HEAVY QUARK SYMMETRY AND WEAK DECAYS OF HEAVY BARYONS* 

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*) Invited talk given at the conference "Quark Cluster Dynamics", Bad Honnef (1992), to be published in the Proceedings

#) Supported in part by BMFT/FRG under contract 06MZ730
Heavy Quark Symmetry and Weak Decays of Heavy Baryons

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Abstract. I give an account of the physics ideas that go into the formulation of Heavy Quark Symmetry (HQS) and use HQS ideas to discuss various aspects of the weak semileptonic decays of heavy baryons.

1. Introduction

Much of the motivation to study the weak decay properties of heavy hadrons can be traced back to the need to determine one of the fundamental constants of nature, the Kobayashi-Maskawa (KM) matrix element $V_{bc}$. It was realized in the last few years that exclusive semileptonic decays of bottom to charm hadrons are much better suited for this purpose than, as had been thought originally, the inclusive semileptonic $b \rightarrow c$ decays [1]. The reason for this is that the KM matrix element $V_{bc}$ can be regarded as a weak transition charge which can be accurately measured at the zero recoil point, at least in the limit when the bottom and charm quark mass become very heavy. This is so since the associated hadron transition form factor is normalized to one at zero recoil [2] just as in the case of electromagnetic transitions where the charge form factor is normalized to one at $q^2 = 0$. Thus the measurement of the weak transition charge $V_{bc}$ acquires the same status as the measurement of the electric charge, at least in the large mass limit. Much better, when corrections to the large mass limit were studied at a later stage, it was realized that the zero recoil normalization condition remains intact at $O(1/m_Q)$ [3,4], where $m_Q$ is the heavy quark’s mass.

Best suited for the determination of the KM element $V_{bc}$ are the mesonic and baryonic ground state to ground state transitions $B \rightarrow D, D^*$ and $\Lambda_b \rightarrow \Lambda_c$, resp., whose flavour diagrams are drawn in Fig.1.

Other decay candidates in the baryon sector are the $1/2^+ \rightarrow 1/2^+$ transitions $\Xi_b(b[su]) \rightarrow \Xi_c(c[su])$ and $\Omega_b(b(ss)) \rightarrow \Omega_c(c(ss))$ and the $1/2^+ \rightarrow 3/2^+$ transition $\Omega_b(b(ss)) \rightarrow \Omega^*_c(c(ss))$, where $[q_1q_2]$ and $\{q_1q_2\}$ refer to flavour-antisymmetric spin 0 and flavour-symmetric spin 1 diquark states, respectively. In this report I will mainly concentrate on heavy baryon transitions and among these, on the $\Lambda_b \rightarrow \Lambda_c$ transitions. I leave the subject of heavy meson transitions to a companion review [5].

Obviously one needs a bridge to connect the physics at the quark level, where theory is formulated and where $V_{bc}$ is defined, to the particle level, where, after all, the experiments are done. Fortunately, there has been significant progress over the last few years in this program (starting with the papers [6-12]) which I want to report on. The progress is related to the fact that now there exists a systematic expansion of QCD in terms of inverse powers of the heavy quark mass termed the "Heavy Quark Effective Theory (HQET)". The leading term in this expansion gives rise to a new symmetry termed the "Heavy Quark Symmetry (HQS)".

Nature has been very kind to us in that it has divided its six flavoured quarks into a heavy and a light quark sector. The "heavy" $c, b, t$-quarks are much heavier than the QCD scale $\Lambda_{QCD} = 300$ MeV whereas the "light" $u, d, s$-quarks are much lighter than $\Lambda_{QCD}$, i.e. one has

$$m_c, m_b, m_t >> \Lambda_{QCD} >> m_u, m_d, m_s$$  \hspace{1cm} (1)
In the heavy quark sector it then makes sense to first consider QCD in the limit where the heavy quark masses become very large and then, in the second stage, to consider power corrections to this limit in terms of a systematic $1/m_Q$ expansion. Likewise one can profitably first study the light quark sector in the zero mass limit, i.e. in the chiral symmetry limit, and then add corrections to the chiral limit at a later stage.

It is quite intriguing that many of the ideas of HQET date back as far as 1937, then of course in the context of QED [13]. In the Block-Nordsieck approach to soft photon radiation it was the electron that was "infinitely" heavy (on the scale of the soft photons) so that the fermionic degrees of freedom could be treated as a classical source of radiation (no $e^+e^-$ pair creation!). In fact, the Block-Nordsieck model was already formulated in terms of an effective theory with the electron degrees of freedom removed from the field theory (see also [14]).

It is quite important to realize that HQS is not a spectrum symmetry but it is a new type of equal velocity symmetry. That one cannot expect a spectrum symmetry to hold in the heavy quark sector should be quite clear from the fact that there are two orders of magnitude difference between the masses of the $c$ and $t$ quarks! On the other hand, the new type of HQS symmetry at equal velocities takes a little bit of getting used to. But once one has gotten into the habit of thinking in terms of quark and particle velocities the HQS will in fact look quite natural.

