Fermilab Library
0 1160 0011875 6

NUP-A-92-4

NUP-A-92-4

# A Minimally-Extended $SU(2)_L \times U(1)$ Electroweak Gauge Model - Possibility of Relatively Heavy Light-Neutrinos and a Relatively Light Heavy-Neutrino -*

Satoshi Momose and S.Y.Tsai†

*Atomic Energy Research Institute and Department of Physics*
*College of Science and Technology, Nihon University*
*Kanda-Surugadai, Chiyoda-ku, Tokyo 101, Japan*

16

## ABSTRACT

The standard $SU(2)_L \times U(1)$ electroweak gauge model is extended so as to incorporate a single right-handed neutrino, a single Higgs singlet and a second Higgs doublet. In such a scheme, four Majorana neutrinos emerge, with two of them ($\omega_3$ and $\omega_4$) forming a sea-saw pair while the rest ($\omega_2$ and $\omega_1$) acquiring mass at one- and two-loop levels respectively. Their masses will therefore exhibit a remarkable hierarchy: $m_1 \ll m_2 \ll m_3 \ll m_4$. By spontaneously breaking the presupposed lepton-number conservation, a Nambu-Goldstone boson (majoron) appears, which allows $\omega_4$ and $\omega_3$ to have a rapid invisible decay mode at tree and one-loop levels respectively. An attempt is made to derive constraints on $m_3$ and $m_4$ from a cosmological consideration. It is found that $m_4$ is restricted to be less than $\sim$100 GeV while $m_3$ is allowed to have any value below the laboratory upper limit ($\sim$35 MeV). In particular, $\omega_3$ may be identified with Simpson's 17 keV neutrino; $m_4$ is then $\lesssim$ 49 GeV. Other phenomenological implications of the model are also discussed.

1992

*Paper contributed to the 26th International Conference on High Energy Physics (Dallas, Texas, August 6-12, 1992).

†E-mail address : tsai@cst.nihon-u.ac.jp



## 1.Introduction

The recent observations by Simpson and others[1,2] that the electron neutrino may contain a component with mass 17 keV have stimulated many people to seriously reconsider the possibility that the neutrinos may have a nonzero rest mass and that some of them may in fact be relatively heavy. To incorporate such possibility, the standard electroweak model, i.e. the minimal $SU(2)_L \times U(1)$ gauge model due to Glashow, Weinberg and Salam[3], has to be modified or extended. Obviously, there are a variety of ways in such extensions. Among the most straightforward is to introduce one or several right-handed neutrino singlets so that one may have a Dirac and/or Majorana mass terms for neutrino fields.[a] In fact, Glashow[4] discussed Simpson's neutrino in a model in which three generations of right-handed neutrino singlets were introduced. He noted that a novel mass hierarchy might exist among the six physical neutrinos, with two of them forming a pseudo-Dirac neutrino of mass 17 keV. He also noted that, by invoking the singlet majoron hypothesis[5], one could reconcile the 17 keV neutrino with relevant cosmological constraints[6].

In the present note, we would like to investigate another extended model which is in some respect more economical than the model investigated by Glashow. The model is characterized by containing only one right-handed neutrino singlet but one more Higgs doublet. This leads to four Majorana neutrinos, with two of them ($\omega_3$ and $\omega_4$) forming a see-saw pair while the other two ($\omega_2$ and $\omega_1$) acqiring mass at one- and two- loop levels respectively. Their masses therefore naturally exhibit a remarkable hierarchy: $m_1 \ll m_2 \ll m_3 \ll m_4$. $\omega_3$ as well as $\omega_4$ may decay into a lighter neutrino by emitting a majoron. As a result, it is found that $m_3$ may take any value below the laboratory upper limit (35MeV), without contradicting the relevant cosmological constraints. $m_4$ is, on the other hand, found to be rather severely restricted: $m_4 \lesssim 100$ GeV. If one identifies $\omega_3$ with the 17 keV neutrino, $m_4$ is then $\lesssim 41.5$ GeV. Constraints from neutrinoless double beta decays will also be discussed.

The paper is organized as follows.We discuss in Sect.2 the neutrino mass spectrum and properties of the lepton mixing matrices in a model in which only one right-handed neutrino singlet is introduced. The number of left-handed lepton doublets $n$ and the number of Higgs doublets $m$ are kept arbitrary. The minimal case,i.e., $n = 3$ and $m = 2$, is considered in Sect.3. Sect.4 is devoted to phenomenology of the minimal model,which is followed by a discussion of the 17 keV neutrino in Sect.5. The paper is ended with some concluding remarks.

