A Measurement of the Gross-Llewellyn Smith Sum Rule
from the CCFR xF3 Structure Function

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We report a measurement of the Gross-Llewellyn Smith Sum Rule:
\[
\int \frac{dx}{x} x F_3(x, Q^2 = 3 \text{ GeV}^2) = 2.50 \pm 0.018 \text{ (stat)} \pm 0.078 \text{ (syst)}.
\]

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The Gross-Llewellyn Smith (GLS) Sum Rule[1] predicts that the integral of \( xF_3 \), weighted by \( 1/x \), equals the number of valence quarks inside a nucleon — three in the naive quark parton model. With next to leading order QCD corrections, the GLS sum rule can be written as

\[
S_{\text{GLS}} = \int_0^1 \frac{dx}{x} x F_3(x, Q^2) = 3 \left[ 1 - \frac{12}{(33 - 2N_f) \ln(Q^2/\Lambda^2)} + O(Q^{-2}) \right], \tag{1}
\]

where \( N_f \) is the number of quark flavors (=4) and \( \Lambda \) is the mass parameter of QCD. Higher twist effects, of the order \( O(Q^{-2}) \), are expected to be small (< 1% of \( S_{\text{GLS}} \) at \( x \approx 0.01 \)).[2] Until now, the most precise measurement of the GLS Sum Rule has come from the Narrow Band Beam (NBB) neutrino data of the CCFR collaboration[3]. The factor of 18 increase in the \( \nu \)-induced charged current (CC) sample of the new data, compared to our earlier experiment, provides a much more precise determination of \( xF_3 \), and an improved measurement of \( S_{\text{GLS}} \). In an accompanying letter, we have reported a high statistics determination of \( F_2(x, Q^2) \) and \( xF_3(x, Q^2) \).[4]

Due to the \( 1/x \) weighting in Eq.1, the small \( x \) region (\( x < 0.1 \)) is particularly important. Accurate measurements of the following ensure small systematic errors:
(a) the muon angle \( (\theta_\mu) \)[5] and (b) the relative \( \nu/\bar{\nu} \) flux. Since \( x F_3 \) is obtained from
the difference of \( \nu \) and \( \bar{\nu} \) cross-sections, small relative normalization errors can become magnified by the weighting in the integral. The absolute normalization uses an average of \( \nu-N \) cross-section measurements.\[^4\] Here we describe procedures for obtaining the relative flux: ratios of neutrino flux from energy to energy, and between \( \nu_\mu \) and \( \nu_\mu \). Two methods have been used in extracting the relative flux \( [\Phi(E)] \): the fixed \( \nu \)-cut method and \( y \)-intercept method.\[^6\] The two techniques yielded consistent measures of \( \Phi(E) \).

The fixed \( \nu \)-cut method uses the most general form for the differential cross section for the \( V-A \) neutrino nucleon interaction (Eq.1 of Ref.\[^4\]) which requires that the number of events with \( \nu < \nu_0 \) in a \( E_\nu \) bin, \( N(\nu < \nu_0) \), is proportional to the relative flux \( \Phi(E_\nu) \) at that bin, up to corrections of order of \( O(\nu_0/E_\nu) \):

\[
N(\nu < \nu_0) = \Phi(E_\nu)\nu_0 \left[ A + \left( \frac{\nu_0}{E_\nu} \right) B + \left( \frac{\nu_0}{E_\nu} \right)^2 C + O\left( \frac{\nu_0}{E_\nu} \right)^3 \right].
\]

(2)
The parameter, \( \nu_0 \), was chosen to be 20 GeV to simultaneously optimize statistical precision while keeping corrections small. There are 426,000 \( \nu \)- and 146,000 \( \bar{\nu} \)-induced events in the fixed \( \nu \)-cut flux analysis.

The \( y \)-intercept method comes from a simple helicity argument: the differential cross sections, \( d\sigma/dy \), for \( \nu \)- and \( \bar{\nu} \)-induced events should be equal for forward scattering, i.e., as \( y \to 0 \).

\[
\left[ \frac{1}{E} \frac{d\sigma^\nu}{dy} \right]_{y=0} = \left[ \frac{1}{E} \frac{d\sigma^{\bar{\nu}}}{dy} \right]_{y=0} = \text{Constant.}
\]

(3)
Thus, in a plot of number of events versus \( y \), the \( y \)-intercept obtained from a fit to the entire \( y \)-region is proportional to the relative flux. The fixed \( \nu \)-cut and \( y \)-intercept methods of \( \Phi(E) \) determination typically agreed to about 1.5% with no
measurable systematic difference. A smoothing procedure was applied to minimize the effects of point-to-point flux variations.[7]

