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— NTUA - 1 - 89

# HELICITY AMPLITUDES FOR FOUR-JET CROSS SECTIONS IN $e^+e^-$ ANNIHILATION

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We give economical expressions for the helicity amplitudes of the processes $e^+e^- \to q\bar{q}gg$ and $e^+e^- \to q\bar{q}\,q'\bar{q}'$ including $\gamma$ and $Z^0$ contributions and apply them to the evaluation of the four-jet cross section in $e^+e^-$ annihilation at LEP I and LEP II energies.

We present exact helicity amplitudes for the processes $e^+e^- \to V \to q\bar{q}gg, q\bar{q}q'\bar{q}'$ (with both $q=q'$ and $q\neq q'$) using spinor-product techniques[1]. Here V stands for either photon or $Z^0$. For such processes, with many particles in the final state, the spinor-product method is the most tractable and most efficient for numerical calculations of multi jet cross sections.

Similar calculations have been done[2] using standard methods (vector products, Dirac algebra and reduce program), which, however, result in cambersome furmulas. We present our results in a concise form, which can easily be used to calculate the complete four-ject cross section in $e^+e^-$ annihilation.

The tree-level Feynman diagrams contributing to $e^-(p_1)+e^+(p_2) \to q(p_3)+\bar{q}(p_4)+g(p_5)+g(p_6)$ are shown in Fig. (1a). These for $e^-(p_1)+e^+(p_2) \to q(p_3)+\bar{q}(p_4)+q'(p_5)+\bar{q}'(p_6)$ are listed in Fig.(1b), where only the first four contribute when $q\neq q'$, whereas all eight contribute when $q=q'$. The general form of the amplitude in both cases (c=g or c=q) is (I=1,2,...8)

$$i\,T^c_I\,(\lambda,p) = C_a \sum_{V=\gamma,Z} e^2 g_s^2\, D^V(\lambda,p)\, D(p)\, M^c_I(\lambda,p) \quad , \qquad (1)$$

where $\lambda=(\lambda_1\lambda_2;\lambda_3\lambda_4\lambda_5\lambda_6)$ are the helicities of the external particles, $\alpha=(\alpha_3\alpha_4\alpha_5\alpha_6)$ are the color indices of the final particles and $p=(p_1p_2;p_3p_4p_5p_6)$ are the external momenta. $C_a$ is the appropriate color matrix for each diagram and

$$D^{V=\gamma}(\lambda,p) = e_q/s \quad , \quad s=(p_1+p_2)^2 \; , \qquad (2a)$$

$$D^{V=Z}(\lambda,p) = \sin^{-2}\theta_w \cos^{-2}\theta_w\; c^e(\lambda_1)c^q(\lambda_q)/(s-M_Z^2+iM_Z\Gamma_{MZ}), \qquad (2b)$$

where the index q refers to the quark that couples to $\gamma$ or $Z^0$ and the couplings $c^e$ and $c^q$ are given in Table I. Also,

$$D(p) = Q_1^{-2}\ Q_2^{-2} \quad , \tag{2c}$$

where $Q_1, Q_2$ are the momenta of the quark or gluon propagators. Finally, $e_q$ is the charge of the quark coupled to the photon, and $\theta_W$ is the weak mixing angle

Following the techniques developped in ref. 1) we reduce the spinor products in each amplitude $M^c_i(\lambda, p)$ by virtue of the identities

$$\not{p} = u_-(p)\bar{u}_-(p) + u_+(p)\bar{u}_+(p) \quad , \tag{3a}$$

$$[\bar{u}_\lambda(p_i)\gamma^\mu u_\lambda(p_j)]\gamma_\mu = 2u_\lambda(p_j)\bar{u}_\lambda(p_i) + 2u_{-\lambda}(p_i)\bar{u}_{-\lambda}(p_j) \quad . \tag{3b}$$

One further has

$$\bar{u}_+(p_i)u_-(p_j) \equiv s_{ij} = -s_{ji} \quad , \quad \bar{u}_-(p_i)u_+(p_j) \equiv t_{ij} = s_{ji}^* \quad , \tag{4}$$

where the quantity

$$s_{ij} = (p^x_i + ip^y_i)\,[(p^0_j - p^z_j)/(p^0_i - p^z_i)]^{1/2} - (p^x_j + ip^y_j)\,[(p^0_i - p^z_i)/(p^0_j - p^z_j)]^{1/2} \tag{5}$$

is expressed in terms of the components of the four-momenta $p_i, p_j$ ($|s_{ij}|^2 = 2p_i \cdot p_j$).



