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NONPERTURBATIVE HIGH-ENERGY SYMMETRIES OF STRING THEORY

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Abstract

By using zero-norm states in the spectrum, we explicitly demonstrate the existence of an infinite number of high-energy symmetry structures of the closed bosonic string theory. Each symmetry transformation (except those generated by massless zero-norm states) *exists only* in the $\alpha' \to \infty$ limit, and relates infinite particles with different masses, thus they are broken spontaneously at the Planck scale. As an application, the results of Das and Sathiapalan which claim σ-model is nonperturbatively nonrenormalizable were reproduced from a stringy symmetry argument point of view. It is conjectured that string theory can be regarded as a spontaneously broken Yang-Mills type theory with infinite dimensional "gauge group" constructed by an infinite number of zero-norm states in the spectrum.

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Analogous to the statement that quantum field theory is a quantum mechanical system with an infinite number degrees of freedom, string theory can be regarded as an infinite generalization of local quantum field theory with consistently self-organized couplings. In going from quantum mechanics to quantum field theory, one suffers from all kinds of high-energy divergences in the perturbation calculation. But instead, in string theory, one removes these unwanted divergences by building in an infinite number of high-energy symmetry structures[1] miraculously when considering the quantum theory of a free string. In fact, there exist many nonrenormalization theorems which have been proved up to the string two-loop order[2]. It is believed that this remarkable property of the string is due to the existence of these infinite symmetry structures of the theory. Thus, from a theoretical point of view, the study of nonperturbative, high-energy ($\alpha' \to \infty$), stringy regime of the theory is as important as, recently developed, 2d quantum gravity[3] which promises to extract nonperturbative (strong-coupling regime) information of the string. One hopes that the understanding of both nonperturbative regimes[4] of the theory may help us to uncover the " unbroken phase" of string theory and shed light on determining its true vacuum.

Gross has shown[1] that there exist an infinite number of linear relations between the scattering amplitudes of different string states as $\alpha' \to \infty$. He then conjectures that an infinite-parameter symmetry group which is broken spontaneously at Planck scale get restored at very high-energy, or $M^2_{Planck} \sim 1/\alpha' \to 0$. On the other hand, it was well known that σ-model can be used to study the dynamics of massless string modes[5]. This has also been generalized to include higher massive modes[6-8]. Based on the formalism in ref[7] , it was proposed[8] that by requiring the decoupling of both types of zero-norm states (for the bosonic string) in the spectrum, one can derive the complete gauge symmetries at each fixed mass level. The usual massless Yang-Mills gauge symmetry and Einstein general covariance can also be generated in this way. It was remarkable to discover that many higher symmetry transformations relate particles with different "spins"[8]. In this formalism, the dimension of the

"symmetry group" is directly related to the (infinite) number of zero-norm states in the spectrum. Instead of using the usual σ-model loop (or α') expansion[5], it turns out that the weak background field approximation (WFA)[8,9], valid to all orders in α', is the appropriate approximation scheme to study the high-energy symmetry of the string. However, the calculation was done only in the lowest order WFA. To this order of approximation, one cannot see the transformation of background fields between different mass sectors, and hence the spontaneously broken symmetries. The difficulty of higher order calculation which involves the operator product of two background fields is closely related to the nonperturbative (all orders in α', hence corresponding to the high-energy regime) nonrenormalizability of two-dimensional σ-model which has been shown by Das and Sathiapalan[10], and one is forced to introduce counterterms which consist of an infinite number of massive tensor fields into the theory. In this letter, we will explicitly demonstrate an infinite number of symmetry transformations between infinite background fields of different mass sectors of the close bosonic string theory. Specifically, we find that for each zero-norm state whose vertex operator can be written as a worldsheet total derivative, one can construct a symmetry generator which generates a symmetry transformation relating an infinite number of particles with different masses. Hence, combined with our previous results in ref[8] where string states at each fixed mass level form a symmetry multiplete was proved, we conclude that all string states are connected as an infinite multiplete. As an interesting application, we also reproduce the results of Das and Sathiapalan from a stringy symmetry argument point of view.