We mention that the implications of HQET and HQS have been vigorously studied in the last two-and-a-half years starting with the 1990 papers by Isgur-Wise [6], Bjorken [7], Georgi [10] and our group at Mainz [12]. In the meantime the field is at full blossom with approximately 300 papers published and new papers coming out every week.

To familiarize oneself with the presence of a spin and flavour symmetry at equal velocity it is quite instructive to consider a bottom and charm baryon at rest as shown in Fig.2. The heavy bottom quark and the charm quark at the center are surrounded by a cloud corresponding to the light diquark system. The only communication between the cloud and the center is via gluons. But since gluons are flavour blind the light cloud knows nothing about the flavour at the center. Also, for infinitely heavy quarks, there is no spin communication between the cloud and the center. Thus one concludes that, in the
heavy mass limit, a bottom baryon at rest is identical to a charm baryon at rest regardless of the spin orientation of the heavy quarks, i.e. one has

Bottom Baryon at rest \( \equiv \) Charm Baryon at rest \( (2) \)

**QED:** \( \bullet \overset{p}{\bullet} \overset{d}{\bullet} \overset{e^+}{\bullet} \)

**Fig. 2:** Portrayal of bottom and charm baryon wave functions at rest. Upper right corner: wave functions of the hydrogen, deuterium and tritium atoms.

One then just needs to boost the rest configuration by a Lorentz boost from velocity zero to velocity \( v \) to conclude

Bottom Baryon at velocity \( v \) \( \overset{v}{\equiv} \) Charm Baryon at velocity \( v \) \( (3) \)

remembering that a Lorentz boost depends only on relative velocities. Eq.(3) exposes the existence of a new spin and flavour symmetry of QCD at equal velocities which holds true in the large mass limit. This is nothing but the advertised Heavy Quark Symmetry HQS.

In fact, everyone should be quite familiar with the existence of such a symmetry in the context of QED. Take a hydrogen, deuterium and tritium atom at rest as also shown in Fig.2. When hyperfine interactions are neglected they possess identical wave function and thus identical atomic properties. The Coulombic interaction between the electron cloud and the nucleus at the centre is sensitive only to the total charge of the nucleus which is the same for all three isotopes.

2. Spin Complexity of Transition Form Factors and Angular Decay Distributions

To start with let us first enumerate the number of form factors that describe the semileptonic \( 1/2^+ \rightarrow 1/2^+ \) and \( 1/2^+ \rightarrow 3/2^+ \) transitions where \( J^P \) denotes the spin (\( J \)) and the parity (\( P \)) of the heavy baryons. This is easily done in the usual covariant expansion. One has (\( q = p_1 - p_2 \))

\[
\frac{1}{2^+} \rightarrow \frac{1}{2^+}: \quad \langle A_\mu(p_1) \rangle V_\mu + A_\mu A_\mu(p_1) =
\]

\[
\bar{u}(p_2)\left[ \gamma_\mu(F^\gamma_1 + F^\gamma_2) + i\sigma_\mu q(F^\gamma_1 + F^\gamma_2) \right. \\
\left. + q_\mu(F^\gamma_1 + F^\gamma_2) \right] u(p_1)
\]

and, equivalently, for the \( 1/2^+ \rightarrow 1/2^+ \) transition \( \Omega_b \rightarrow \Omega_c \). For the \( 1/2^+ \rightarrow 3/2^+ \) transition one has
\[ 1/2^+ \rightarrow 3/2^+ : \quad \left\langle \Omega^- \left| (p_2) V + A_\mu \right| \Omega^+(p_1) \right\rangle = \]

\[
\bar{u}^\mu(p_2) \left\{ g_{\alpha\alpha}(G_1^V + G_1^A) + p_{\alpha\alpha} Y_\alpha(G_2^V + G_2^A) \right\} + p_{\alpha\alpha} P_{\alpha\alpha}(G_3^V + G_3^A) + p_{\alpha\alpha} q_{\alpha\alpha}(G_4^V + G_4^A) \gamma_5 u(p_1) \] \tag{5}

There are thus \((4+2)\) and \((6+2)\) form factors for the \(1/2^+ \rightarrow 1/2^+\) and \(1/2^+ \rightarrow 3/2^+\) transitions, respectively.

The first number in the brackets counts the number of form factors that can be measured in the zero lepton mass case (typically \(e\) and \(\mu\)) whereas a measurement of the form factors multiplying \(q_{\alpha\alpha}(F_3^V,A\text{ and } G_4^V,A)\) require non-zero lepton masses (typically the \(\tau\)). When one wants to define physical observables it is more advantageous to linearly transform the invariant amplitude \(F_i\) defined in (7) to helicity amplitudes \(H_i\) (see e.g. [15-18]). These again split into the two \((4+2)\) and \((6+2)\) sets mentioned above.

