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Abstract

We have obtained a new approximate general formula for radiative cooling of the accretion disk which is valid for all optical depths τ, including τ of the order of unity. Using this formula and following Björnsson and Svensson (1991) for determination of the equilibrium structure of the geometrically thin α-disk, we analyze the general global disk solutions for fixed viscosity parameter α. We have determined the values of the maximal possible rate of accretion, at which there are formal global solutions, for various values of the viscosity parameter α.

1.Introduction

It is well known that disk accretion plays a crucial role in many astrophysical objects and phenomena. Linden-Bell (1969) was the first who proposed the model of gaseous disk accretion onto black holes as a source of energy of quasars and active galactic nuclei (AGN). Shakura (1972), Pringle and Rees (1972), Shakura and Sunyaev (1973) have built the Newtonian models of accretion disk. Later Novikov and Thorne (1973), Page and Thorne (1974), gave the theory of disk accretion in the framework of General Relativity. This theory is important for the structure of innermost parts of the disk.

In the subsequent period a lot of works were devoted to this problem. In the most early papers the emphasis has mainly been on the existence and analysis of the local disk solutions at a given radius r. Of greater importance is whether or not these local solutions at different radii connect smoothly to form physically self-consistent global disk solution that is continuous from large radii to the inner disk edge.

It is known that for a given values of the accretion rate $\dot{M}$ and viscosity parameter α, there are two different solutions for a steady state disk structure equations at the given radius r. One of them corresponds to optically thick, low-temperature ($T \sim 10^5 \div 10^7$K) state. Another solution corresponds to the optically thin, high-temperature ($T \sim 10^9$K) state.

The physics and properties of these solutions, stability for example, are quite different. The formulae for radiation flux and radiation pressure conventionally used are valid only in limiting cases either for the optically thick region, or for the optically thin one, but not for the intermediate zone. So far only a few papers with the discussion of the intermediate zone were published (Maraschi and Molendi (1990), Liang and Wandel (1991), Wandel and Liang (1991), Lasota and Pelat (1991), Kusunose and Mineshige (1992), Luo and Liang (1994)).

Here we study the disk behavior at all τ, including $\tau_{eff} \approx 1$, and derive a new approximate formula for the radiative flux which is valid for all optical depths.

Using this formula we follow Björnsson and Svensson (1991) for determination of the equilibrium structure of the geometrically thin disk and analyse the general topology of the family of the global disk structure solutions for a fixed viscosity parameter α. Each curve corresponds to a given rate of accretion $\dot{M}$. From this analyses it is clearly seen the bifurcation of the solutions into different sub-families. In order to specify the main behavior of global solutions we analyze the simplest accretion disk model with the pure bremsstrahlung proton-electron release of the radiation energy $\dot{E}$.

In section 2 we obtain an approximate formula for radiative flux, valid for all τ. In the section 3 we present the disk structure equations. In the section 4 we discuss the results of numerical solutions of the disk structure equations.

2.General formula for radiative flux

For obtaining the formula for the vertical radiative flux F_{rad} in the disk and radiative pressure P_{rad} for any optical depth, we consider the Eddington approximation in a grey, isothermal, scattering medium in LTE conditions. The first momentum of the equation of transfer for plane-parallel geometry gives

$$\frac{dF_{rad}}{dz} = -\rho\dot{E}\left(\frac{S}{aT^4} - 1\right), \tag{1}$$

where z is a vertical coordinate ($z = 0$ at the symmetry plane of the disk), ρ is the matter density, S is the energy density, $\dot{E}$ is the source function. Using the mean value ρ one can write down the following approximate formula for radiation flux:

$$F_{rad} = 2\frac{F_0}{\Sigma_0}\rho z, \tag{2}$$

where F_0 is the surface value of F_{rad}, $F_0 = F_{rad}|_{z=h}$, h is the half height of the disk, $\Sigma_0 = 2\rho h$ is the surface density. Using (1) in (2) we get the expressions for S and P_{rad} in Eddington approximation

$$3P_{rad} = S = aT^4\left(1 - \frac{F_0}{\dot{E}\rho h}\right), \tag{3}$$

From the second momentum of the transfer equation we get

$$c\frac{dP}{dz} = -\kappa\rho F_{rad} = -2\frac{F_0}{\Sigma_0}\rho\kappa\rho z, \tag{4}$$

where $\kappa = \frac{\sigma_t}{m_p}$, σ_t is the Tomson's scattering depth, c is the speed of light. One can write eq (6) in the following form ($\tau = \int_z^\infty \kappa\rho\, dz$, ρ is taken constant)

$$c\frac{dP}{d\tau} = F_{rad} = 2\frac{F_0}{\kappa\Sigma_0}\tau. \tag{5}$$

We solve (5) with the following boundary condition:

$$F_{rad}|_{\tau=0} = F_0 = \frac{cS}{2} = \frac{3cP_{rad}}{2}.$$

what gives

$$cP_{rad} = \frac{cS}{3} = \frac{2F_0}{3} + \frac{F_0\tau^2}{\kappa\Sigma_0} = F_0(\frac{2}{3} + \frac{\tau^2}{2\tau_0}), \tau_0 = \kappa\rho h. \tag{6}$$