In the generalized σ-model formalism, let T_Φ define a conformal field theory (CFT) with the most general background field couplings consistent with the string vertex operator consideration in the WFA, ($\alpha' \equiv 1$)

$$T_\Phi = -1/2\, \eta_{\mu\nu}\partial X^\mu \partial X^\nu + h_{\mu,\nu}\partial X^\mu \bar\partial X^\nu + M_{\mu\nu,\alpha\beta}\partial X^\mu \partial X^\nu \bar\partial X^\alpha \bar\partial X^\beta + D_{\mu\nu,\alpha}\partial X^\mu \partial X^\nu \bar\partial^2 X^\alpha + E_{\mu,\alpha\beta}\partial^2 X^\mu \bar\partial X^\alpha \bar\partial X^\beta + A_{\mu,\alpha}\partial^2 X^\mu \bar\partial^2 X^\alpha + \cdots + \text{higher order terms} \qquad (1)$$

with background fields (Φ) equations of motion

$$\beta_i [\Phi] = 0 \tag{2}$$

where β_i are the renormalization group β function for each background coupling. We have used the worldsheet light-cone coordinates in (1) and neglected a similar left-moving equation [7,8]. In the first order WFA, we have calculated many examples which involve lower massive states[8]. On the other hand, it has been demonstrated[7] that if one can find an infinitesimal operator Q such that

$$T_\Phi + [Q, T_\Phi] = T_{\Phi+\delta\Phi} , \tag{3}$$

then the worldsheet generator Q generates a space-time symmetry transformation. In that case, $T_{\Phi+\delta\Phi}$ specifies a new CFT with $\beta_i [\Phi + \delta\Phi] = 0$. In the first order WFA, it can be shown that the integral of each worldsheet (1,0) or (0,1) primary field corresponds to a Q which fulfills the criterion in (3). A key step, made in ref[8], was to realize that the complete gauge symmetries of the string which include those in (3) can be systematically constructed by using the well-known zero-norm states in the spectrum. Hence, as one expects for a unitary theory, all space-time symmetries are directly related to the decoupling of zero-norm states in the spectrum. In this paper, however, we will calculate the important higher order correction of the symmetry transformations. It is from this second order correction (first order in the background fields and first order in the transformation parameters) that one begins to see the nonperturbative character (high-energy character) of the symmetry transformations, and hence the spontaneously broken symmetries. We will use equation (3) to do the calculation for Q constructed by those zero-norm states whose vertex operators can be written as a worldsheet total derivative. As an example, we give a class of type I [8] zero-norm states of the following form (omit all spin indices)

$$\theta\, (\alpha_{-1})^{n+1}(\widetilde{\alpha}_{-1})^{n-1}\widetilde{\alpha}_{-2} \mid 0,k > + k\, \theta\, (\alpha_{-1})^{n+1}(\widetilde{\alpha}_{-1})^{n+1} \mid 0,k > \tag{4}$$

where θ is a $2n + 1$ spin index parameter that is symmetric on the first $n+1$ and last n indices. It is orthogonal to k_μ on each index, and traceless on any pair of the first $n+1$, or any pair of the last n indices. In the second term, the index of k_μ is symmetrized with the last n indices of θ. The mass of the state is

$$- k^2 = M^2 = 2n. \tag{5}$$

The corresponding worldsheet generator is

$$Q_n = \int d\sigma\, \theta(X)(\partial X)^{n+1}(\bar{\partial} X)^n , \tag{6}$$

where θ is now promoted to become a function of X, and the orthogonality condition stated above becomes divergence free on each index. Also, Equation (5) is replaced by $(\Box - 2n)\,\theta = 0$. Under these constraints, it can be shown that the integrand in equation (6) is a (1,0) primary field. The lowest order (zero order in the background fields and first order in the θ parameters) calculation of $[\, Q_n , T_\Phi]$ gives

$$\partial_\nu\theta(X)(\partial X)^{n+1}(\bar{\partial} X)^{n+1} + \theta(X)(\partial X)^{n+1}(\partial^2 X)(\bar{\partial} X)^{n-1} , \tag{7}$$

where the index ν is symmetrized with the last n indices of θ. Comparing with equation (1) and using the constraints on θ, we find that equation (7) generates a symmetry transformation for a single nth massive level particle. The nonperturbative effects begin to show up at the second order calculation. In the following, we will use the lowest order results to calculate the second order correction of the symmetry transformation law. In general, one has to calculate terms of the following type, (we use complex coordinate in this calculation)

$$[\, Q_n , M(X)\, (\partial X)^{m+1}(\bar{\partial} X)^{m+1}] =$$

$$\oint \frac{d\omega}{2\pi i}\, \theta(X)(\partial X)^{n+1}(\bar{\partial} X)^n(\omega) \cdot M(X)(\partial X)^{m+1}(\bar{\partial} X)^{m+1}(z) , \tag{8}$$

and compare them with the first order terms of the background fields in equation (1) to see whether they satisfy equation (3) or not. It can be checked that the only dangerous terms which might violate (3) consist of operator contraction of the form $< \theta(X(\omega)) M(X(z)) >$ in the integrand. To prove that those terms vanish, let