It is quite remarkable that, in the infinite mass limit, HQS tells us that the six form factors in the \(A_b \rightarrow A_c\) case are all related to one reduced form factor \(F_A(\omega)\) which is a function of the "scaling" velocity transfer variable \(\omega = v_1 \cdot v_2\) and which is normalized to one at zero recoil \(F_A(\omega=1)=1\). For the transitions involving spin 1 diquarks, the 14 form factors describing the \(\Omega_b \rightarrow \Omega_c\) and \(\Omega_b \rightarrow \Omega^*\) transitions are all related to two reduced form factors \(F_{1,2}(\omega)\) and \(F_{3,4}(\omega)\) which satisfy the zero recoil normalization conditions \(F_{1,2}(1) = F_{3,4}(1) = 1\). I have intentionally chosen the phrase "reduced form factor" in analogy to the corresponding phrase "reduced matrix element" used in the Wigner-Eckart theorem. We shall later on describe how one actually determines the "Clebsch-Gordan" coefficients that project the general sets of form factors onto the respective reduced form factor \(F_A(\omega), F_{1,2}(\omega)\) and \(F_{3,4}(\omega)\). HQS by itself can say nothing about the actual functional form of the reduced form factors except for their normalization at zero recoil \(\omega=1\). To obtain their functional form one needs additional dynamical input.

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Fig. 3: Definition of hadron side polar angle \(\theta_A\), lepton-side polar angle \(\theta\), and azimuthal angle \(\chi\) in the decay \(A_b \rightarrow A_c \rightarrow \pi^+ + W^- (\rightarrow \ell^- \bar{\nu}_\ell)\).

In the following I want to delineate how the form factors can actually be measured in the semileptonic decay processes and how the predictions of HQS can thus be tested. To be specific I shall discuss the \(A_b \rightarrow A_c\) transition. Fig. 3 shows the decay configuration \(A_b \rightarrow A_c(\rightarrow \pi^+) + W^-\text{off-shell} (\rightarrow \ell^- \bar{\nu}_\ell)\) in the \(A_b\) rest system. I view the decay process as a two-step process. In the first step the \(A_b\) decays into the \(W^-\text{off-shell} on one side and the \(A_c\) on the other side\(^1\) (back-to-back to the \(W^-\)). In the second step these

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\(^1\) For reasons of conciseness the \(W^-\text{off-shell}\) will be referred to as \(W^-\) in the following.
further cascade via $W \to \ell^- \bar{\nu}_\ell$ (lepton side) and, via $\Lambda_c \to \Lambda_c \pi$ (hadron side). The second-step decays are again analyzed in their respective rest systems in terms of a lepton-side polar angle $\theta$ and a hadron-side polar angle $\theta_A$. Finally, the relative orientation of the two decay planes defines an azimuthal angle $\chi$ as Fig.3 shows. The first step is governed by the weak decay amplitudes $H_i$ or $F_i$ describing the "decay" $\Lambda_b \to \Lambda_c + W^-$. The decay products $\Lambda_c$ and $W^-$ emerge in highly polarized states. Their polarization density matrices are given in terms of bilinear forms of the decay amplitudes. The second step decays $\Lambda_c \to \Lambda_c \pi$ and $W^- \to \ell^- \bar{\nu}_\ell$ can then in turn be used to analyze the polarization states of the $\Lambda_c$ and the $W^-$. In this regard the decay $W^- \to \ell^- \bar{\nu}_\ell$ is an optimal analyzer since it possesses 100% analyzing power.

What has been described in words can be surmised in the form of a joint angular decay distribution for the decay $\Lambda_b \to \Lambda_c (\to \Lambda_c \pi) + W^-(\to \ell^- \bar{\nu}_\ell)$. The joint angular decay distribution will involve the lepton side polar angle $\theta$, the hadron-side polar angle $\theta_A$ and the relative azimuth $\chi$ of the two decay planes. Collecting all kinematical factors one has [17-19]

$$\frac{d\Gamma(\Lambda_b \to \Lambda_c (\Lambda_c \pi) + \ell^- \bar{\nu}_\ell)}{dq^2 \cos \theta \cos \theta_A d\chi} = \frac{G^2}{(2\pi)^2} |V_{bc}|^2 |q^2|^p \cdot \frac{24M}{24M}$$

$$B(\Lambda_c \to \Lambda_c \pi) \cdot$$

$$\left\{ \begin{array}{l}
\frac{3}{8} (1 + \cos \theta)^2 (1 + \alpha_{\Lambda_c} \cos \theta_H) |H_{1/2}^+|^2
+ \frac{3}{8} (1 - \cos \theta)^2 (1 - \alpha_{\Lambda_c} \cos \theta_H) |H_{1/2}^-|^2
+ \frac{3}{4} \sin^2 \theta (1 + \alpha_{\Lambda_c} \cos \theta_H) |H_{2/2}^0|^2
+ \frac{3}{4} \sin^2 \theta (1 - \alpha_{\Lambda_c} \cos \theta_H) |H_{-2/2}^0|^2
- \frac{3}{2\sqrt{2}} \alpha_{\Lambda_c} \cos \chi \sin \theta \sin \theta_H (1 + \cos \theta) \Re(H_{1/2}^- o H_{1/2}^+)
\left[ \begin{array}{c}
- \frac{3}{2\sqrt{2}} \alpha_{\Lambda_c} \cos \chi \sin \theta \sin \theta_H (1 - \cos \theta) \Re(H_{2/2}^0 o H_{-2/2}^0)
\end{array} \right]
\end{array} \right\}$$

