroweak form factor $\kappa$, so that their effect on the asymmetries is particularly significant. As mentioned before, the difference between $(\Delta r)_{II}$ and $(\Delta r)_{III}$ for small $m_t$ arises because the latter, based on Eq.(3), does not incorporate the subleading logarithms of $O(\alpha^2 \ln(m_Z/m_f))$. Thus, $(\Delta r)_{II}$ is much to be preferred.

On the other hand, as mentioned before, $(\Delta r)_I - (\Delta r)_{II}$ involves mainly subleading terms of $O((\alpha/2\pi s^2)(c^2/s^2)\Delta\bar{\rho})$ or, equivalently, $O((c^2/s^2)(\alpha_2/2\pi)(\kappa_t/2\pi))$ where $\alpha_2 \equiv g^2/4\pi$ is the $SU(2)$ coupling and $\kappa_t \equiv (G_t)^2/4\pi$ is the Yukawa coupling of the top quark. These are very small for low $m_t$ but they are expected to be $\approx 6.5 \cdot 10^{-4}$ for $m_t = 300$GeV. The tables display a somewhat smaller difference $\lesssim 3.8 \cdot 10^{-4}$. A cursory examination of the Tables shows that the proposed on-shell expression of Eq.(7) and the $\overline{MS}$ relation of Eq.(10) lead to $s^2$ and $m_W$ values which are in very close agreement over a large range of $m_H$ and $m_t$ values.

Following the discussions of Refs.7,10 , it is useful to remark that the relation

$$\frac{m_W^2}{m_Z^2} = \frac{1}{2}\left\{1 + \left[1 - \frac{4A^2}{m_Z^2(1 - \Delta r)}\right]^{1/2}\right\} , \tag{12}$$

which expresses the $m_W$-$m_Z$ interdependence and follows from Eq.(11), can be expressed in alternative forms. On the basis of the on-shell formula of Eq.(7) one obtains

$$\frac{m_W^2}{m_Z^2} = \frac{\rho}{2}\left\{1 + \left[1 - \frac{4A^2}{\rho m_Z^2 C}\right]^{1/2}\right\} , \tag{13}$$

where $\rho \equiv (1-\Delta\bar{\rho})^{-1}$ and

$$C = 1 - \Delta\alpha - (\Delta r)_{rem}/(1+(c^2/s^2)\Delta\bar{\rho}) \ . \tag{14}$$

On the other hand, on the basis of the $\overline{MS}$ relation of Eq.(10), one finds[10]

$$\frac{m_W^2}{m_Z^2} = \frac{\hat{\rho}}{2}\left\{1+\left[1-\frac{4A^2}{\hat{\rho}\, m_Z^2(1-\Delta\hat{r}_W)}\right]^{1/2}\right\} \ . \tag{15}$$

The similarity between Eq.(13) and Eq.(15) is apparent. It should be stressed, however that these expressions, although theoretically interesting, are exactly equivalent to Eq.(12). Thus, Eq.(13) and Eq.(15) are equivalent to Eq.(12) when $1-\Delta r$ is identified with Eq.(7) and Eq.(10), respectively. It is worthwhile to note, however, that Eq.(13) and Eq.(15) can be readily generalized to models with $\rho \neq 1$ at the tree level.

Finally, we briefly discuss the difference between Eq.(3) and Eq.(7) from the point of view of mass singularities. According to the analysis of Ref.3, aside from small two-loop irreducible contributions, the mass singularities in $\Delta r$ are induced by the renormalization of $e_0^2$, i.e. by the relation $e_0^2 = e^2/(1-e^2\Pi(0))$. This implies, however, that in the perturbative expansion each power of $\alpha$ must be accompanied by a factor $(1-\Delta\alpha)^{-1}$ (or equivalently $(1+(\frac{2\delta e}{e})_{\overline{MS}})^{-1}$ in the $\overline{MS}$ formulation). If one inserts Eq.(7) into Eq.(1), factors out $(1-\Delta\alpha)$ and recalls that $A^2$ and each term in $(\Delta r)_{rem}$ contains one power of $\alpha$, one readily verifies that this requirement is indeed satisfied. The same holds true if one employs Eq.(10) as can be verified using the explicit relations of Ref.10. On the other hand, this is not true in the case of Eq.(3): terms in $(\Delta r)_{rem}$ involving $G_\mu$ and $\alpha$ are deficient in one and two powers of $(1-\Delta\alpha)^{-1}$, respectively.