Structure functions were extracted from the CC data in the kinematic domain $E_{\text{had}} > 10 \text{ GeV}, Q^2 > 1 \text{ GeV}^2$ and $E_\nu > 30 \text{ GeV}$. In this sample, there were 1,050,000 $\nu$- and 180,000 $\bar{\nu}$-induced events. Accepted events were separated into twelve $x$ bins and sixteen $Q^2$ bins from 1 to 600 GeV$^2$. Integrating the $\nu$-N differential cross-section (Eq.1 of Ref.[4]) times the flux over each $x$ and $Q^2$ bin gives two equations for the number of neutrino and antineutrino events in the bin in terms of the structure functions at the bin centers, $x_0$ and $Q^2_0$.

$$\Delta N^{\nu,\bar{\nu}} = \left( \int a\Phi(E)^{\nu,\bar{\nu}} dE \right) (F_2(x_0, Q^2_0)) \pm \left( \int b\Phi(E)^{\nu,\bar{\nu}} dE \right) (xF_3(x_0, Q^2_0))$$

(4)

where $a$ and $b$ are known functions of $x$, $y$, $E$ and $R(x, Q^2)$.[4] and $\Phi(E)$ is the flux. The observed numbers of events, $N^{\nu}$ & $N^{\bar{\nu}}$, were corrected with an iterative Monte Carlo procedure for acceptance and resolution smearing.

To solve these equations for $F_2$ and $xF_3$ certain known corrections have to be applied. We assumed a parameterization of $R(x, Q^2)$ determined from the SLAC measurements,[8] and applied corrections for the 6.85% excess of neutrons over protons in iron. We used the magnitude and the $x$-dependence of the strange sea determined from our opposite-sign dimuon analysis.[9] The threshold dependence of charm quark production was corrected with the slow rescaling model,[10] where the relevant charm quark mass parameter, $m_c = 1.34 \pm 0.31 \text{ GeV}$, was determined from our data.[9] Radiative corrections followed the calculation by De Rújula et al.[11] and the cross-sections were corrected for the massive W-boson propagator. The
charm-threshold, strange sea, and radiative corrections were largely independent of $Q^2$. For $F_2$, they ranged from $\pm 10\%$ at $x = 0.15$, to $\pm 3\%$ at $x = 0.125$, to $\pm 5\%$ at $x = 0.65$ over our $Q^2$ range. For $xF_3$, they ranged from $\pm 1\%$ at $x = 0.15$, to $\pm 1.5\%$ at $x = 0.125$, to $\pm 2\%$ at $x = 0.65$. Resolution smearing was corrected using a Monte Carlo calculation which incorporated the measured resolution functions from dedicated test run data.[5] We have excluded the highest $x$-bin, $0.7 \leq x \leq 1.0$, due to its susceptibility to Fermi motion (which was not included in the smearing correction).

To measure $S_{GLS}$, the values of $xF_3$ were interpolated or extrapolated to $Q^2_0 = 3$ GeV$^2$, which is the mean $Q^2$ of the data in the lowest $x$-bin which contributes most heavily to the integral. Figure 1 shows the data and the $Q^2$-dependent fits used to extract $xF_3(x, Q^2 = 3)$ in three $x$-bins. The resulting $xF_3$ is then fit to a function of the form: $f(x) = Ax^b(1 - x)^c (b > 0)$. The best fit values are $A = 5.976 \pm 0.148$, $b = 0.766 \pm 0.010$, and $c = 3.101 \pm 0.036$. The integral of the fit weighted by $1/x$ gives $S_{GLS}$. Figure 2 shows the measured $xF_3(x)$ at $Q^2 = 3$ GeV$^2$, as a function of $x$, the fits and their integrals. The measurement of the sum rule yields:[12]

$$S_{GLS} = \int_0^1 xF_3(x) dx = 2.50 \pm 0.018 \text{(stat.)}$$

Fitting different functional forms to our data,[7] gives answers within $\pm 1.5\%$ of the above. We estimate $\pm 0.040$ to be the systematic error on $S_{GLS}$ due to fitting. The dominant systematic error of the measurement comes from the uncertainty in determining the absolute level of the flux, which is $2.2\%$. The other two systematic errors are $1.5\%$ from uncertainties in relative $\nu$ to $\nu$ flux measurement and $1\%$ from uncertainties in $E_\mu$ calibration.[7] The systematic errors are detailed in Table 1. Our
value for $S_{\text{GLS}}$ is:

$$S_{\text{GLS}} = \int_0^1 \frac{x F_2}{x} dx = 2.50 \pm 0.018 \text{ (stat.)} \pm 0.078 \text{ (syst.)} \quad (5)$$

The theoretical prediction of $S_{\text{GLS}}$, for the measured $\Lambda = 210 \pm 50$ MeV from the evolution of the non-singlet structure function,[7] is $2.66 \pm 0.04$ (Eq.1). The prediction assumes negligible contributions from higher twist effects, target mass corrections,[13] and higher order QCD corrections. The world status of $S_{\text{GLS}}$ measurements is shown in Fig.3.