where $q^2=(p_1-p_2)^2$ is the invariant momentum transfer squared and $p$ is the CM momentum of the $\Lambda_c$. The $H_{\lambda_{\Lambda_c} \lambda_{W}}$ are the aforementioned helicity amplitudes of the decay $\Lambda_b \to \Lambda_c + W^-$ where $\lambda_W$ is the helicity of the $W^-$ and $\lambda_{\Lambda_c}$ is the helicity of the daughter baryon. The decay distribution (6) holds for zero lepton masses. If lepton mass effects are included there are ten more terms in (6) [17]. Furthermore, if one includes also the so-called T-odd contributions that could arise from CP and/or final state interaction effects there are even three more additional terms in (6) when $m_{\ell} \neq 0$. Thus, when $m_{\ell} = 0$ and T-odd effects are included, there are altogether 19 observables in the decay distribution $\Lambda_b \to \Lambda_c (\to \Lambda_c \pi) + W^- (\to \ell^- \bar{\nu}_\ell)$. Since there are only six independent amplitudes in the decay process a complete or even a partial measurement of the observables would considerably over determine the form factor amplitudes.

Let me remind the reader that the analysis of joint angular decay distributions such as the one given in Eq.(6) has by now become a standard fare in the analysis of weak decays of heavy mesons. For example, the well-known amplitude analysis of the decay $D \to K^*+ \ell \nu_\ell$ by E691 was based on a full three-fold angular fit to an event sample of $\approx$ 200 events [20]. A similar analysis was done by E653 for the same decay $D \to K^*(\to K\pi)+ \mu \nu_\mu$ ($\approx$ 300 events) where lepton mass effects ($m_\mu \neq 0$) were included in the
analysis [21]. For $b \to c$ decays, ARGUS ($\approx 400$ events) [22] and CLEO ($\approx 200$ events) [23] have done a full amplitude analysis based on the threefold angular decay distribution in the decay $B \to D^*(\to D\pi) + \ell \nu$. 

On the theoretical side various aspects of joint angular decay distributions in semileptonic decays have been discussed in the literature. I cite refs. [24-31] for semileptonic meson decays and refs. [18,19,32] for semileptonic baryon decays. Recently there has been a very comprehensive, almost encyclopedic analysis of joint angular decay distributions in the weak semileptonic and nonleptonic decays of heavy mesons and baryons including non-zero lepton mass effects as well as polarization effects [17].

Instead of analyzing the full three-fold angular decay distribution (6) one can also consider single angle distributions. For example, the lepton-side polar angle distribution reads

$$W(\theta) = 1 + 2\alpha' \cos \theta + \alpha^* \cos^2 \theta$$

(7)

where the asymmetry parameters $\alpha'$ and $\alpha^*$ are given in terms of bilinear forms of the helicity amplitudes. They can be read off from the decay distribution Eq.(6) and read

$$\alpha' = \frac{H_{1/2} \left[ -H_{1/2, -1} - 2H_{1/2, 0} \right] - H_{1/2} - H_{1/2, -1} - 2H_{1/2, 0} + H_{1/2, 0} \left[ -H_{1/2, 0} - H_{1/2, 0} \right]}{H_{1/2} - H_{1/2, -1} + 2H_{1/2, 0} + H_{1/2, 0}}$$

(8)

$$\alpha^* = \frac{H_{1/2} - H_{1/2, -1} + 2H_{1/2, 0} - H_{1/2, 0}}{H_{1/2} - H_{1/2, -1} + 2H_{1/2, 0} + H_{1/2, 0}}$$

(9)

On the hadron-side one has the polar angle distribution [18]

$$W(\theta_\lambda) = 1 + \alpha_{\lambda} \cos \theta$$

(10)

where

$$\alpha'_{\Lambda} = \left\{ -0.95^{+0.42}_{-0.42} \right\}$$

CLEO [33] collaborations and is given by

$$\alpha_{\Lambda} = \left\{ -0.95^{+0.42}_{-0.42} \right\}$$

(12)

The two asymmetry parameters $\alpha'$ in Eq.(7) and $\alpha_{\Lambda}$ in Eq.(10) are sensitive to parity-violating effects, i.e. sensitive to the differences

$$|H_{1/2, \Lambda_w} - H_{1/2, -\Lambda_w}| > 0 \quad (\Lambda_w = 1, 0)$$

They can in fact be utilized to extract information on the chirality of the $b \to c$ transition. In the left-chiral case, as predicted by the Standard Model, the $c$-quark emerges from the weak interaction with dominant negative helicity. This information is handed over to the $\Lambda_c$ into which it hadronizes. Thus one has

$$|H_{1/2, -\Lambda_w} - H_{1/2, \Lambda_w}| > 0 \quad (\Lambda_w = 1, 0)$$

and consequently the asymmetry parameters $\alpha'$ and $\alpha_{\Lambda}$ are predicted to be negative irrespective of the details of the underlying quark model dynamics.