Excluding S from (5) and (6), we obtain the final expression for the temperature distribution over the thickness of the disc

$$aT^4 = \frac{3}{2}\frac{F_0}{c}\left(\frac{\tau^2}{\tau_0} + \frac{4}{3} + \frac{2}{3\tau_\alpha}\right). \tag{7}$$

Here we have introduced the optical depth to the absorption $\tau_f \ll \tau$,

$$\tau_f = \frac{\dot{E}}{aT_0^4 c}\rho z\ , \tau_\alpha = \frac{\dot{E}}{aT_0^4 c}\rho h,$$

where T_0 is the temperature in the plane of the disc. Introducing the effective optical depth:

$$\tau_* = (\tau_0\tau_\alpha)^{1/2}.$$

We get from (7) the connection betwen the flux F_0 and T_0

$$F_0 = \frac{2aT_0^4 c}{3\tau_0}\left(1 + \frac{4}{3\tau_0} + \frac{2}{3\tau_*^2}\right)^{-1}. \tag{8}$$

The formula (8) has the following properties:
1) In the optically thick limit $\tau_0 \gg 1$, $\tau_* \gg 1$ we have the diffusion limit

$$F_0 = \frac{2aT^4 c}{3\kappa\rho h},$$

2) In the effective optically thin case $\tau_* \ll 1$, any $\tau_0 \gg \tau_*$ we have

$$F_0 = \dot{E}\rho h.$$

Thus we can use the formula (8) for any optical depth.

3.Disk structure equations

Now we can investigate the geometrically thin disk structure without any restriction on the optical depth. To the equation (8) we add the following disk structure equations: the total pressure:

$$P_{tot} = P_{gas} + P_{rad}, \tag{9}$$

the equation for the gas pressure:

$$P_{gas} = \frac{m_e c^2}{r_g\sigma_t}\overline{n}_p\left(\theta_p + \theta_e\right), \tag{10}$$

where m_e is the mass of the electron, $r_g = \frac{GM}{c^2}$, $\overline{n}_p$ is the dimension-less proton concentration

$$\overline{n}_p = n_p r_g \sigma_t.$$

Here θ_p and θ_e are the dimensionless proton and electron temperatures

$$\theta_p = \frac{T_p}{m_e c^2},\ \theta_e = \frac{T_e}{m_e c^2}$$

The equation for the radiation pressure with account of (6),(8) is

$$P_{rad} = \frac{a}{3}\frac{m_e c^2}{r_g\sigma_t}\theta_e^4\left(1 + \frac{4}{3\tau_0}\right)\left(1 + \frac{4}{3\tau_0} + \frac{2}{3\tau_0\tau_\alpha}\right)^{-1}, \tag{11}$$

Hydrostatic equilibrium in the vertical direction is described by the equation

$$P_{tot} = \frac{m_p c^2}{r_g\sigma_t}\overline{n}_p\frac{h}{r}r_*^{-1}, \tag{12}$$

where $r_* = \frac{r}{r_g}$ is the dimensionless radius. Conservation of the angular momentum is presented by the equation

$$\alpha P_{tot} = \frac{m_p c^2}{r_g\sigma_t}\dot{m}\left(\frac{h}{r}\right)^{-1} r_*^{-2.5}J(r_*), \tag{13}$$

where $J(r_*) = 1 - \left(\frac{6r_*}{r}\right)^{1/2}$ and $\dot{m}$ is the dimensionless accretion rate $\dot{m} = \frac{\dot{M}c^2}{L_{edd}}$; $\dot{M}$ is the accretion rate, L_{edd} is the Eddington luminosity. The rate of release of the gravitational energy per unit volume is

$$\dot{E}_+ = \frac{3m_pc^2}{2r_g\sigma_t}\dot{m}\left(\frac{h}{r}\right)^{-1} r_*^{-4}J(r_*). \tag{14}$$

The radiative cooling rate is

$$\dot{E}_{rad} = \frac{3\alpha_f m_e c^2}{8\pi r_g^2\sigma_t}\bar{n}_p^2 F_{br}, \tag{15}$$

where $\alpha_f = \frac{1}{137}$ is the constant of thin structure, F_{br} is the dimensionless radiation rate due to bremsstrahlung. The last two relations are the energy balance equation

$$\dot{E}_+ = \left(\frac{F_0}{h}\right). \tag{16}$$

and the expression for free-free optical depth

$$\tau_\alpha = \frac{r_g}{ac}\left(\frac{k}{m_ec^2}\right)^4\left(\frac{h}{r}\right) r_*\dot{E}_{rad}\theta_e^{-4}, \tag{17}$$

with the Boltzman's constant k

4.Results

We have obtained the solutions to the system of equations discussed above numerically using the technique described in Björnsson and Svensson (1991). In the Figure (1) we plot the dependences of the optical depth τ_0 (which is proportional to the surface density Σ_o) on the radius r_* for the case $M_{BH} = 10^8 M_\odot$, $\alpha = 1.0$ and various fixed $\dot{m}$. One can clearly see the bifurcation of the global solutions into two (optically thick and optically thin) families.