The antiparticle helicity spinors are eliminated by using $v_\lambda(-p) = u_{-\lambda}(p)$. Defining $\delta^\pm_\lambda = 1$ if $\lambda = \pm$ and $\delta^\pm_\lambda = 0$ if $\lambda = \mp$ , we obtain:

**Case (a): $e^+e^- \to q\bar{q}gg$**

$$M^g_1 = -2(p_6 \cdot p_8 p_5 \cdot p_7)^{-1/2}\, \delta_{\lambda_1\lambda_2}\, \delta_{\lambda_3\lambda_4}$$

$$\times\Big\{\Big[\sum_{j=3,5}\ \sum_{i=4,6}\ \delta^+_{\lambda_3}\,(\delta^-_{\lambda_6}\, s_{46}\, t_{8i} + \delta^+_{\lambda_6}\, s_{48}\, t_{6i})$$

$$\times(\delta^-_{\lambda_1}\, s_{i1} t_{2j} + \delta^+_{\lambda_1}\, s_{i2} t_{1j})$$

$$\times(\delta^-_{\lambda_5}\, s_{j5}\, t_{73} + \delta^+_{\lambda_5}\, s_{j7}\, t_{53})\Big] + [\delta^+ \leftrightarrow \delta^-,\ s \leftrightarrow t]\Big\} \quad , \tag{6a}$$

$$M^g_2 = M^g_1\,(p_5 \leftrightarrow p_6,\ p_7 \leftrightarrow p_8,\ \lambda_5 \leftrightarrow \lambda_6\ ) \quad , \tag{6b}$$

$$M^g_3 = 2(p_6 \cdot p_8 p_5 \cdot p_7)^{-1/2}\, \delta_{\lambda_1\lambda_2}\, \delta_{\lambda_3\lambda_4}$$

$$\times\Big\{\Big[\sum_{j=3,5}\ \sum_{i=3,5,6}\ \delta^+_{\lambda_3}\,(\delta^-_{\lambda_1}\, s_{41} t_{2i} + \delta^+_{\lambda_1}\, s_{42} t_{1i})(\delta^-_{\lambda_6}\, s_{i6} t_{8j} + \delta^+_{\lambda_6}\, s_{i8} t_{6j})$$

$$\times(\delta^-_{\lambda_5}\, s_{j5} t_{73} + \delta^+_{\lambda_5}\, s_{j7} t_{53})\Big] + [\delta^+ \leftrightarrow \delta^-\ ,\ s \leftrightarrow t]\Big\} \quad , \tag{6c}$$

$$M^g_4 = M^g_3(p_5 \leftrightarrow p_6, p_7 \leftrightarrow p_8,\ \lambda_5 \leftrightarrow \lambda_6) \quad , \tag{6d}$$

$$M^g_5 = M^g_3\,(p_3 \leftrightarrow p_4, p_5 \leftrightarrow p_6, p_7 \leftrightarrow p_8,\ \lambda_3 \to -\lambda_3,\ \lambda_4 \to -\lambda_4,\ \lambda_5 \leftrightarrow \lambda_6) \quad , \tag{6e}$$

$$M^g_6 = M^g_5(p_5 \leftrightarrow p_6, p_7 \leftrightarrow p_8,\ \lambda_5 \leftrightarrow \lambda_6) \quad , \tag{6f}$$

$$M^g_7=2\varepsilon_{\lambda5}\cdot\varepsilon_{\lambda6}\Big\{\Big[\sum_{i=3,5,6}\delta^+_{\lambda3}(\delta^-_{\lambda1}s_{4i}t_{2i}+\delta^+_{\lambda1}s_{42}t_{1i})$$

$$(s_{i5}t_{53}-s_{i6}t_{63})\Big]+[\delta^+\leftrightarrow\delta^-,\ s\leftrightarrow t]\Big\}$$

$$+2\Big\{\Big[\sum_{i=3,5,6}\delta^+_{\lambda3}(\delta^-_{\lambda1}s_{4i}t_{2i}+\delta^+_{\lambda1}s_{42}t_{1i})(2p_6\cdot\varepsilon_{\lambda5}/\sqrt{p_8\cdot p_6})$$