2 As mentioned before the $\Lambda_c$ is made from a $c$-quark and a spin-zero diquark and thus the helicity of the $c$-quark is the helicity of the $\Lambda_c$. In fact, in the HQS limit, the transfer of the helicity information from the $c$-quark to the $\Lambda_c$ is 100% irrespective of whether the fragmentation is direct or indirect [35].
The asymmetry $\alpha$ and $\alpha_a$ can be conveniently projected out by defining forward-backward asymmetries. One averages over the events in the respective forward (F) and backward (B) hemispheres of the two decays and then takes the ratios $A_{FB} = (F-B)/(F+B)$. One then has

lepton side:

$$A_{FB} = -\frac{3}{4} \frac{\left[H_{1/21}^F - H_{1/21}^B\right]^2}{\left[H_{1/21}^F + H_{1/21}^B\right]^2 + \left[H_{1/21}^F - H_{1/21}^B\right]^2 + \left[H_{1/20}^F + H_{1/20}^B\right]^2}$$  \hspace{1cm} (13a)

hadron side:

$$A_{FB} = \frac{1}{2} \alpha \alpha_a$$  \hspace{1cm} (13b)

where the forward hemispheres are defined w.r.t. the momentum direction of the $W$ and $\Lambda_c$, i.e. $\pi/2 \leq \Theta < \pi$ and $0 \leq \Theta_a < \pi/2$, respectively.

As $\alpha$ and $\alpha_a$ are negative both the lepton-side and hadron-side FB asymmetries (13a) and (13b) are predicted to be positive in the Standard Model. In fact in the diquark model of Ref.[36] one finds

$$A_{FB}^{(\text{lepton side})} = 0.18$$  \hspace{1cm} (14a)

$$A_{FB}^{(\text{hadron side})} = -0.35 \alpha_a$$  \hspace{1cm} (14b)

The hadron-side FB-asymmetry is predicted to be relatively large on account of the two facts that there are large longitudinal contributions (see Eq.(11)) and that the analyzing power of the decay $\Lambda_c \rightarrow \Lambda_k + \pi$ is large (see Eq.(12)). In addition, the hadron-side FB asymmetry has the advantage of being a true parity-odd spin momentum correlation measure ($\langle \vec{G} \cdot \vec{p}\rangle$-type) and thus does not suffer from the criticism recently raised against using the lepton-side FB asymmetry (parity-even momentum-momentum correlation $\langle \vec{p}_l \cdot \vec{p}_a\rangle$-type) to conclude for the handedness of the $b \rightarrow c$ current [37].

3. Heavy Quark Symmetry and Heavy Baryon Transition Form Factors

Consider the semileptonic decay of a bottom baryon to a charm baryon as drawn in Fig.4. The bottom quark at four-velocity $v_1$ makes a transition to a charm quark at four-velocity $v_2$ by emitting a virtual $W^-$. The light "spectator" quark system which propagates independently is dragged along to expedite it from the velocity $v_1$ to $v_2$ without, however, touching its spin.\(^3\)

The spin neutral velocity kick (or alignment) can be conceived of to result from the exchange of many soft gluons between the $c$-quark at velocity $v_2$ and the spectator system which starts off at velocity $v_1$ and ends up with velocity $v_2$ in order to align its velocity with the $c$-quark. Compared to the time scale of the $b \rightarrow c$ transition the alignment process is slow. The exchanged gluons are all of the longitudinal non flip type, i.e. there is no spin information transferred from the heavy side to the light side. This can be made manifest on the heavy side by splitting the gluon's $\gamma_5$ coupling into its spin flip and spin non flip components, viz.

$$\gamma_5 = \left(\frac{\gamma_5}{\text{flip}} - \frac{\gamma_5}{\text{non flip}}\right) + \frac{\gamma_5}{\text{non flip}}$$  \hspace{1cm} (15)

\(^3\) Remember that in the case of the $\Lambda_Q$ and the $(\Omega_Q, \Omega_Q^+)$ the light quark system has spin zero (scalar diquark) and spin 1 (vector diquark).
The spin flip coupling \((\gamma^\mu - v^\mu)\) vanishes in the heavy mass limit and one remains with the Bloch-Nordsieck type non flip coupling \(v^\mu\). From what was said it is clear that the weak amplitudes \(\Lambda_b \to \Lambda_c + W^-\) and \(\Omega_c \to (\Omega_c, \Omega_c^*) + W^-\) factorize into a heavy-side and into light-side transition amplitudes. The only information that is exchanged between the heavy- and the light-side is velocity information necessitated by the requirement to reassemble the final charm quark and the light diquark system in the same final baryon. The dynamics of the heavy-side transition \(b \to c + W^-\) is known. It is specified by the usual SM left-chiral weak coupling with a coupling strength proportional to \(V_{bc}\). The light-side transition involves the three unknown transition probabilities

\[
I_{\text{scalar diquark}}; v_1, \lambda \rightarrow I_{\text{scalar diquark}}; v_2
\]

\[
I_{\text{vector diquark}}; v_1, \lambda_1 \rightarrow I_{\text{vector diquark}}; v_2, \lambda_2
\]