## 2. Model with a single right-handed neutrino singlet

Let us begin with a discussion of the neutrino mass spectrum and properties of the lepton mixing matrices in the case that only one right-handed neutrino singlet is present.[b] Ingredients of the model, together with their $SU(2)_L \times U(1)$ quantum numbers (weak isospin $I$ and weak hypercharge $Y$) and lepton number $L$, are listed in Table I. The

number of generations $n$ and the number of Higgs doublets $m$ will be kept arbitrary for the moment, while the quark sector is left untouched throughout. The Yukawa coupling reads

Table I

$$-L_Y = \sum_{i,j}^{n}\sum_{\alpha}^{m}(c_\alpha)_{ij}\bar{L}_i\phi_\alpha e_{jR} + \sum_{i}^{n}\sum_{\alpha}^{m}d_{\alpha i}\bar{L}_i\tilde{\phi}_\alpha\nu_R + \frac{1}{2}f\bar{\nu}_R^c\sigma\nu_R + h.c., \tag{1}$$

where $\tilde{\phi} = i\tau_2\phi^*$ and $\psi^c$ denotes the field charge-conjugate to $\psi$. ($\nu_R^c$ is left-handed in our notation.) With the substitution[c]

$$\phi_\alpha \to \begin{pmatrix} \phi_\alpha^+ \\ \sqrt{\frac{1}{2}}v_\alpha + \phi_\alpha^0 \end{pmatrix}, \sigma \to \sqrt{\frac{1}{2}}u + \sigma^0,$$

we write $L_Y$ as

$$L_Y = L_{mass}^\nu + L^\nu(\phi^0,\sigma^0) + L_Y' \tag{2}$$

$$-L_{mass}^\nu = m_D\bar{\nu}_{nL}'\nu_R + \frac{1}{2}m_R\bar{\nu}_R^c\nu_R + h.c. \tag{2a}$$

$$-L^\nu(\phi^0,\sigma^0) = \sum_{i}^{n}\sum_{\alpha}^{m}d_{\alpha i}\bar{\nu}_{iL}\nu_R\phi_\alpha^{0*} + \frac{1}{2}f\bar{\nu}_R^c\nu_R\sigma^0 + h.c. \tag{2b}$$

$$\begin{aligned} -L_Y' &= \sum_{\alpha}^{m}[\sum_{i,j}^{n}(c_\alpha)_{ij}\bar{\nu}_{iL}e_{jR} - \sum_{i}^{n}d_{\alpha i}^*\bar{\nu}_R e_{iL}]\phi_\alpha^+ \\ &\quad + \sum_{\alpha}^{m}\sum_{i,j}^{n}(c_\alpha)_{ij}\bar{e}_{iL}e_{jR}\phi_\alpha^0 + \sum_{i,j}^{n}m_{ij}\bar{e}_{iL}e_{jR} + h.c., \end{aligned} \tag{2c}$$

where

$$\nu_{nL}' = \sum_{i}^{n}V_{ni}\nu_{iL} \tag{3a}$$

$$V_{ni} = m_D^{-1}\sum_{\alpha}^{m}\sqrt{\frac{1}{2}}d_{\alpha i}v_\alpha \qquad (i = 1,\cdots,n) \tag{3b}$$

$$m_D = [\sum_{i}^{n}|\sum_{\alpha}^{m}\sqrt{\frac{1}{2}}d_{\alpha i}v_\alpha|^2]^{1/2}, \qquad m_R = \sqrt{\frac{1}{2}}fu, \tag{3c}$$

$$m_{ij} = \sum_{\alpha}^{m}\sqrt{\frac{1}{2}}(c_\alpha)_{ij}v_\alpha. \tag{3d}$$

Although we have $n$ $\nu_L$'s at hand, it is seen from Eq.(2a) that only one particular linear combination given by $\nu_{nL}'$ couples to $\nu_R$ and conspires with the $\bar{\nu}_R^c\nu_R$ term to result in two Majorana neutrinos. In fact, Eq.(2a) can be rewritten as

$$-L_{\rm mass}^\nu = \frac{1}{2}m_n\bar{\omega}_n\omega_n + \frac{1}{2}m_N\bar{\omega}_N\omega_N, \tag{4}$$



where

$$\omega_n = \omega_{nL} + \eta_n\omega_{nL}^c, \qquad \omega_N = \omega_{NL} + \eta_N\omega_{NL}^c, \tag{5a}$$

$$\begin{aligned} \omega_{nL} &= \nu_{nL}'\cos\phi - \nu_R^c\sin\phi, \\ \omega_{NL} &= \nu_{nL}'\sin\phi + \nu_R^c\cos\phi, \end{aligned} \tag{5b}$$

$$\left.\begin{matrix} m_n \\ m_N \end{matrix}\right\} = \frac{1}{2}[(m_R^2 + 4m_D^2)^{1/2} \mp |m_R|], \tag{5c}$$