$$\times(\delta^-_{\lambda6}s_{i6}t_{83}+\delta^+_{\lambda6}s_{i8}t_{63})$$

$$-2(p_5\cdot\varepsilon_{\lambda6}/\sqrt{p_5\cdot p_7})\ (\delta^-_{\lambda5}s_{i5}t_{73}+\delta^+_{\lambda5}s_{i7}t_{53})\Big]+[\delta^+\leftrightarrow\delta^-,\ s\leftrightarrow t]\Big\}, \tag{6g}$$

$$M^g_8=-M^g_7\,(p_3\leftrightarrow p_4,\ \lambda_3\to-\lambda_3,\ \lambda_4\to-\lambda_4) \quad . \tag{6h}$$

In the above formulas $\varepsilon^\mu_\lambda(p,q)$ is the gluon polarization vector, which is given by

$$\varepsilon^\mu_\lambda(p,q)=u_\lambda(q)\gamma^\mu u_\lambda(p)\ /\ 2\sqrt{p\cdot q} \quad , \tag{7}$$

where p is the gluon four-momentum and q is an arbitrary, auxiliary momentum restricted to be $q\neq p$. Also $p_7,p_8$ are the auxiliary momenta of the polarization vectors of the gluons with momenta $p_5,p_6$ respectively. Explicitly

$$\varepsilon_{\lambda5}\cdot\varepsilon_{\lambda6}=2(p_5\cdot p_7p_6\cdot p_8)^{-1/2}\ ([\delta^+_{\lambda5}(\delta^-_{\lambda6}s_{76}t_{85}+\delta^+_{\lambda6}s_{78}t_{65})]+$$

$$[\delta^+\leftrightarrow\delta^-,\ s\leftrightarrow t]) \quad , \tag{8a}$$

$$p_6\cdot\varepsilon_{\lambda5}=2(p_5\cdot p_7)^{-1/2}(\delta^+_{\lambda5}s_{76}t_{65}+\delta^-_{\lambda5}t_{76}s_{65}) \tag{8b}$$



$$p_5\cdot\varepsilon_{\lambda6}=2(p_6\cdot p_8)^{-1/2}(\delta^+_{\lambda6}s_{85}t_{56}+\delta^-_{\lambda6}t_{85}s_{56}) \tag{8c}$$

**Case (b): $e^+e^-\to q\bar q\,q'\bar q'$**

$$M^q_1=4\delta_{\lambda1\lambda2}\delta_{\lambda3\lambda4}\delta_{\lambda5\lambda6}\Big\{\Big[\sum_{i=3,5,6}\delta^+_{\lambda3}(\delta^-_{\lambda1}s_{4i}t_{2i}+\delta^+_{\lambda1}s_{42}t_{1i})$$

$$(\delta^-_{\lambda5}s_{i5}t_{63}+\delta^+_{\lambda5}s_{i6}t_{53})\Big]+[\delta^+\leftrightarrow\delta^-,\ s\leftrightarrow t]\Big\} \quad , \tag{9a}$$

$$M^q_2=-M^q_1(p_3\leftrightarrow p_4,\ \lambda_3\to-\lambda_3,\lambda_4\to-\lambda_4) \quad , \tag{9b}$$

$$M^q_3=M^q_1(p_3\leftrightarrow p_5,p_4\leftrightarrow p_6,\lambda_3\leftrightarrow\lambda_5,\lambda_4\leftrightarrow\lambda_6) \quad , \tag{9c}$$

$$M^q_4=-M^q_3(p_5\leftrightarrow p_6,\ \lambda_5\to-\lambda_5,\ \lambda_6\to-\lambda_6) \quad . \tag{9d}$$

These are all the amplitudes for $q\neq q'$. If $q=q'$, in addition to the above four amplitudes, we have four more, which are given by

$$M^q_{i+4}=M^q_i\ (p_3\leftrightarrow p_5,\ \lambda_3\leftrightarrow\lambda_5)\ ,\quad i=1,2,3,4. \tag{10}$$