(\(\lambda_1 = \lambda_2 = 0, 1\)) where the \(\lambda_{1,2}\) are the helicities of the vector diquark. We parametrize the three form factor functions by \(F_A(\omega)\), \(F_L(\omega)\) and \(F_T(\omega)\), where \(L\) and \(T\) refer to the longitudinal \((\Lambda_1 = \Lambda_2 = 0)\) and transverse \((\Lambda_1 = \Lambda_2 = \pm 1)\) vector diquark transitions. They can only be a function of \(\omega = v_1, v_2\) since the velocity transfer variable \(\omega\) is the only Lorentz invariant variable that can be constructed in the light-side transitions. At zero recoil, when \(v_1 = v_2\) and \(\omega = 1\), the diquark goes through unhindered with amplitude 1 and thus we have the normalization condition \(F_A(\omega) = F_L(\omega) = F_T(\omega) = 1\). It is clear that one has to identify the \(F_i(\omega)\) \((i = A, L, T)\) with the reduced form factor function \(F_i(\omega)\) mentioned in Sect.1. One expects the \(F_i(\omega)\) to fall when \(\omega\) moves away from the zero recoil limit as it costs to provide the velocity kick. Pole-type form factors and explicit model calculations confirm this expectation.

Since the zero recoil normalization condition of HQS is of such central importance let us have another look at it from a different point of view. Replace the final-state c-quark in Fig.4 by a b-quark with the same velocity. This is a symmetry operation as shown in Sect.1. At zero recoil, and for the vector current part of the transition, the normalization \(F_i(\omega=1)=1\) now is nothing but the well-familiar charge form factor normalization at \(q^2=0\) applied to the elastic \(\Lambda_b \to \Lambda_c\) and \(\Omega_c \to \Omega_b\) transitions. Still

\[\text{Parity relates the helicity transitions } \lambda_1 = \lambda_2 = 1 \text{ and } \lambda_1 = \lambda_2 = -1.\]
another way of looking at the zero recoil normalization condition is afforded by considering the b-quark rest configuration in Fig.2. Replacing the b-quark at rest by a c-quark at rest, as happens in the decay at zero recoil, will not affect the wave function of the light diquark system. Thus, the overlap between the wave functions before and after the b→c transition is complete, giving again the zero recoil normalization condition.

Let us now turn to the spin properties of bottom to charm transitions as implied by the spectator diquark picture Fig.4. As has been emphasized before there is complete spin factorization of the heavy-side and light-side transitions. This factorization property was exploited in the helicity matching approach of [15,16,38,39] to derive the HQS heavy baryon form factor structure. The algebraic approach of [40], using spin commutation relations, is quite similar to the helicity matching approach. Finally, the group theoretic approach [41,42] and the Bethe-Salpeter approach [43,44] employ tensor techniques to derive the same heavy baryon form factor structure.

All the above four approaches [15,16,38-44] are of course equivalent. Technically the group theoretic approach of [41,42] is the simplest. The spin wave functions of the Λ-type and Ω-type JP = 1/2+ ground state baryons are represented by the spinor u and by ψ(ψ*=ψ) and the ground state JP = 3/2+ Ω*-type baryon is represented by its Rarita-Schwinger spinor u. The HQS form factor structure can then be written down immediately by considering the independent ways of contracting Lorentz indices. One has

\[ \langle \Lambda,|\bar{\psi}|\Lambda,\rangle = \bar{u} F_3(\omega) \gamma_\mu (1 - \gamma_\nu) u \]

\[ \langle \Omega,|\bar{\psi}|\Omega,\rangle = \left( \frac{1}{F_3(\omega)} \right) \gamma_\mu (\gamma_5 \mathbf{\gamma}_\nu + \mathbf{\gamma}_\nu \mathbf{\gamma}_5) \]

\[ F_1(\omega) g_{\mu\nu} - F_2(\omega) \gamma_\mu \gamma_\nu (1 - \gamma_5) \frac{1}{F_3(\omega)} (\gamma_5 \mathbf{\gamma}_\nu + \mathbf{\gamma}_\nu \mathbf{\gamma}_5) \] (16a)

Note that one may not use \( \gamma \)-matrices for the contraction as they would bring in spin interactions on the heavy quark legs which are absent in the static approximation. One thus has three universal form factors. The normalization condition for the Λ-type transition is \( F_3(\omega=1)=1 \) as before. The normalization condition for the (Ω, Ω*)-type transitions can be obtained by relating the two form factors \( F_1(\omega) \) and \( F_2(\omega) \) to the longitudinal and transverse form factors \( F_L(\omega) \) in Eq.(16) and \( F_T(\omega) \) introduced earlier. One has

\[ F_1(\omega) = F_T(\omega) \] (17a)

and thus the normalization reads \( F_1(\omega)=1 \). As Eqs.(16b) or (17b) show the form factor \( F_2(\omega) \) does not contribute at zero recoil.