$$\tan^2\phi = m_n/m_N = (m_n/m_D)^2. \tag{5d}$$

$\eta_n$ and $\eta_N$ are the charge-conjugation parity of $\omega_n$ and $\omega_N$, which are opposite in sign (e.g. $\eta_n = -1$ and $\eta_N = +1$ if $m_R > 0$ ). The other $n-1$ independent fields $\nu_{iL}'(i = 1,\cdots,n-1)$ related to the original $\nu_{iL}(i = 1,\cdots,n)$ by

$$\nu_{iL}' = \sum_{j}^{n}V_{ij}\nu_{jL} \qquad (i = 1,\cdots,n-1), \tag{6}$$

$V = (V_{ij})$ being an $n \times n$ unitary matrix with $V_{ni}$ given by Eq.(3b), remain massless at tree-level. Combining Eqs.(3a), (5b) and (6), one has

$$\omega_{aL} = \sum_{b}^{N}U_{ab}^\nu\nu_{bL}\,(a = 1,\cdots,N), \tag{7}$$

where $N \equiv n+1, \nu_{NL} \equiv \nu_R^c, \omega_{aL} \equiv \nu_{aL}'(a = 1,\cdots,n-1)$ and $U^\nu = (U_{ab}^\nu)$ is an $N \times N$ unitary matrix:

$$\begin{aligned} U_{ij}^\nu &= V_{ij}, \qquad U_{iN}^\nu = 0, \\ U_{nj}^\nu &= V_{nj}\cos\phi, \qquad U_{nN}^\nu = -\sin\phi, \\ U_{Nj}^\nu &= V_{nj}\sin\phi, \qquad U_{NN}^\nu = \cos\phi. \\ &\quad (i = 1,\cdots,n-1; j = 1,\cdots,n) \end{aligned} \tag{8}$$

If one denotes diagonalizing unitary matrices for the charged-lepton mass matrix $(m_{ij})$ by $U^L$ and $U^R$ and the physical (mass-eigenstate) charged-lepton fields by $l_k(k = 1,\cdots,n)$, i.e.

$$\sum_{ij}^{n}U_{ki}^L m_{ij}(U_{lj}^R)^* = m_l\delta_{kl}, \tag{9a}$$

$$\begin{aligned} l_{kL} &= \sum_{i}^{n}U_{ki}^L e_{iL}, \qquad l_{kR} = \sum_{i}^{n}U_{ki}^R e_{iR}, \\ &\quad (k,l = 1,\cdots,n) \end{aligned} \tag{9b}$$

the gauge couplings of lepton fields read

$$\begin{aligned} -L_{\rm W} &= \frac{g}{\sqrt{2}}\sum_{i}^{n}\bar{\nu}_{iL}\gamma^\lambda e_{iL}W_\lambda^+ + h.c. \\ &= \frac{g}{\sqrt{2}}\sum_{a}^{N}\sum_{k}^{n}\bar{\omega}_{aL}\gamma^\lambda U_{ak}l_{kL}W_\lambda^+ + h.c. \end{aligned} \tag{10a}$$

$$
\begin{aligned}
-L_Z &= \frac{g}{2\cos\theta_\omega}\sum_i^n[\bar\nu_{iL}\gamma^\lambda\nu_{iL} - \bar e_{iL}\gamma^\lambda e_{iL} + 2\sin^2\theta_\omega \bar e_i\gamma^\lambda e_i]Z_\lambda \\
&= \frac{g}{2\cos\theta_\omega}\left\{\sum_{a,b}^N \bar\omega_{aL}\gamma^\lambda U^{\nu\nu}_{ab}\omega_{bL} + \sum_k[-\bar l_{kL}\gamma^\lambda l_{kL} + 2\sin^2\theta_\omega \bar l_k\gamma^\lambda l_k]\right\} Z_\lambda,
\end{aligned} \tag{10b}
$$

where $\theta_\omega$ is the Weinberg angle and

$$
\begin{aligned}
U_{ak} &= \sum_i^n U^\nu_{ai}(U^L_{ki})^*, \qquad U^{\nu\nu}_{ab} = \sum_i^n U^\nu_{ai}(U^\nu_{bi})^*, \\
&(a,b = 1,\cdots,N; k = 1,\cdots,n \text{ or } e,\mu,\tau,\cdots)
\end{aligned} \tag{11}
$$

It is important to realize that a mixing matrix appears in the charged current as well as in the neutrino part of the neutral current and that they have the following properties:

$$
\sum_a^N U_{ak}U^*_{al} = \delta_{kl}; \tag{12a}
$$

$$
\sum_k^n U_{ak}U^*_{bk} = U^{\nu\nu}_{ab} = \begin{pmatrix} 1 & & 0 \\ 0 & \cos^2\phi & \cos\phi\sin\phi \\ & \cos\phi\sin\phi & \sin^2\phi \end{pmatrix}, \tag{12b}
$$

1 being the $(n-1)\times(n-1)$ unit matrix; and

$$
{U_{Nk}}^2/{U_{nk}}^2 = \tan^2\phi = m_n/m_N, \tag{12c}
$$

independent of $k$. From Eq.(12b), one has

$$
\sum_k^n |U_{Nk}|^2 = n - \sum_{i,k}^n |U_{ik}|^2 = \sin^2\phi = m_n/(m_n+m_N). \tag{13}
$$

This is an interesting equation which, in the case of $n=3$, relates low energy physics to physics beyond the standard model.