# Acknowledgments

This paper is dedicated to the cherished memory of M. A. B. Bég, distinguished physicist, dear friend and collaborator. A man of great generosity, Baqi transformed his laboratory at the Rockefeller University into a very special and inspiring place for many physicists. In particular, it seems proper to acknowledge that the research cited in Ref.3, which plays a significant role in our present considerations, was carried out while one of us (A.S.) was a visitor at the Bég laboratory. We will miss his sharp intelligence, his forceful presence, his generosity and strong sense of loyalty, but Baqi will live in his writings and in the deep memories he has left on us.

One of us (A.S.) would like to thank M. Consoli, W. Hollik and F. Jegerlehner for useful discussions at the Topical Conference on Precise Measurements of Electroweak Parameters as Probes for New Physics, Institute for Theoretical Physics, Santa Barbara, California, February 21-23, 1991. The authors would also like to acknowledge the partial support of this research by the National Science Foundation under Grant No. PHY-9017585.

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Table 1: Values of $\Delta r$ obtained on the basis of Eq.(10) ($(\Delta r)_I$), Eq.(7) ($(\Delta r)_{II}$) and Eq.(3) ($(\Delta r)_{III}$), as a function of $m_t$ (GeV) for $m_Z = 91.177$ GeV and $m_H = 50.0$ GeV. Values of ($\Delta r_{rem}$) are also displayed.

| $m_t$ | $10^2(\Delta r)_I$ | $10^2(\Delta r)_{II}$ | $10^2(\Delta r)_{III}$ | $10^2(\Delta r)_{rem}$ |
|---|---|---|---|---|
| 90.0 | 5.894 | 5.897 | 5.824 | 0.674 |
| 100.0 | 5.576 | 5.579 | 5.522 | 0.548 |
| 120.0 | 4.936 | 4.939 | 4.905 | 0.359 |
| 140.0 | 4.246 | 4.248 | 4.230 | 0.212 |
| 160.0 | 3.480 | 3.481 | 3.474 | 0.088 |
| 180.0 | 2.623 | 2.622 | 2.623 | -0.023 |
| 200.0 | 1.662 | 1.659 | 1.663 | -0.124 |
| 220.0 | 0.584 | 0.578 | 0.581 | -0.220 |
| 240.0 | -0.625 | -0.635 | -0.638 | -0.313 |
| 260.0 | -1.979 | -1.994 | -2.011 | -0.404 |
| 280.0 | -3.496 | -3.517 | -3.557 | -0.496 |
| 300.0 | -5.197 | -5.226 | -5.300 | -0.590 |

Table 2: same as in Table 1 for $m_H = 100.0$ GeV.

| $m_t$ | $10^2(\Delta r)_I$ | $10^2(\Delta r)_{II}$ | $10^2(\Delta r)_{III}$ | $10^2(\Delta r)_{rem}$ |
|---|---|---|---|---|
| 90.0 | 6.081 | 6.083 | 5.989 | 0.857 |
| 100.0 | 5.765 | 5.766 | 5.689 | 0.732 |
| 120.0 | 5.129 | 5.130 | 5.077 | 0.545 |
| 140.0 | 4.442 | 4.443 | 4.408 | 0.400 |
| 160.0 | 3.680 | 3.680 | 3.659 | 0.277 |
| 180.0 | 2.828 | 2.826 | 2.816 | 0.168 |
| 200.0 | 1.872 | 1.867 | 1.865 | 0.069 |
| 220.0 | 0.801 | 0.792 | 0.792 | -0.025 |
| 240.0 | -0.401 | -0.414 | -0.415 | -0.116 |
| 260.0 | -1.748 | -1.766 | -1.774 | -0.206 |
| 280.0 | -3.256 | -3.282 | -3.304 | -0.295 |
| 300.0 | -4.947 | -4.982 | -5.029 | -0.387 |

Table 3: same as in Table 1 for $m_H = 250.0$ GeV.

| $m_t$ | $10^2(\Delta r)_I$ | $10^2(\Delta r)_{II}$ | $10^2(\Delta r)_{III}$ | $10^2(\Delta r)_{rem}$ |
|---|---|---|---|---|
| 90.0 | 6.394 | 6.395 | 6.265 | 1.164 |
| 100.0 | 6.081 | 6.082 | 5.969 | 1.041 |
| 120.0 | 5.450 | 5.451 | 5.366 | 0.857 |
| 140.0 | 4.770 | 4.770 | 4.706 | 0.714 |
| 160.0 | 4.015 | 4.013 | 3.968 | 0.594 |
| 180.0 | 3.170 | 3.166 | 3.136 | 0.488 |
| 200.0 | 2.222 | 2.216 | 2.199 | 0.391 |
| 220.0 | 1.160 | 1.150 | 1.143 | 0.299 |
| 240.0 | -0.032 | -0.046 | -0.046 | 0.212 |
| 260.0 | -1.366 | -1.387 | -1.383 | 0.125 |
| 280.0 | -2.862 | -2.890 | -2.888 | 0.039 |
| 300.0 | -4.538 | -4.576 | -4.582 | -0.049 |