$$\theta(X) = \int d\vec{k}\, \theta(k)\, e^{ikx} \ , \quad M(X') = \int d\vec{k}'\, M(k')\, e^{ik'x'} \ , \tag{9}$$

where $k = (k_0, \vec{k})$ is the 26d momentum. From the lowest order calculation[8], we have

$$(\ \Box - 2n\ )\,\theta = 0 \ , \quad (\ \Box - 2m\ )\, M = 0 \ , \tag{10}$$

which means

$$k^2 = 2n \ , \quad k'^2 = 2m \ . \tag{11}$$

Equation (11) looks like on-shell conditions although we are not calculating scattering amplitude.They are valid only in the lowest order calculation. So, for each fixed $n \geq 1$, one has to deal with integral of the following form ($s \leq \mathrm{Min}\,[n+1, m+1]$)

$$I = \int dk\, dk' \theta(k) M(k') \oint \frac{d\omega}{2\pi i} <e^{ikx(\omega)}\, e^{ik'x(z)}> <\partial X(\omega)\partial X(z)>^s\, (\partial X(\omega))^{n+1-s}\, (\bar{\partial} X(\omega))^n$$

$$(\partial X(z))^{m+1-s}\, (\bar{\partial} X(z))^{m+1} \ , \tag{12}$$

where m is any nonnegative integer. In the background field method, $<e^{ikx(\omega)}\, e^{ik'x(z)}>$ is defined to be

$$<e^{ikx(\omega)}\, e^{ik'x(z)}> = \int [d\xi]\, e^{ik(x_0(\omega)+\xi(\omega))}\, e^{ik'(x_0(z)+\xi(z))}\, e^{-S[x_0+\xi]-S[x_0]} \ , \tag{13}$$

where $X^\mu = X_0^\mu + \xi^\mu$ is expanded around a classical background X_0^μ and ξ^μ is the quantum fluctuation. The worldsheet action is

$$S = \int d^2z\, \partial X^\mu \bar{\partial} X^\nu \eta_{\mu\nu} \ . \tag{14}$$

The calculation of (13) is straightforward, one gets

$$e^{ikx_0(\omega)}\, e^{ik'x_0(z)}\, \mu^{k^2+k'^2} (\mu |\, \omega - z\, |)^{2kk'} \tag{15}$$

where μ is an infrared cutoff. The factor $\mu^{k^2+k'^2}$ comes from the tadpole divergences which occured already in the V.E.V. of single vertex function. To cancel the infrared cutoff dependence μ, we must require $k+k' = 0$ or equation (15) will vanish as μ goes to zero.Hence

$$\langle e^{ikx(\omega)}\, e^{ik'x(z)} \rangle = e^{ikx_0(\omega)}\, e^{ik'x_0(z)}\, |\, \omega - z\, |^{k^2+k'^2} \qquad \text{for } k+k' = 0\ , \tag{16a}$$

$$= \qquad 0 \qquad\qquad\qquad\qquad \text{for } k+k' \neq 0\ . \tag{16b}$$

By using equation (11) which is the result of the lowest order calculation, we note that equation (16a) contributes a factor $|\, \omega - z\, |^{2(n+m)} = |\, \omega - z\, |^{4n}$ in the integrand of I. But $\langle \partial X(\omega) \partial X(z) \rangle^s$ contributes $|\, \omega - z\, |^{-2s}$ to I. Since $s \leq n+1$, we conclude that I vanishes for $n \geq 1$. It is important to note that equation (11) is crucial to prove our final result, or I can be divergent for some range of (k,k'). In the concreteness, we give the $n=1$ case as an example which corresponding to the worldsheet generator

$$Q_1 = \oint \frac{d\omega}{2\pi i}\, \theta_{\mu\nu,\alpha} \partial X^\mu \partial X^\nu \bar{\partial} X^\alpha\ . \tag{17}$$

The calculation of $[\, Q_1\, ,\, h_{\mu,\nu} \partial X^\mu \bar{\partial} X^\nu + \cdots\cdots\,]$ with all first order background fields included is straightforward. One gets the following infinite symmetry transformation

$$\delta M_{(\alpha\beta),(\lambda\nu)} = \partial_\nu \theta_{\alpha\beta,\lambda} - 2\partial_\beta \theta_\alpha{}^\gamma{}_{,\lambda} h_{\gamma,\nu} - 2\theta_\alpha{}^\gamma{}_{,\lambda} \partial_\gamma h_{\beta,\nu} - \partial^\mu \theta_{\alpha\beta,\lambda} h_{\mu,\nu} + \partial_\alpha \partial_\beta \theta^{\gamma\mu}{}_{,\lambda} \partial_\gamma h_{\mu,\nu} +$$