Next turn our attention to the Yukawa interaction of neutrinos, Eq.(2b). Since the $m$ particular combinations of $\nu_{1L},\cdots,\nu_{nL}, \hat\nu_{\alpha L} = \sum_i^n d_{\alpha i}\nu_{iL}(\alpha = 1,\cdots,m)$, couple to $\phi^0_\alpha(\alpha = 1,\cdots,m)$ respectively and since $\nu'_{nL}$ is itself a combination of $\hat\nu_{1L},\cdots,\hat\nu_{mL}$ (see Eqs. (3a) and (3b)), we realize that $m-1$ fields, to be identified with $\omega_{iL}(i = n-m+1,\cdots,n-1)$, have a coupling with $\phi^0_1,\cdots,\phi^0_m$, while $\omega_{nL}$ and $\omega_{NL}$ have a coupling with $\sigma^0$ in addition. These $m-1$ left-handed fields will acquire a radiative Majorana mass and a coupling with $\sigma^0$ at one-loop level (see Fig.1), while the remaining $n-m$ fields, $\omega_{iL}(i = 1,\cdots,n-m)$, will do so only at two-loop level.

Fig. 1

Since $\phi^0_\alpha(\alpha = 1,\cdots,m)$ and $\sigma^0$ are complex fields, we have altogether $2(m+1)$ real



neutral scalar fields and they will in general mix with one another. Denoting mass-eigenstate fields by $\psi^0_\mu(\mu = 1,\cdots,2(m+1))$, we have

$$
\phi^0_\alpha = \sum_\mu^{2(m+1)} b_{\alpha\mu}\psi^0_\mu/\sqrt{2}, (\alpha = 1,\cdots,m+1) \tag{14}
$$

where $\phi^0_{m+1} \equiv (\sigma^0)^*$ and $b_{\alpha\mu}$ are complex coefficients satisfying the appropriate orthonormality and completeness conditions. One of $\psi^0$'s is the would-be Goldstone boson to be eaten up by the $Z^0$ boson:

$$
\phi_Z = \sqrt{2}\sum_\alpha^m (v_\alpha/v)\mathrm{Im}(\phi^0_\alpha), \tag{15}
$$

where $v = (\sum_\alpha^m v_\alpha^2)^{1/2}$. The requirement that the lepton number conservation breaks down spontaneously through $<\sigma>= u/\sqrt{2} \neq 0$ leads to another Goldstone boson, i.e. the majoron,

$$
\phi_M = \sqrt{2}\mathrm{Im}(\sigma^0), \tag{16}
$$

to which only $\omega_n$ and $\omega_N$ couple:

$$
\begin{aligned}
-L^\nu(\phi_M) &= \frac{1}{2}f\bar\nu^c_R\nu_R(i\phi_M/\sqrt{2}) + h.c. \\
&= \frac{1}{2}\sqrt{\frac{1}{2}}if(-\bar\omega_n\gamma_5\omega_n\sin^2\phi + \bar\omega_N\gamma_5\omega_N\cos^2\phi \\
&\qquad -\bar\omega_n\omega_N\sin 2\phi)\phi_M.
\end{aligned} \tag{17}
$$

$\phi_Z$ and $\phi_M$ may be identified with $\psi^0_{2m+1}$ and $\psi^0_{2(m+1)}$, respectively; $\psi^0_\mu(\mu = 1,\cdots,2m)$ are then massive neutral Higgs bosons.

## 3. The minimal model

Now let us consider the special case: $n = 3$ and $m = 2$. Our foregoing discussion indicates that, without loss of generality, one may take

$$
\begin{aligned}
&v_1 = v \neq 0, \qquad v_2 = 0, \\
&d_{11} = d_{12} = 0, \qquad d_{13} \neq 0, \\
&d_{21} = 0, \qquad d_{22} \neq 0, \qquad d_{23} \neq 0.
\end{aligned} \tag{18}
$$