Ref.[43,44] contains a derivation of the form factor structure (16) using Bethe-Salpeter amplitudes for the heavy baryon bound state systems. The form factors are thereby related to wave function overlap integrals which are computable for any given model of the bound state wave functions. Further assumptions on the spin structure of the bound states reduces the number of independent form factors in (16) from three to two and three to one [43].

We mention that the heavy baryon to light baryon form factor structure may be obtained from (16) by allowing for spin interactions of the light active quark [44]. This amounts to the replacement \( F_\Lambda \rightarrow F_\Lambda + \gamma F_\Lambda, \ F_1 \rightarrow F_1 + \gamma F_1 \) and \( F_2 \rightarrow F_2 + \gamma F_2 \). Now there is no normalization condition for the form factors. Also the \( \Omega_b \rightarrow \Omega_{light} \) and \( \Omega_b \rightarrow \Omega_{light}^* \) form factors are not related. Phenomenological
consequences of the heavy to heavy (including $1/m_Q$ corrections) and the heavy to light baryonic form factor structure are presently being worked out [15,18,36,38].

Eq. (16) provides a covariant form of the "Clebsch-Gordan" coefficients that tell us how to project the transition form factors onto the reduced form factors $F_i(\omega)$ ($i = \Lambda, 1, 2$). HQS by itself can say nothing about the actual functional form of the reduced form factors $F_i(\omega)$. To obtain their $\omega$-dependencies one needs additional dynamical input as e.g. provided by the QCD sum rule approach, by lattice calculations or by explicit quark model calculations. In the following I shall briefly discuss an explicit diquark model of heavy $\Lambda$-type baryons which, when evaluated in the infinite momentum frame, provides an explicit form of the HQS reduced form factor $F_\Lambda(\omega)$ in the low recoil regime and also $1/m_Q$ corrections to the heavy mass limit [36]. In Sec.4, finally, we consider the large $\omega$ or $q^2$-behaviour of the reduced form factors which can be conveniently studied within the Brodsky-Lepage hard scattering formalism. It is quite remarkable that one retains a modified form of heavy quark symmetry in the large recoil regime [45,46]. The large $\omega$-behaviour of the reduced form factors in the large recoil regime can again be studied within particular models [45,46].

Returning to the low recoil regime the $\omega$-dependence of the mesonic reduced form factor $F(\omega)$ has recently been obtained by Neubert and Rieckert [47] using the heavy meson relativistic oscillator light-cone wave functions of Bauer, Stech and Wirbel (BSW). The $q^2=0$ values of the $(Q_{\pi})\rightarrow(Q_{\pi})$ transition form factors were obtained by calculating the wave function overlap integrals for different current components. The overlap integrals were then expanded in a $1/m_Q$ power series with the coefficient functions depending on the mass ratio $M_1/M_2$ only. Now, since at $q^2=0$ one has $\omega = (M_1/M_2 + M_2/M_1)/2$, the $\omega$-dependence of the coefficient form factor functions can be obtained by varying the mass ratio $M_1/M_2$. They found their quark model results to be consistent with HQET up to and including the $O(1/m_Q)$ corrections [3], yielding, of course, explicit functional forms for the five $O(1)$ and $O(1/m_Q)$ reduced form factor functions and a value for the dimensionful constant $\bar{\Lambda}$ that appear in the general HQET analysis [3].

Together with B. König, M. Krämer and P. Kroll I have recently extended the Neubert-Rieckert approach to the baryon sector using BSW-type quark-diquark wave functions for the $\Lambda$-type heavy baryons [36]. Again the quark model calculation of the $\Lambda_b\rightarrow\Lambda_c$ transitions was found to be consistent with the $1/m_Q$ structure of the HQET [48]. Contrary to the mesonic case, though, one has to restrict oneself to the use of the "good" components of the current transitions only. To illustrate our results I show a plot of the $O(1)$ form factor behaviour of the $\Lambda_b\rightarrow\Lambda_c$ transition form factor $F_\Lambda(\omega)$ in Fig.5.

The diquark form factor is appropriately normalized to 1 at zero recoil. However, it falls off much faster than the dipole-type form factor as one moves away from the zero recoil point (see Fig.4). The rapid fall-off can be traced back to the rather narrow infinite-momentum-frame wave functions used in [36] that result from adapting conventional three-quark baryon wave functions to the quark-diquark case. The $1/m_Q$ corrections to the $O(1)$ results were found to be quite small, as was the case in mesonic transitions [47]. I refer to [36] for a discussion of phenomenological implications for rates, spectra and asymmetries in $\Lambda_b\rightarrow\Lambda_c$ transitions.
4. Heavy Quark Symmetry at Large Recoil

The heavy quark symmetry predictions are expected to be rather good close to the zero recoil point where not much momentum is transferred to the spectator system. However, as one moves away from the zero recoil point, more momentum gets transferred to the spectator system, and hard gluon exchange including spin flip interactions becomes more important and the low recoil heavy quark symmetry discussed in the previous sections can be expected to break down. This is illustrated in Fig.6 where the mismatch between the "kicked" heavy quark momentum and the momentum of the light spectator system becomes progressively larger as one moves away from the zero recoil point.