Table 4: same as in Table 1 for $m_H = 1000.0$ GeV.

| $m_t$ | $10^2(\Delta r)_I$ | $10^2(\Delta r)_{II}$ | $10^2(\Delta r)_{III}$ | $10^2(\Delta r)_{rem}$ |
|---|---|---|---|---|
| 90.0 | 6.971 | 6.971 | 6.769 | 1.731 |
| 100.0 | 6.662 | 6.663 | 6.482 | 1.612 |
| 120.0 | 6.042 | 6.043 | 5.893 | 1.432 |
| 140.0 | 5.372 | 5.372 | 5.250 | 1.294 |
| 160.0 | 4.628 | 4.627 | 4.530 | 1.178 |
| 180.0 | 3.796 | 3.793 | 3.720 | 1.076 |
| 200.0 | 2.862 | 2.857 | 2.808 | 0.983 |
| 220.0 | 1.816 | 1.807 | 1.780 | 0.896 |
| 240.0 | 0.642 | 0.629 | 0.625 | 0.813 |
| 260.0 | -0.673 | -0.692 | -0.674 | 0.732 |
| 280.0 | -2.146 | -2.172 | -2.134 | 0.651 |
| 300.0 | -3.797 | -3.833 | -3.775 | 0.569 |

Table 5: Values of $\sin^2\theta_W$ and $m_W$ (GeV) obtained on the basis of Eq.(10) ( $s_I^2$ and $(m_W)_I$), Eq.(7) ( $s_{II}^2$ and $(m_W)_{II}$) and Eq.(3) ( $s_{III}^2$ and $(m_W)_{III}$), as a function of $m_t$ (GeV) for $m_Z = 91.177$ GeV and $m_H = 50.0$ GeV.

| $m_t$ | $s_I^2$ | $s_{II}^2$ | $s_{III}^2$ | $(m_W)_I$ | $(m_W)_{II}$ | $(m_W)_{III}$ |
|---|---|---|---|---|---|---|
| 90.0 | 0.23103 | 0.23103 | 0.23078 | 79.954 | 79.954 | 79.967 |
| 100.0 | 0.22992 | 0.22992 | 0.22973 | 80.012 | 80.011 | 80.022 |
| 120.0 | 0.22772 | 0.22773 | 0.22761 | 80.126 | 80.126 | 80.132 |
| 140.0 | 0.22540 | 0.22541 | 0.22535 | 80.246 | 80.246 | 80.249 |
| 160.0 | 0.22289 | 0.22289 | 0.22287 | 80.376 | 80.376 | 80.377 |
| 180.0 | 0.22015 | 0.22015 | 0.22015 | 80.518 | 80.518 | 80.517 |
| 200.0 | 0.21717 | 0.21716 | 0.21717 | 80.671 | 80.672 | 80.671 |
| 220.0 | 0.21393 | 0.21391 | 0.21392 | 80.838 | 80.839 | 80.839 |
| 240.0 | 0.21042 | 0.21039 | 0.21038 | 81.018 | 81.020 | 81.020 |
| 260.0 | 0.20663 | 0.20660 | 0.20655 | 81.212 | 81.214 | 81.217 |
| 280.0 | 0.20257 | 0.20251 | 0.20241 | 81.420 | 81.423 | 81.428 |
| 300.0 | 0.19821 | 0.19814 | 0.19795 | 81.642 | 81.646 | 81.656 |

Table 6: Same as in Table 5 for $m_H = 100.0$ GeV.