$$\partial_\alpha \theta^{\gamma\delta}{}_{,\lambda} \partial_\gamma \partial_\delta h_{\beta,\nu} - 2\partial_\alpha \partial^\mu \theta^\delta{}_{\beta,\lambda} \partial_\delta h_{\mu,\nu} + 1/2\ \partial_\alpha \partial_\beta \partial^\mu \theta^{\gamma\delta}{}_{,\lambda} \partial_\gamma \partial_\delta h_{\mu,\nu}\ ,$$

$$\delta D_{(\alpha\beta),\lambda} = \theta_{\alpha\beta,\lambda}\ ,$$

$$\delta E_{\alpha,(\lambda\nu)} = -2\theta_\alpha{}^\beta{}_{,\lambda} h_{\beta,\nu} + \partial_\alpha \theta^{\gamma\mu}{}_{,\lambda} \partial_\gamma h_{\mu,\nu} - 2\partial^\mu \theta^\gamma{}_{\alpha,\lambda} \partial_\gamma h_{\mu,\nu} + 1/2\ \partial_\alpha \partial^\mu \theta^{\gamma\delta}{}_{,\lambda} \partial_\gamma \partial_\delta h_{\mu,\nu}\ ,$$

$$\delta A_{\alpha,\beta} = 0\ ,$$

..... (18)

where the zero order (in background fields) terms have been calculated in equation (7).The symmetric property of the spin indices on the r.h.s. of equation (18) is understood. One can calculate the transformations corresponding to all higher massive modes as well [11]. Each single transformation in equation (18) relates particles with mass difference 1. Thus, this n=1 massive zero-norm state can be used to generate a symmetry transformation which relates all particles in the bosonic string spectrum (except tackyon). Similar argument goes for general Q_n cases. The symmetry generated by Q_n relates particles with mass difference n. For the n = 0 case, $I \neq 0$ if m = 0. In that case, one can still check that the contribution of I does not violate equation (3). Indeed, an explicit calculation gives

$$\delta h_{\mu,\nu} = \partial_\nu \theta_{\mu,} + \partial^\alpha \theta_{\mu,} h_{\alpha,\nu} - 2\partial_\mu \theta^\alpha{}_{,} h_{\alpha,\nu} - \theta^\lambda{}_{,} \partial_\lambda h_{\mu,\nu} - \partial_\mu \partial^\alpha \theta^\lambda{}_{,} \partial_\lambda h_{\alpha,\nu} \ ,$$

..... . (19)

One can also calculate the transformations of all higher massive modes[11]. Note that, to any finite order calculation in WFA, the usual general covariance for graviton is lost . The symmetry transformation corresponding to Q_0 is the only symmetry which relates particles with the same mass. Therefore, one is tempted to argue that this infinite-parameter " symmetry group" constructed from Q_n is broken spontaneously down to Q_0 , leaving the corresponding gauge particle massless. Presumable the Higgs mechanism is operating as suggested by the inhomogeneous terms of equation (18). All Goldstone bosons become the longitudinal parts of higher massive modes in the string spectrum. In addition to Q_n, one can also consider other types of zero-norm states [11].

There is another interesting implication of the present work. If one believes that the symmetry structures presented in this paper is crucial in the study of the quantum theory of string, then any truncation of massive modes would inevitably lose these important symmetry structures, and thus leads to meaningless results. This is just like

the case of Kuluza-Klein truncation[12]. In fact, when one proved the nonrenormalization theorems for massless external legs, the massive modes effects have been included in the string-loop diagram. Unfortunately until now, most researches of the theory of string are confined to the low energy regime. The results of this paper strongly suggest that nonperturbative string vacuum should be seriously considered. We believe that there are still many fundamental structures in the high-energy regime which remain to be uncovered even in the critical string theory. An interesting application of the symmetry presented in equation (18) is the following : If one naively includes only the massless mode in equation (1), then symmetry induced by Q_1 in equation (18) will force one to include all higher massive modes. This simple observation is consistent with the results of Das and Sathiapalan[10] and the fact that there are infinite couplings between infinite number of states of the string[9,13]. Finally, there are still many zero-norm states which cannot be written as a worldsheet total derivative[8]. A further studies are in progress.

From the BRST point of view, the *zeroth-order WFA* of our approach is equivalent to the fact that the state $Q_{BRST}|\psi>$ which is a zero-norm state should be decoupled from the physical S-matrix. This is analogous to the BRST formulation of usual Yang-Mills theory where we know exactly a priori what classical action is from the symmetry principle. It is thus easy to convince oneself that string theory can be regarded as a spontaneously broken Yang-Mills type theory with infinite dimensional "gauge group" constructed by an infinite number of zero-norm states in the spectrum.

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