For the rate $\dot{m} \leq 9.39$ there are two families of the global solutions which are continuous from the large radii to the inner disk edge. At $\dot{m} \approx 9.39$ two global solutions cross each other at $r_* = 10.67$ and for this case we observe the transmutation of the optically thick solutions into optically thin and vise verse. For $\dot{m} > 9.39$ and $\alpha = 1, M_{BH} = 10^8 M_\odot$ there are no global solutions.
For each fixed $\dot{m} > 9.39$ ther are two formal branches of solutions which both start and end either at large radii or at the inner disk edge but without any solutions for the values r_* in some region around $r_* = 32/3$. For these formal solutions at the turning points the gradient of all physical values (as functions of r_*) tends to infinity and physical restrictions for correctness the used equations are violated.

The value $\dot{m} \equiv \dot{m}_{max} = 9.39$ is the maximum possible value of the rate of accretion $\dot{m}$ when there are global solutions for the disk structure. Note that the point of bifurcation $r_* = 32/3$ corresponds to the place of the maximum realize of the gravitational energy in the disk. In the Figure (2) we have plotted the dependence of $\dot{m}_{max}$ on the value of the parameter α. It is obviously that $\dot{m}_{max}$ can be both greater and smaller then the value of $\dot{m}$ which corresponds to the Eddington luminosity. The dependence of critical value $\dot{m}_{max}$ on the mass of a Black Hole is weak and will be represented elsewere. Let us stress that our results reflect the formal properties of the accretion disc equations in Sec.(3), which has physical meaning only at $\frac{\dot{m}}{\varepsilon} < 1$ for global solution, or $r_* > \frac{2\dot{m}}{\varepsilon}$ for $\frac{\dot{m}}{\varepsilon} > 1$. Formally at $\frac{\dot{m}}{\varepsilon} > 1$ the ratio h/r is becoming large in the inner parts of the disc and the disc approximation fails. Here ε is the efficiency of accretion $\varepsilon = \frac{L}{Mc^2}$, $\varepsilon \approx 0.1$. That indicates to the begining of the intensive mass loss from the inner parts of the disc in combination with the disc accretion outside (G.S. Bisnovatyi-Kogan and S.I. Blinnikov, 1977).
We do not discuss here the problems of stability of the solutions. In the subsequent papers we shall discuss the same problems for more complicated physics of the disk including the Comptonization effect, two temperature, pair creation and advection.

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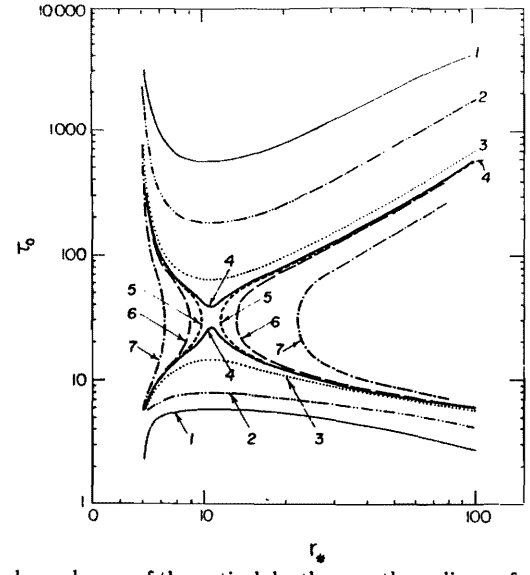

Figure 1: The dependences of the optical depth τ_0 on the radius r_* for the case $M_{BH} = 10^8 M_\odot$, $\alpha = 1.0$ and various fixed $\dot{m}$. Lines corresponde: $1 - \dot{m} = 1.0$, $2 - \dot{m} = 3.0$, $3 - \dot{m} = 8.0$, $4 - \dot{m} = 9.35$, $5 - \dot{m} = 10.0$, $6 - \dot{m} = 11.0$, $7 - \dot{m} = 15.0$

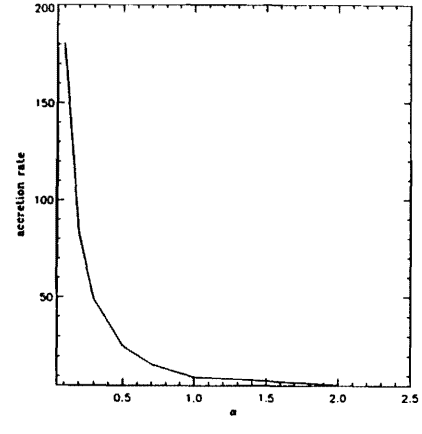

Figure 2: The dependence of $\dot{m}_{max}$ on the value of the parameter α.