With the above explicit expressions we can calculate the averaged squared amplitudes

$$\overline{|m|^2}_{q\bar q gg}=\langle 1/8\rangle\sum_{(\lambda_i)}\sum_{i,j=1}^{8}T^g_i\ T^{g*}_j\ C^g_{ij} \quad , \tag{11a}$$

$$\overline{|m|^2}_{q\bar q q'\bar q'}=\langle 1/4\rangle\sum_{(\lambda_i)}\sum_{i,j=1}^{4}T^q_i\ T^{q*}_j\ C^q_{ij}\ ,\ (q\neq q'), \tag{11b}$$

$$\overline{|m|^2}_{q\bar{q}q\bar{q}} = \langle 1/16 \rangle \sum_{(\lambda_i)} \sum_{i,j=1}^{8} T^q_i \, T^{q*}_j \, C^q_{ij} \quad . \tag{11c}$$

The physical four-jet cross section is obtained from the above expressions by summing over quark flavors, and using the expressions for the color matrices $C^q$ and $C^q$ given in Table II. In programming these formulas the sum over helicity configurations need not exhaust the whole set since $T^c_i(\lambda,p)=T^c_i{}^*(-\lambda,p)$ due to parity conservation. Note also that a considerable simplification results from the Kronecker functions $\delta_{\lambda i \lambda j}$, which come from the fact that the helicity along a massless fermion line does not change.

The fact that an auxiliary arbitrary momentum enters the expression for $\varepsilon^\mu_\lambda$ (see Eq.7) provides us with a powerful check of the whole numerical procedure. Namely, the results should be independent of the particular choice for the auxiliary momentum q ($p_7$ and $p_8$ in our formulas). We have checked that this is the case with several choices for $p_7$ and $p_8$ ($p_7=p_3,p_6$ and $p_8=p_4,p_5$). This is a consequence of gauge invariance. Moreover, we checked that in the purely electromagnetic case ($V=\gamma$) our calculation and that of Ali et al[2] give the same results for $e^+e^- \to q\bar{q}gg$.

As an application of our matrix elements we calculate the azimuthal-angle $\varphi$ distribution of four-jet events at LEP I and LEP II energies, where $\varphi$ is the angle of the planes formed by jets 1,2 and 3,4 respectively - See Fig.2. The jets are here numbered according to their total energy and these two planes are



conveniently reconstructed experimentally. To ensure that all four jets are detectable at LEP we apply the cuts $\eta_i < \eta_0$ ($\eta_0=3$) and $m_{ij}^2 > y_c s$ ($y_c=0.02$) [3], where $m_{ij}$ is the invariant mass of any pair of jets, i,j=1,2,3,4, and $\eta_i$ is the rapidity of jet i. Note that this distribution is expected [4], to be significantly deformed, due to the presence of a non-abelian gluon. The contributions of the various subprocesses to the total cross section are given in Table III.

Finally, in Fig. 3 we present the $\chi$-distribution of the fastest jet, where $\chi=(1+\cos\vartheta)/(1-\cos\vartheta)$ applying the same cuts as above, at LEP I and LEP II energies. This distribution is expected to be deformed by the presence of new contributions, leading to relatively more sperical multi jet configurations, such as those arising from (non-minimal) Higgs production or from a heavy diquark system or even from composite structures.

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## TABLE AND FIGURE CAPTIONS

**Table I** Helicity structure of the couplings of the $Z^0$ (eq.2b) to the electron and the "up" - and "down" - quark families.

**Talbe II** Products of colour matrices required for the construction of the averaged squared amplitudes (11).

**Table III** Absolute contributions of the various final states to the cross section for $e^+e^- \to 4$ jets (with the cuts defined in the text) at LEP I and LEP II. $M_Z$=92.6 GeV, $\Gamma_Z$=2.55 GeV, $\sin^2\vartheta_W$= 0.23.

**Fig. 1** (a) The eight diagrams contributing to $V \to q(p_3)\bar{q}(p_4)g(p_5)g(p_6)$

(b) The eight diagrams contributing to $V \to q(p_3)\bar{q}(p_4)q'(p_5)\bar{q}'(p_6)$

**Fig.2** Azimuthal-angle $\varphi$ distribution of 4-jet events

**Fig.3** Angular $\chi$-distribution of the leading jet for 4-jet events.

TABLE I

| | $c^f(+)$ | $c^f(-)$ |
|---|---|---|
| $f = e$ | $-1/2+\sin^2\theta_W$ | $\sin^2\theta_W$ |
| $f = u_i$ | $1/2-2/3\sin^2\theta_W$ | $-2/3\sin^2\theta_W$ |
| $f = d_i$ | $-1/2+1/3\sin^2\theta_W$ | $1/3\sin^2\theta_W$ |