There are three complex neutral Higgs fields ($\phi^0_1, \phi^0_2$ and $\phi^0_3 \equiv (\sigma^0)^*$) related to six real mass-eigenfields $\psi^0_\mu$ ($\mu = 1,\cdots,6$; $\psi^0_5 = \phi_Z$ and $\psi^0_6 = \phi_M$ ) as in Eq.(14). Equation (2b) reads

$$
-L^\nu(\phi^0,\sigma^0) = \sum_\mu^6(\sum_{i=2}^3 B_{\mu i}\bar\nu_{iL}\nu_R + \frac{1}{2}B_{\mu 4}\bar\nu^c_R\nu_R)\psi^0_\mu + h.c., \tag{19}
$$

where

$$
\begin{aligned}
B_{\mu i} &= \sum_\alpha^2 b^*_{\alpha\mu}d_{\alpha i}/\sqrt{2}, \\
B_{\mu 4} &= b^*_{3\mu}f/\sqrt{2}, \\
&(i = 2,3; \quad \mu = 1,\cdots,6)
\end{aligned} \tag{20}
$$

Evaluating, with the aid of Eqs.(17) and (19), dominant contributions from the one-loop diagram explicitly, one obtains[d]

$$-L^{\nu(1)}_{mass} = \frac{1}{2}\sum_{a,b=2}^{4} m^{(1)}_{ab}\bar{\omega}_{aL}\omega^c_{bL} + h.c., \tag{21}$$

$$-L^{\nu(1)}(\phi_M) = \frac{1}{2}i\sum_{a,b=2}^{4} A^{(1)}_{ab}\bar{\omega}_{aL}\omega^c_{bL}\phi_M + h.c., \tag{22}$$

where, e.g.,

$$m^{(1)}_{22} = \frac{1}{16\pi^2}m_R\sum_{\mu}^{4} B_{\mu 2}B_{\mu 2}\frac{m^2_\mu}{m^2_R - m^2_\mu}\ln\frac{m^2_R}{m^2_\mu}, \tag{21a}$$

$$A^{(1)}_{23} = -\frac{1}{16\pi^2}\sqrt{\frac{1}{2}}f\sum_{\mu}^{4} B_{\mu 2}B_{\mu 3}\frac{m^2_\mu}{m^2_R - m^2_\mu}\ln\frac{m^2_R}{m^2_\mu}. \tag{22a}$$

To go one step further, we assume

$$\begin{aligned} d_{22} \approx d_{23} &\approx d_{13} \equiv d, \\ b_{\alpha\mu} &\approx O(1). \end{aligned} \tag{23}$$

One then gets, as an order-of-magnitude estimate,

$$m^{(1)}_{ij} \approx m^{(1)}, \quad m^{(1)}_{i4} \approx (f/d)m^{(1)}, \quad m^{(1)}_{44} \approx (f/d)^2 m^{(1)}, \quad (i,j = 2,3) \tag{21b}$$

$$A^{(1)}_{23} \approx -\sqrt{\frac{1}{2}}f m^{(1)}/m_4, \tag{22b}$$

where

$$m^{(1)} = m_3(m_4/v)^2(m^2_4/m^2_H - 1)^{-1}\frac{1}{16\pi^2}\ln(m^2_4/m^2_H). \tag{24}$$

Here $m_H$ is a representative value of $m_\mu(\mu = 1, \cdots, 4)$, and use has been made of Eqs.(3c), (18) and (20).

From Eqs. (4), (5c), (21) and (21b), after rediagonalization, four physical neutrinos emerge, with three of them, approximately identical to $\omega_a = \omega_{aL} + \omega^c_{aL}$ $(a = 2,3,4)$, having mass $m_2 \approx m^{(1)}$, $m_3 \approx m^2_D/m_R$ and $m_4 \approx m_R$ and charge-conjugation parity $\eta_2 = 1$, $\eta_3 = -1$ and $\eta_4 = 1$, respectively, while the fourth one remaining massless at one-loop level. Their masses therefore naturally exhibit a remarkable hierarchy:

$$m_1 \ll m_2 \ll m_3 \ll m_4. \tag{25}$$

## 4. Phenomenology

We now confront our minimal model with observations. Since it is known that there are three flavor eigenstates $\nu_e$, $\nu_\mu$ and $\nu_\tau$ and that, experimentally [10],

$$m(\nu_e) < 17\text{eV}, \quad m(\nu_\mu) < 270\text{keV}, \quad m(\nu_\tau) < 35\text{MeV}, \tag{26}$$



it follows, as a necessary condition, that

$$m_1 < 17\text{eV}, \quad m_2 < 270\text{keV}, \quad m_3 < 35\text{MeV}. \tag{27}$$

$\omega_4$ is most likely a hitherto unobserved heavy neutrino. The negative results of neutrino oscillation experiments [10] then suggest that

$$\nu_e \approx \omega_1, \quad \nu_\mu \approx \omega_2, \quad \nu_\tau \approx \omega_3, \tag{28}$$

implying that

$$\begin{aligned} U_{1e}, U_{2\mu}, U_{3\tau} &\approx 1, \\ \text{all other elements of } U &: \text{small}. \end{aligned} \tag{29}$$

It is to be stressed however that the identification (28) is by no means unique; it is merely a naive expectation.