| $m_t$ | $s_I^2$ | $s_{II}^2$ | $s_{III}^2$ | $(m_W)_I$ | $(m_W)_{II}$ | $(m_W)_{III}$ |
|---|---|---|---|---|---|---|
| 90.0 | 0.23168 | 0.23169 | 0.23136 | 79.920 | 79.920 | 79.937 |
| 100.0 | 0.23057 | 0.23058 | 0.23031 | 79.978 | 79.977 | 79.991 |
| 120.0 | 0.22837 | 0.22838 | 0.22820 | 80.092 | 80.092 | 80.101 |
| 140.0 | 0.22605 | 0.22605 | 0.22594 | 80.212 | 80.212 | 80.218 |
| 160.0 | 0.22354 | 0.22354 | 0.22347 | 80.342 | 80.343 | 80.346 |
| 180.0 | 0.22080 | 0.22079 | 0.22076 | 80.484 | 80.484 | 80.486 |
| 200.0 | 0.21781 | 0.21780 | 0.21779 | 80.638 | 80.639 | 80.639 |
| 220.0 | 0.21457 | 0.21455 | 0.21455 | 80.805 | 80.806 | 80.806 |
| 240.0 | 0.21106 | 0.21102 | 0.21102 | 80.986 | 80.987 | 80.988 |
| 260.0 | 0.20727 | 0.20722 | 0.20720 | 81.180 | 81.182 | 81.183 |
| 280.0 | 0.20320 | 0.20313 | 0.20307 | 81.388 | 81.391 | 81.394 |
| 300.0 | 0.19884 | 0.19875 | 0.19863 | 81.610 | 81.615 | 81.621 |

Table 7: Same as in Table 5 for $m_H = 250.0$ GeV.

| $m_t$ | $s_I^2$ | $s_{II}^2$ | $s_{III}^2$ | $(m_W)_I$ | $(m_W)_{II}$ | $(m_W)_{III}$ |
|---|---|---|---|---|---|---|
| 90.0 | 0.23279 | 0.23280 | 0.23233 | 79.862 | 79.862 | 79.886 |
| 100.0 | 0.23168 | 0.23169 | 0.23129 | 79.920 | 79.920 | 79.940 |
| 120.0 | 0.22948 | 0.22948 | 0.22919 | 80.035 | 80.034 | 80.050 |
| 140.0 | 0.22715 | 0.22715 | 0.22694 | 80.155 | 80.155 | 80.166 |
| 160.0 | 0.22463 | 0.22463 | 0.22448 | 80.286 | 80.286 | 80.294 |
| 180.0 | 0.22189 | 0.22188 | 0.22178 | 80.428 | 80.428 | 80.433 |
| 200.0 | 0.21890 | 0.21888 | 0.21882 | 80.582 | 80.583 | 80.586 |
| 220.0 | 0.21565 | 0.21562 | 0.21560 | 80.750 | 80.751 | 80.752 |
| 240.0 | 0.21213 | 0.21208 | 0.21209 | 80.931 | 80.933 | 80.933 |
| 260.0 | 0.20833 | 0.20827 | 0.20828 | 81.126 | 81.129 | 81.128 |
| 280.0 | 0.20425 | 0.20417 | 0.20418 | 81.334 | 81.338 | 81.338 |
| 300.0 | 0.19987 | 0.19978 | 0.19976 | 81.558 | 81.563 | 81.563 |

Table 8: Same as in Table 5 for $m_H = 1000.0$ GeV.

| $m_t$ | $s_I^2$ | $s_{II}^2$ | $s_{III}^2$ | $(m_W)_I$ | $(m_W)_{II}$ | $(m_W)_{III}$ |
|---|---|---|---|---|---|---|
| 90.0 | 0.23487 | 0.23488 | 0.23414 | 79.754 | 79.754 | 79.792 |
| 100.0 | 0.23376 | 0.23376 | 0.23311 | 79.812 | 79.812 | 79.846 |
| 120.0 | 0.23154 | 0.23155 | 0.23102 | 79.927 | 79.927 | 79.954 |
| 140.0 | 0.22921 | 0.22921 | 0.22879 | 80.049 | 80.049 | 80.070 |
| 160.0 | 0.22668 | 0.22667 | 0.22635 | 80.180 | 80.180 | 80.197 |
| 180.0 | 0.22392 | 0.22391 | 0.22367 | 80.323 | 80.323 | 80.336 |
| 200.0 | 0.22091 | 0.22089 | 0.22074 | 80.478 | 80.479 | 80.487 |
| 220.0 | 0.21764 | 0.21761 | 0.21753 | 80.647 | 80.648 | 80.653 |
| 240.0 | 0.21410 | 0.21406 | 0.21405 | 80.829 | 80.831 | 80.832 |
| 260.0 | 0.21028 | 0.21023 | 0.21028 | 81.025 | 81.028 | 81.026 |
| 280.0 | 0.20618 | 0.20611 | 0.20621 | 81.236 | 81.239 | 81.234 |
| 300.0 | 0.20178 | 0.20169 | 0.20184 | 81.460 | 81.465 | 81.457 |