TABLE II

$$C^q_{IJ} = \begin{vmatrix} \alpha & \beta & \alpha & \beta & \alpha & \beta & \gamma & \gamma \\ & \alpha & \beta & \alpha & \beta & \alpha & -\gamma & -\gamma \\ & & \alpha & \beta & \alpha & \beta & \gamma & \gamma \\ & & & \alpha & \beta & \alpha & -\gamma & -\gamma \\ & & & & \alpha & \beta & \gamma & \gamma \\ & & & & & \alpha & -\gamma & -\gamma \\ & & & & & & \delta & \delta \\ & & & & & & & \delta \end{vmatrix}$$

$\alpha=16/3$, $\beta=-2/3$, $\gamma=6$, $\delta=12$

$$C^q_{IJ} = \begin{vmatrix} \varepsilon & \varepsilon & \varepsilon & \varepsilon & \zeta & \zeta & \zeta & \zeta \\ & \varepsilon & \varepsilon & \varepsilon & \zeta & \zeta & \zeta & \zeta \\ & & \varepsilon & \varepsilon & \zeta & \zeta & \zeta & \zeta \\ & & & \varepsilon & \zeta & \zeta & \zeta & \zeta \\ & & & & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ & & & & & \varepsilon & \varepsilon & \varepsilon \\ & & & & & & \varepsilon & \varepsilon \\ & & & & & & & \varepsilon \end{vmatrix}$$

$\varepsilon=2$, $\zeta=-2/3$

TABLE III

| | Final state | $Q=M_Z$ σ (nb) | $Q=200$ GeV σ (pb) |
|---|---|---|---|
| 1. | $u_iu_igg$ | 0.700 | 0.424 |
| 2. | $d_id_igg$ | 0.923 | 0.247 |
| 3. | $u_iu_id_jd_j$ | 0.127 | 0.0414 |
| 4. | $u_iu_iu_ju_j$  $i\neq j$ | 0.0290 | 0.0124 |
| 5. | $u_iu_iu_iu_i$ | 0.00940 | 0.00398 |
| 6. | $d_id_id_jd_j$  $i\neq j$ | 0.0342 | 0.00830 |
| 7. | $d_id_id_id_i$ | 0.0110 | 0.00267 |
| | **total** | 1.83 | 0.740 |