In the large recoil limit the limiting behaviour of the form factors can be conveniently studied in the Brodsky-Lepage formalism [49]. As it turns out the form factors exhibit a new heavy quark symmetry in the large recoil limit which is reminiscent but not identical to the heavy quark symmetry at low recoil. One finds that the transition form factors have the correct large momentum transfer power behaviour as expected from dimensional counting rules.

According to Brodsky and Lepage (BL) [48] the large $\omega$- or $q^2$-behaviour of the form factors is obtained by convoluting the initial and final state hadron's distribution amplitude with a hard scattering
amplitude as shown in Fig.7 for heavy meson transitions \((Q_1\bar{q}) \rightarrow (Q_2\bar{q})\).

The hard scattering amplitude \(T_\mu\) is computed in perturbative QCD in the collinear approximation, whereas the distribution amplitudes \(\phi_i\) contain the nonperturbative long distance dynamics.

For the \((Q_1\bar{q}) \rightarrow (Q_2\bar{q})\) transitions one obtains
\[
\langle (Q_1\bar{q})|V_\mu + A_\mu (Q_2\bar{q}) = -\sqrt{m_1m_2}\, \epsilon f_1 f_2
\]
\[
\int dx_1dy_1\, \Phi^*_i(y_1)T_\mu(x_1,y_1,\omega)\phi_i(x_1)
\]
(18)

where \(x_1\) and \(y_1\) are the longitudinal momentum fractions of the heavy quarks \(Q_1\) and \(Q_2\), \(\epsilon = M_1 - m_1 = M_2 - m_2\) is the flavor-independent heavy meson-heavy quark mass difference and the \(\phi_i\) are the usual wave function at the origin (or meson decay constants) that scale as \(f_i \sim 1/\sqrt{M_i}\) [2,50].

To leading order in the heavy mass one obtains
\[
\langle (Q_1\bar{q})|V_\mu + A_\mu (Q_2\bar{q}) = \frac{4\pi\alpha e^2}{\epsilon^2(\omega - 1)^2} f_1 f_2
\]
\[
\frac{1}{4\sqrt{M_1M_2}}\text{Tr}\left\{\gamma_5 + \gamma_5 \gamma_2 + 1\right\}\gamma_\mu(1-\gamma_i\gamma_i + 1)\gamma_5\}
\]
(19)
The second line in Eq.(19) is nothing but the well known HQS *trace* formula for heavy meson transitions (see e.g. [5]) at low recoil.

\[\text{Fig.7: Hard scattering contributions to } (Q_1\bar{q}) \rightarrow (Q_2\bar{q}) \text{ mesonic transition form factors.}\]

In order to exhibit the heavy mass structure of the large recoil amplitude Eq.(19) I define Clebsch-Gordan coefficients \(\tilde{f}_i(\omega)\) that project onto a given transition amplitude \(F_i(\omega)\) according to the trace in Eq.(19). In the low recoil regime of HQS one then has generically
\[
F_1^{\text{HQS}}(\omega) = \tilde{f}_1(\omega) F_1^{\text{HQS}}(\omega)
\]
(20)
where \(F_1^{\text{HQS}}(\omega)\) is the mesonic HQS reduced form factor function. The large recoil amplitudes (denoted by "BL") have the generic structure
\[
\frac{1}{\sqrt{M_1M_2}} F_1^{\text{BL}}(\omega) = \tilde{f}_1(\omega) F_1^{\text{BL}}(\omega) + O(\omega / m_Q)
\]
(21)
where the large \(\omega\)-behaviour of the reduced BL form factor is given by
\[
F_1^{\text{BL}}(\omega) - (\omega - 1)^2
\]
(22)

\[\text{We choose heavy meson transitions to discuss the large recoil behaviour of heavy hadron transitions because mesons are simpler. The large recoil behaviour of heavy baryon transitions is discussed in [46].}\]
In Eq. (21) we have already substituted for the heavy mass scaling behaviour of the wave function at the origin factors $f_i$. The $1/m_Q$ contributions in (21) contain, among others, spin flip contributions proportional to $\omega$ which, when $\omega \gg M_Q$, provide for the correct large $\omega$- or $q^2$-behaviour of the transition form factors.

The leading terms of the large recoil form factors $F^{BL}_L(\omega)$ in Eq. (21) can be seen to possess the spin and flavor symmetry of HQS if the wave function at origin factors $f_i$ are divided out. We mention that the large recoil heavy baryon transition form factors have a structure identical to Eq. (21) and a dipole-type large $\omega$-behaviour of the reduced BL form factor identical to (22) \cite{46}. In this context it is quite intriguing that the first analysis of the experimental $B \to D, D^*$ data in the low recoil regime indicates that the reduced form factor has a dipole-type behaviour even at low recoil.

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