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Quantum Phase in Action*

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We are gathered here to pay homage to the quantum phase. Out of classical nothingness something quantum emerges.

One of the deepest mysteries in physics is the existence of two kinds of particles, bosons and fermions. We now know that in $2+1$ dimensional spacetime there are also anyons, such that when two anyons are exchanged, the wave function acquires a phase. In particular, when two semions are exchanged, the wave function changes by a factor of i. $2+1$ dimensional spacetime is not just less of a good thing compared to $3+1$ dimensional spacetime. It is homotopically different: a new physical concept, that of "going around", appears. It makes sense to say that a particle has gone around another. This basic fact is what makes the notion of anyons and fractional statistics possible.

Shortly after Wilczek, and earlier, Leinaas and Myrheim, proposed the existence of anyons, the question naturally arose as to whether these hypothetical particles can be incorporated into quantum field theory. The answer is yes, and the concept of gauge potential enters naturally. One simply couples a gauge potential to a conserved current of interest, and have the dynamics of the gauge potential governed by the Chern-Simons term.[1] In Maxwell dynamics, the spactime derivatives of the gauge field are related to the current. In Chern-Simons dynamics, the gauge field is directly related to the current. Life is simpler because one doesn't have to solve any partial differential equations. This is possible in $2+1$ spactime. In any dimensions, the current is of course a vector, the gauge field an antisymmetric tensor, but in $2+1$ dimensions, an antisymmetric tensor is also a vector, thanks to the Levi-Civita antisymmetric symbol.

This means that a charged particle would have a magnetic flux attached to it. Here the terms electric charge and magnetic flux refer of course to the quantities associated with the gauge potential we have introduced and not to the quantities studied by Coulomb, Faraday, Oersted and their friends. Long ago, Aharonov and Bohm told us that when a charged particle goes around a flux tube, the wave function acquires a phase. Thus if we have particles carrying both charge and flux, then when one such particle goes around another, the wave function acquires a phase. Fractional statistics is just a slice of the Aharonov-Bohm effect. Thus, two of the greatest names in physics meet two of the greatest names in mathematics.

In hindsight, this connection between Aharonov-Bohm and Chern-Simons appears so natural and so obvious that some workers in this field now think that it was known since the beginning of time. In fact, this connection only became clear in the fall and winter of 1983.

Over the last ten years, there have been many interesting applications using this formalism. Here I would like to talk about a recent discussion of tunnelling effect in double layered Hall systems.[2]

In this formalism, in the quantum Hall effect electrons are coupled to gauge potentials obeying Chern-Simons dynamics. As explained above, the electrons then carry magnetic flux. In a special state in the double-layered quantum Hall system (technically this corresponds to a certain matrix having a zero eigenvalue so that one of the gauge potential is liberated from being governed by Chern-Simons dynamics), the electrons in layer 2 act like flux tubes carrying flux -2π to the electrons in layer 1. Thus, an electron in layer 1 does not see the magnetic field imposed by the experimentalist, but an effective magnetic field equal to the magnetic field imposed

by the experimentalsit minus 2π times the local density of electrons in layer 2. Now consider a long wavelength density wave such that as the density of electrons in layer 1 goes up the density of electrons in layer 2 goes down correspondingly. But then the effective magnetic field seen by the electrons in layer 1 also goes up. Thus, things can be arranged to work out very neatly. Even as the density of electrons in layer 1 goes up and down, those electrons can be made to believe that they are still just filling the first Landau level, not one too many, not one too few. Similarly, the electrons in layer 2 are also living under the illusion that they are filling just the first Landau level. Thus, as the wavelength of the density fluctuation goes to infinity, the energy cost of the fluctuation goes to zero. This is the physics behind the appearance of a gapless mode: the gaplessness is a consequence of an exquisitely balanced cooperation between the electrons in layer 1 and layer 2. The same physics is in fact responsible for anyon superfluidity. Technically, the gauge field liberated from being governed by Chern-Simons dynamics is now happily massless and governed by Maxwell dynamics.

The appearance of a gapless mode is consistent with symmetry considerations. In the absence of tunnelling, there are two separate $U(1)$ symmetries, corresponding to the conservation of the sum and difference of the electron numbers in the layers. In the special state described above, the $U(1)$ corresponding to the conservation of the difference of the electron numbers in the two layers is spontaneously broken and thus we expect a Nambu- Goldstone gapless mode.

Tunnelling, that is, interlayer hopping, corresponds to the explicit breaking of this $U(1)$ symmetry and thus according to general considerations, the Nambu-Goldstone boson becomes pseudo and acquires mass.