On the other hand, it is known from cosmology that a stable neutrino must be lighter than $\sim$100eV [11]. Since our $\omega_2$ is stable up to one-loop level, we must require that

$$m_2 \lesssim 100\text{eV}. \tag{30}$$

In contrast, an unstable neutrino can have mass $m$ greater than 100 eV, if its lifetime $\tau$ satisfies the cosmological lifetime constraint, which is, according to Kolb and Turner [6],

$$\tau < 2.3(m/\text{keV})^{-2}(\Sigma_0 h^2)^{3/2}10^{15}\text{sec}, \tag{31}$$

where $\Sigma_0$ is the total energy density of the present universe in unit of the critical density for closing the universe and $h$ is the Hubble constant in unit of 100 km $s^{-1}$ Mpc$^{-1}$. Our $\omega_3$, having decay mode, $\omega_3 \to \omega_2 + \phi_M$, at one-loop level, is indeed farely unstable. Noting that the relevant amplitude $A_{23}$ is $A^{(1)}_{23}$ (as given by Eq.(22b)) times $m_3/m_4$, its lifetime $\tau_3$ is readily estimated:

$$\tau_3 = (\frac{1}{16\pi}|A_{23}|^2 m_3)^{-1} = (\frac{1}{32\pi}f^2 m^3_3 m^2_2/m^4_4)^{-1} \tag{32}$$

Combining Eqs.(24), (30), (31) and (32) and inputting[e]

$$m_H > 42\text{GeV}, \tag{33}$$

$$v = 250\text{GeV}, \tag{34}$$

$$\Sigma_0 h^2 \lesssim 1, \tag{35}$$

$$f < 1, \tag{36}$$

one derives constraints on $m_3$ and $m_4$, as shown in Fig.2. It is seen that the mass of the heavy neutrino, $\omega_4$, is rather severely restricted:

Fig.2

$$m_4 \lesssim 100\,\text{GeV}, \tag{37}$$

and that, amusingly, only if $m_3$ takes a value close to 270 keV, the value quoted as the laboratory upper limit for $m(\nu_\mu)$, $m_4$ can be as large as $\sim$100GeV. If one regards it plausible to take $m_D = m_\mu$ so that $m_3 m_4 = m_\mu^2$, then

$$m_4 < 80\text{GeV}, \quad m_3 > 130\text{keV}. \tag{38}$$

One may wonder if $\omega_2$, $\omega_3$ and $\omega_4$, with $2\text{eV} < m_2 \ll m_3 \ll m_4$, contradict the experimental result [12]

$$< m(\nu) >< 2\text{eV}, \tag{39}$$

where $< m(\nu) >$ is the weighted sum of neutrino masses contributing to the neutrinoless double beta decay. In our case, $< m(\nu) >$ is given by

$$< m(\nu) >= |m_1(U_{1e})^2 + m_2(U_{2e})^2 - m_3(U_{3e})^2 + \epsilon m_4(U_{4e})^2|, \tag{40}$$

where we have taken account that the contribution from the heavy $\omega_4$ may be suppressed by a facter $\epsilon(0 < \epsilon \leqq 1)$. The minus sign in front of $m_3$ comes from the opposite charge-conjugation parity of $\omega_3$ relative to those of the others. Thus, one may expect that a natural cancellation would bring $< m(\nu) >$ to satisfy Eq.(39). Without trying estimation of $\epsilon$, we note the following:

(1)If $\epsilon = 1$, the contributions from $\omega_3$ and from $\omega_4$ cancel against each other exactly owing to the property (12c). Combining Eq.(39) with Eq.(29) then gives $m_1 \lesssim 2$ eV.

(2)If $\epsilon = 0$, it is expected that the contributions from $\omega_2$ and from $\omega_3$ cancel against each other, indicationg that $U_{3e} \ll U_{2e}$ and $m_1 \lesssim 2$ eV.

## 5. The 17 keV neutrino

We proceed to see if it is possible to identify $\omega_3$ with Simpson's neutrino. With $m_3 = 17$keV, one finds from Fig.2 that

$$m_4 \lesssim 49 \quad \text{GeV}. \tag{41}$$

It is to be noted that our $\omega_3$ with mass $< m_Z/2$ does not contradict any results from the LEP experiments including neutrino counting and direct search for heavy neutral leptons [10,13]. If $\epsilon = 1$, one wuold have no trouble at all with the experimental results on $< m(\nu) >$, as already mentioned. On the other hand, if $\epsilon = 0$, there would be no counter term to cancel $m_3 U_{3e}^2$ as far as $U_{3e}^2 \approx 0.01$ [1,2]. A cancellation is possible, however, if, for example,

$$m_2 \approx 17\,\text{eV}, \; U_{2e} \approx 1 \quad \text{and} \quad U_{3e}^2 \approx 0.001, \tag{42}$$

suggesting that the mixing probability of the 17 keV neutrino may not be so large as reported [1,2] and that, contrary to the naive expectation, Eq.(28), the electron neutrino may not be the lightest of the three light neutrinos.