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<sup>8</sup>An next-to-next-to-leading order calculation predicts $S_{\text{GLS}} = 2.63 \pm 0.04$.[14].
References


[5] Small x events have small angles. The muon angle resolution of the CCFR detector is about 1.3 mrad at the mean $E_\mu$ of 100 GeV; for details see W.K.Sakumoto et al., Nucl. Inst. Meth., A294, 179(1990).


Neutrino '00.


[11] A.De Rújula et al., Nucl. Phys., B154:394, 1979. We have estimated the effect of using the more detailed radiative correction calculation by D.Yu.Bardin et al., JINR-E2-86-260 (1986). The difference between the two corrections was generally very small, except at the lowest \((x = 0.015)\) and the highest \((x = 0.65)\) \(x\)-bins. Our structure function results would, thus, change by a few percent if the Bardin's instead of the De Rújula's calculation were used. In a future publication we shall present our results with Bardin's calculation.

[12] Our present value of \(S_{GLS}\) (2.50) is lower than the earlier preliminary presentations (2.66) [see S.R.Mishra, Talk at Lepton-Photon, 1991, Geneva]. Two small changes in the assumptions of the analysis lowered \(S_{GLS}\). These changes, we believe, are more accurate than those employed earlier. See Ref.[7].


Table 1: Error on the Gross-Llewellyn Smith sum rule: The statistical and systematic errors on \( S_{GLS} \) are presented.

<table>
<thead>
<tr>
<th>Error</th>
<th>Variation</th>
<th>( \Delta S_{GLS} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statistical</td>
<td></td>
<td>( \pm .018 )</td>
</tr>
<tr>
<td>Systematic</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fit</td>
<td>different fits</td>
<td>( \pm .040 )</td>
</tr>
<tr>
<td>( \sigma^{\nu N} ) Level</td>
<td>( \pm 2.1% )</td>
<td>( \mp .056 )</td>
</tr>
<tr>
<td>( \frac{\sigma^{\nu N}}{\sigma^{\nu P}} ) Level</td>
<td>( \pm 1.0% )</td>
<td>( \mp .034 )</td>
</tr>
<tr>
<td>Energy Scale</td>
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<td>( \pm .001 )</td>
</tr>
<tr>
<td>Rel. Calibr.</td>
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<td>( \mp .010 )</td>
</tr>
<tr>
<td>Flux Shape</td>
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</tr>
<tr>
<td>Total</td>
<td></td>
<td>( \pm .078 )</td>
</tr>
</tbody>
</table>

Figure Captions

Figure 1: Fits to \( Q^2 \)-dependence of \( xF_3 \) in 3 \( x \)-bins (the 2 lowest \( x \)-bins and a middle \( x \)-bin). \( xF_3 \) at \( Q^2_0 = 3 \text{ GeV}^2 \) (squares) is obtained by interpolation, as in the 2 lowest \( x \)-bin and shown by a dark symbol, or by extrapolation as in the middle \( x \)-bin (dark symbol).

Figure 2: The GLS sum rule: The squares are \( xF_3(x, Q^2 = 3) \) and the dashed line is the fit to \( xF_3(x, Q^2 = 3) \) by \( Ax^4(1-x)^c \). The solid line is the integral of the fit, \( \int_0^1 xF_3 \). The diamonds are an approximation to the integral computed by a weighted sum \([S(x_j)]\) of \( xF_3 = xF_3(x_i, Q^2 = 3) \), i.e., \( S(x_j) = \sum_i \Delta x_i xF_3 \).
Figure 3: GLS sum rule as measured by previous experiments and these data. The references for other measurements are: CDHS[15a], CHARM[15b], CCFRR[15c], WA25[15d], and CCFR-NBB[3].
GLS Sum Rule: CCFR Data at $Q^2 = 3$ GeV$^2$

\[ \int_0^1 F_3 dx = 2.50 \pm 0.018 \pm 0.078 \]
The Status of $S_{GLS}$ Measurement