In the present formalism, the current describing the difference of the currents in the two layers is written as a curl of a gauge potential. When an electron tunnels from one layer to the other, this current is no longer conserved. When the divergence of the curl of a gauge potential does not vanish, we know that there is a magnetic monopole lurking in the vicinity. The spacetime integral of the magnetic flux coming out of the monopole is the spacetime integral of the divergence of the current, and hence the change in the difference of numbers of electrons in the two layers, equal to ± 2 in the tunnelling event. Thus, the monopole in our formalism is quantized a la Dirac because electrons are discrete.

Dirac quantization of magnetic monopoles represents of course another manifestation of the Aharonov-Bohm effect. Dirac obtained magnetic quantization by requiring that the Aharono-Bohm phase acquired by a particle going around his string vanishes. Indeed, Coleman explains Dirac quantization by arguing in reverse. He describes a prankster trying to trick an experimentalist into believing that he or she has found the fabled magnetic monopole. The prankster introduces an arbitarily thin flux tube into the lab. The experimentalist can detect the flux tube by letting a charged particle move around and measure the resulting Aharonov-Bohm phase. It is precisely when the flux going through the tube is such that the monopole has the Dirac magnetic charge that the flux tube becomes undetectable. The experimentalist can then become very excited and proclaim the discovery of the magnetic monopole.

Thus, we have a Euclidean 3-space filled with a plasma of magnetic monopoles and anti-monopoles. Wherever there is a monopole, an electron tunnels from layer

1 to layer 2 at the corresponding point in spacetime. Whereever there is an anti-monopole, an electron tunnels back from layer 2 to layer 1. Now we get to re-live the golden days of quantum field theories. One of the most celebrated results of the 1970's was the realization by Polyakov that in the presence of a dilute plasma of magnetic monopoles the photon acquires a mass.

This is completely consistent with our expectation from sysmmetry considerations. To summarize, we have the following "life story" of a gauge quantum. When it was governed by Chern-Simons dynamics, it was massive. After being liberated into a life of Maxwell dynamics, it becomes massless. But then non-perturbative tunnelling effects made it massive again, Technically, the plasma of monopoles is a Coulomb gas, and a Coulomb gas can be represented by a sine-Gordon theory. Expanding the cosine in the Lagrangian to quadratic order, one sees immediately that the sine-Gordon field is massive.

For his purposes Polyakov did not have to exploit the fact that the sine-Gordon field is in fact an angular order parameter. But we know that there is very interesting physics associated with angular order parameters! Incidentally, the order parameter is angular precisely because the magnetic monopole is quantized by Dirac. Wen and I are thus led to make the perhaps a priori rather surprising prediction that when a DC voltage V is applied across a double-layered Hall system, for certain special filling factors, there is an oscillating tunnelling current. In a word, there is a superfluid lurking in the system and hence there is Josephson-like current. Note however that the frequency is only half of the Josephson frequency because we don't have pairing here. We may entertain the hope that this effect will be experimentally detectable in the near future.

Perhaps even more interestingly, with an arrangement of three double- layered Hall systems, it may be possible to observe, in a phenomenon we call "sideways tunnelling," tunnelling currents which oscillate at a frequency equal to an odd integer fraction of the Josephson frequency.[3]

I hope to have conveyed the impression that the circle of theoretical ideas appearing in this subject are among the deepest in theoretical physics.

We encounter here quantum statistics, homotopic property of space, gauge potential, Chern-Simons and Maxwell dynamics, Aharonov-Bohm phase, Dirac quantization of magnetic monopole, quantum tunnelling, Nambu-Goldstone bosons, cooperative density and flux fluctuation and anyon superconductivity, discreteness of the electron, Coulomb gas, angular order parameter, and Josephson ocsillation. In the end, we can attribute all these strikingly beautiful notions to the fact that when we move from classical physics to quantum physics the complex number mysteriously appears on the scene.

With your indulgence, I will end by entertaining a speculation, in fact the same speculation[4] I made here in South Carolina a few years ago at another conference celebrating the Aharonov-Bohm effect. The appearance of statistics in quantum physics is one of the deepest mysteries in physics and in some ways is responsible for the current difficulties in particle theory. As Weisskopf discovered ages ago, fermions are nice and bosons are nasty. The self energy of a boson diverges quadratically. It is partly to cure this problem that supersymmetry was invented, to solve the so-called naturalness problem. We all know down what glorious paths supersymmetry has taken particle physics: from supersymmetry to supergravity to superstrings

to supermathematics to superphysicists. Might it not be possible that quantum statistics is a composite notion? In the end, there are only fermions (or perhaps, only bosons.) After all, we know that a bound state of a boson and a magnetic monopole is a fermion (and vice versa.) There is an additional phase when two such bound states are interchanged. Indeed, it is possible to obtain reasonable quantum numbers for the observed quarks and leptons.[5] Some of the theoretical ideas I listed above are so deep that they ought to have further consequences for particle physics as well as for condensed matter physics.

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