## 6. Concluding remarks

Our minimal model, if viable indeed, indicates, among other things, richness of the Higgs sector and hierarchical structure of the neutrino mass spectrum. From a cosmological consideration, it has been suggested that the heavy neutrino may be relatively light and that one or two of the three familiar light neutrinos can be relatively heavy. To accommodate the 17 keV neutrino without contradicting the absence of neutrinoless double beta decays, it has been suggensted that its mixing probability must be much lower than what is reported and that the electron neutrino appears to constist mainly of a component which is not the lightest of the three light neutrinos. We hope that all these predictions will be tested in current and future experiments.

To conclude, just because of the hierarchical structure of the neutrino mass spectrum, the MSW mechanism [14] as an explanation of deficit of solar neutrinos is not applicable to our minimal model.

## Footnotes

[a] One may have, by introducing a triplet Higgs field, a Majorana mass term for left-handed neutrinos. This possibility will not be considered here.

[b] New physics implied by presence of right-handed neutrinos was stressed by Jarlskog [7], properties of the lepton mixing matrices in the presence of a single right-handed neurtino singlet was discussed in [8], and neutrino mass hierarchy in the presence of multiple Higgs doublets was studied by Grimus and Neufeld [9].

[c] The vacuum expectation values $v_\alpha(\alpha = 1, \cdots, m)$ and $u$, and all coupling constants as well, are assumed to be real for simplicity throughout.

[d] Here and in the following, we assume $m_D \ll m_R$ so that $m_3 \approx m_D^2/m_R \ll m_4 \approx m_R$. The other limiting case $m_R = 0$ (i.e. the case in which $\omega_3$ and $\omega_4$ fuse to form a Dirac neutrino), a possibility by no means excluded phenomenologically, will not be considered.

[e] Without reanalyzing relevant data in the light of our model, we take $\approx$42GeV as the experimental lower limit on $m_H$.



## References


1. J.J. Simpson, Phys. Rev. Lett. 54 (1985) 1891.
   J.J. Simpson and A. Hime, Phys. Rev. D 39 (1989) 1825.
   A. Hime and J.J. Simpson, Phys. Rev. D 39 (1989) 1837.

2. B. Sur et al., Phys. Rev. Lett. 66 (1991) 2444.
   A. Hime and N.A. Jelley, Phys. Lett. B 257 (1991) 441.
   See also I. Žlimen et al., Phys. Rev. Lett. 67 (1991) 560.

3. S.L. Glashow, Nucl. Phys. 22 (1961) 579.
   S. Weinberg, Phys. Rev. Lett. 19 (1967) 1264.
   A. Salam, in *Elementary Particle Theory*, ed. N. Svarthalm(Almqvist and Wiksells, 1969), p367.

4. S.L. Glashow, Phys. Lett. B 256 (1991) 255.

5. Y. Chikashige, R.N. Mohapatra and R.D. Peccei, Phys. Lett. B 98 (1981) 265.

6. E.W. Kolb and M.S. Turner, Phys. Rev. Lett. 67 (1991) 5.

7. C. Jarlskog, Phys. Lett. B 241 (1990) 579.

8. S.Y. Tsai, lecture given at the Graduate School, Academia Sinica (Beijing), March 25-26, 1991.

9. W. Grimus and H. Neufeld, Nucl. Phys. B 325 (1989) 18.

10. Particle Date Group. Phys. Lett. B 239 (1990).

11. R. Cowsik and J. McClelland. Phys. Rev. Lett. 29 (1972) 669.

12. D.O. Caldwell, Int. J. Mod. Phys. A 4 (1989) 1851.

13. The LEP collaborations, Phys. Lett. B 276 (1992) 247.

14. S.P. Mikheyev and A.Yu. Smirnov. Sov.J. Nucl. Phys. 42 (1986)1441.
    L. Wolfenstein. Phys. Rev. D 17 (1979) 2369.

## Table and figure captions

**Table I.** Ingredients of the model. $I$, $Y$ and $L$ stand for weak isospin, weak hypercharge and lepton number, respectively.

**Fig. 1.** One-loop diagrams contributing to mass and majoron coupling of neutrinos.

**Fig. 2.** Constraints on $m_3$ and $m_4$ derived from a cosmological consideration. The region below the curve (a) and the straight line (b) is allowed. Also shown is the straight line (c) corresponding to $m_3 m_4 = m_\mu^2$.