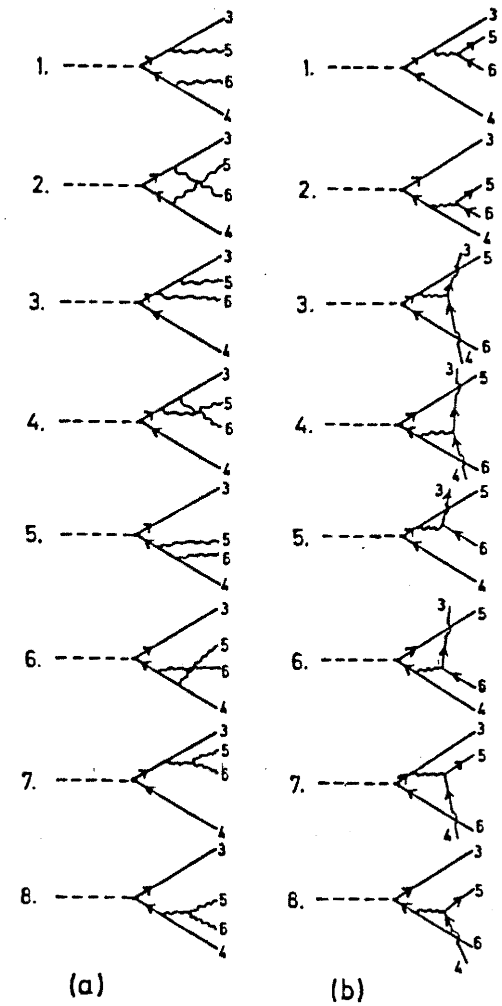
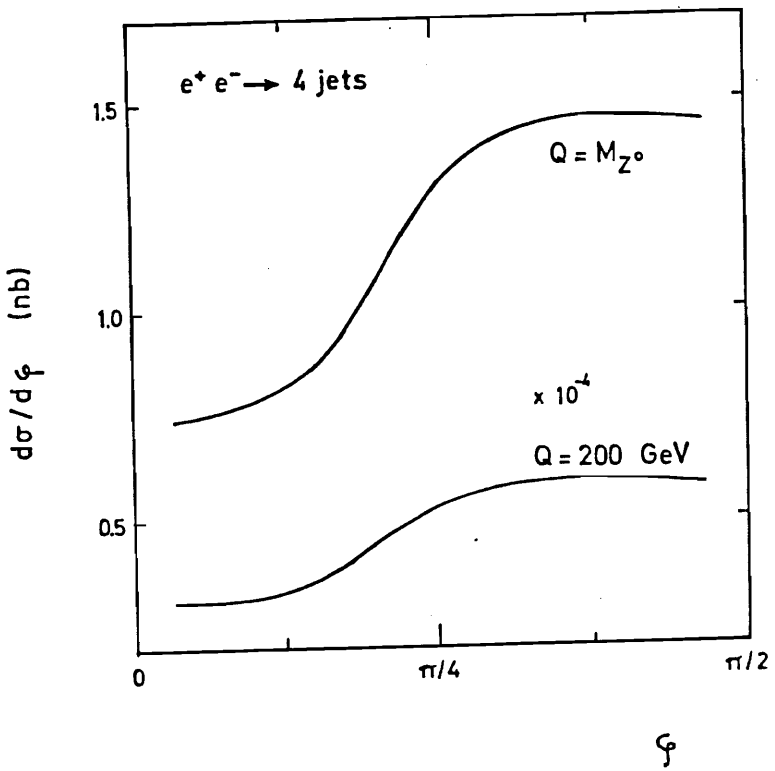
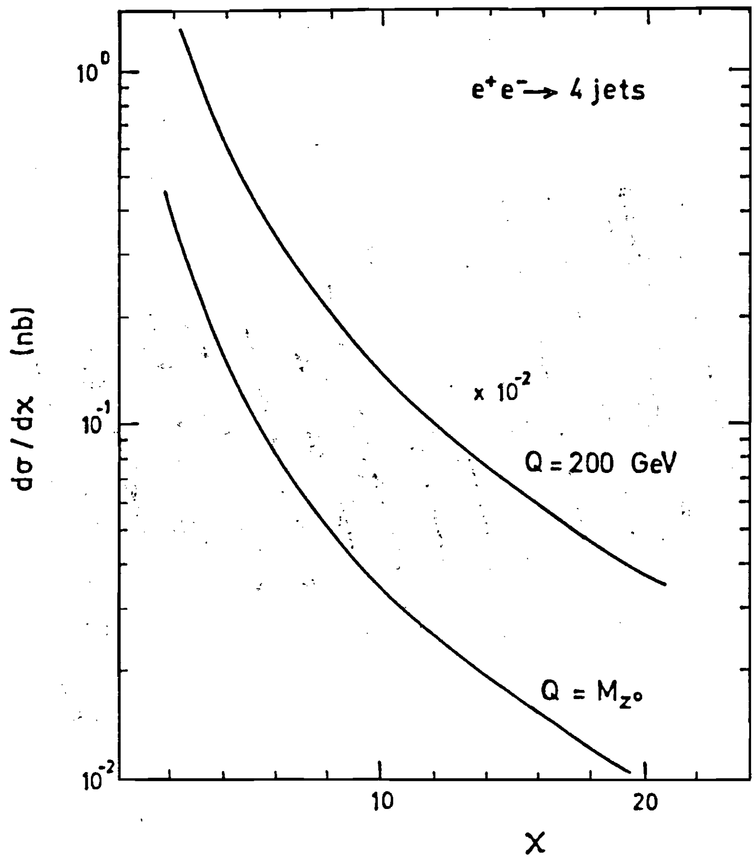



Fig. 1



e⁺e⁻ → 4 jets
1.5
1.0
0.5
0
π/4
π/2
dσ/dφ (nb)
φ
Q = M_Z°
x 10⁻⁴
Q = 200 GeV


Fig. 2



e⁺e⁻ → 4 jets
10⁰
10⁻¹
10⁻²
10
20
X
dσ/dx (nb)
x 10⁻²
Q = 200 GeV
Q = M_Z°


Fig. 3