Table I. Ingredients of the model. $I$, $Y$ and $L$ stand for weak isospin, weak hypercharge and lepton number, respectively.

| | | | | $I$ | $Y$ | $L$ |
|---|---|---|---|---|---|---|
| $L_i$ | $=$ | $\begin{pmatrix} \nu_{iL} \\ e_{iL} \end{pmatrix}$ | $(i = 1, \cdots, n)$ | 1/2 | -1 | 1 |
| $e_{iR}$ | | | $(i = 1, \cdots, n)$ | 0 | -2 | 1 |
| $\nu_R$ | | | | 0 | 0 | 1 |
| $\phi_\alpha$ | $\rightarrow$ | $\begin{pmatrix} \phi_\alpha^+ \\ \sqrt{\frac{1}{2}} v_\alpha + \phi_\alpha^0 \end{pmatrix}$ | $(\alpha = 1, \cdots, m)$ | 1/2 | 1 | 0 |
| $\sigma$ | $\rightarrow$ | $\sqrt{\frac{1}{2}} u + \sigma^0$ | | 0 | 0 | -2 |

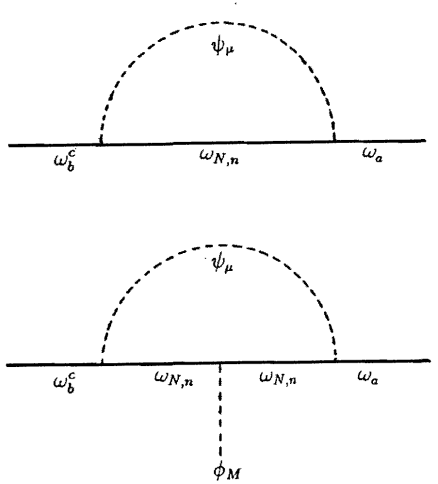


**Fig. 1.** One-loop diagrams contributing to mass and majoron coupling of neutrinos.



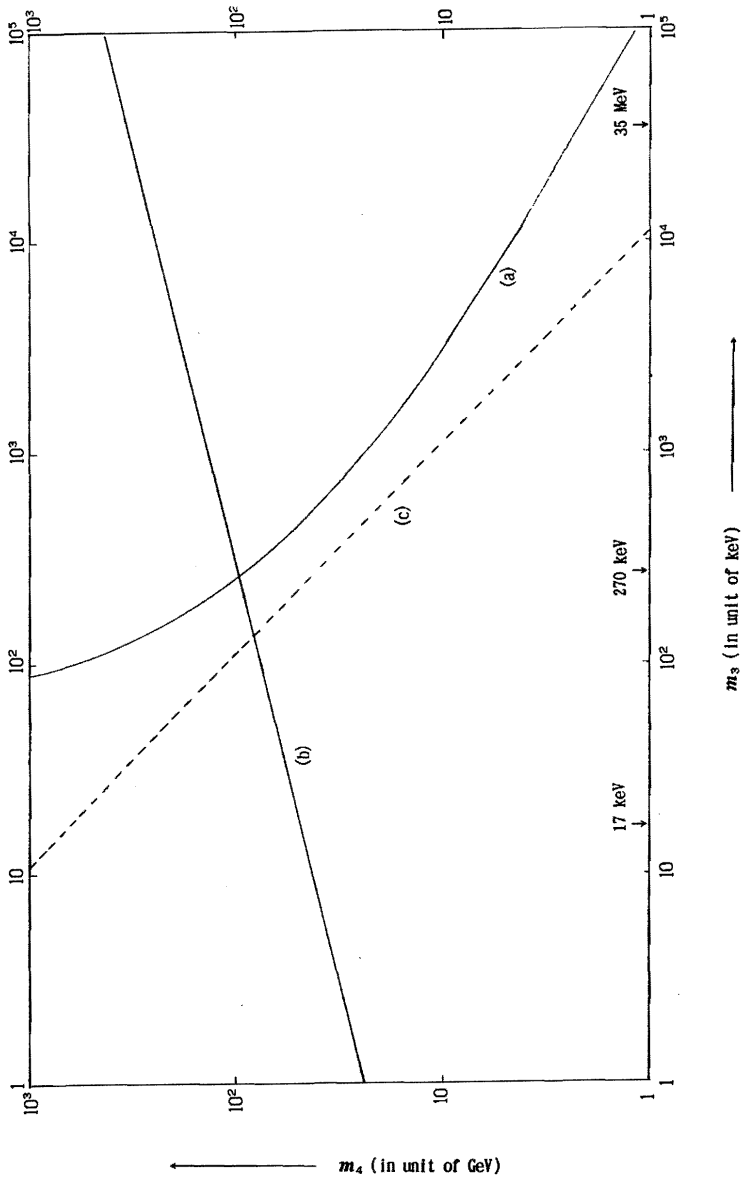


**Fig. 2.** Constraints on $m_3$ and $m_4$ derived from a cosmological consideration. The region below the curve (a) and the straight line (b) is allowed. Also shown is the straight line (c) corresponding to $m_3 m_4 = m_\mu^2$.