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LEPTON-HADRON CORRELATIONS TO $O(\alpha_s^2)$ IN (2+1) JET PRODUCTION PROCESSES AT HERA*

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LEPTON-HADRON CORRELATIONS TO $O(\alpha_s^2)$ IN (2+1) JET PRODUCTION PROCESSES AT HERA

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We give a qualitative account of some of the technical features that go into the calculation of the one-loop improved $O(\alpha_s^2)$ (2+1) jet production rates in deep inelastic scattering which was completed only recently. We present some numerical results on (2+1) jet production with an emphasis on lepton-hadron correlation effects by exhibiting the contribution of the longitudinal structure function.

1. Introduction

At the time of the meeting at Teupitz in April 1992 HERA had not yet started to operate. This has changed in the meantime while this report is being written up as the first events have been registered at HERA in the beginning of June. While the subject of this talk does bear on HERA physics the deep inelastic ($\doteq$ high Q^2) production of (2+1)-jet events that we are concerned with will certainly not be one of the first physics items to be investigated at HERA but will have to wait a few years until enough luminosity will have been collected at HERA [#1].

Looking back in time the discovery of bona fide 3-jet events at the e^+e^--collider PETRA was the big news in 1979 after PETRA's inception. These 3-jet events and their production characteristics had been predicted to exist from QCD by a number of authors [1–3] due to hard gluon bremsstrahlung from the originally produced quark-antiquark pair. A few years later the $O(\alpha_s)$ calculation of 3-jet production was one-loop improved by calculating the 3-jet production rate at $O(\alpha_s^2)$ [4,5]. At a somewhat later stage these results were appended by calculating also lepton-hadron correlation effects [6–8] including parity violation [9].

Returning to deep inelastic scattering the corresponding $O(\alpha_s)$ calculation of the (2+1)-jet production rates was done as early as 1978 by a number of authors [10–13]. It then took a number of years before first results were obtained on the $O(\alpha_s^2)$ improvement [14–17] of the earlier $O(\alpha_s)$ results. For once there had been no pressing experimental need to undertake such a complicated theoretical calculation. The situa-

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[#1] Much higher effective luminosity will be achieved for (low Q^2) quasi-real photoproduction of jets. The theoretical interpretation of such events, however, is more difficult, since there is added complication of the direct and the resolved photon contribution to the cross section, which are hard to separate.

tion changed, of course, as HERA drew closer to completion. As concerns exclusive three jet production, we are in a better theoretical shape for HERA now in terms of higher order QCD calculations than was the case back in 1979 when PETRA started to operate.

2. Parton Model Calculation

Compared to the e^+e^-–case the computation of one–loop corrected jet production rates in deep inelastic scattering (DIS) is technically more involved. For once, one has a parton in the initial state which brings in additional initial state singularities due to collinear divergencies that have to be dealt with. Tree graph integrations in phase space regions close to these initial state singularities are more difficult to handle than integrations close to final state singularities. Also, the issue of lepton–hadron correlations has to be dealt with from the very beginning in DIS, whereas one can afford to study lepton–hadron correlation effects in the e^+e^-–case as an entirely separate issue [6–9]. In this report we attempt to provide a qualitative description of some of the features of the calculation of one–loop corrected (2+1)–jet production rates in DIS.

Technically the physics of the lepton–hadron correlations in DIS is described by the contraction of the lepton tensor $L^{\mu\nu}$ with the hadron tensor $H_{\mu\nu}$. When one of the hadronic final state momenta in the lepton scattering process is measured, the physics of lepton–hadron correlations in the one–photon approximation is described by altogether five hadronic structure functions. They divide into the following pieces

$$5 = 4(\text{disp.}) + 1(\text{abs.})$$

i.e. four dispersive (disp.) and one absorptive (abs.) contribution. We note in passing that the

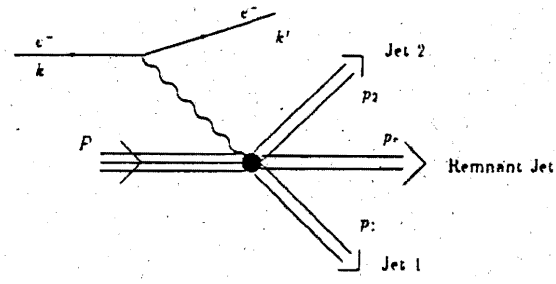

Fig. 1. *(2+1) jet production in deep inelastic scattering*

dispersive and absorptive parts contribute to so–called T–even and T–odd observables. This structure can readily be exhibited by writing down the most general covariant expansion of the hadron tensor. One has:

$$\begin{aligned} H^{pc}_{\mu\nu} &= H_1\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) \\ &+ H_2 \tilde{P}_\mu \tilde{P}_\nu \\ &+ H_3 \tilde{p}_{1\mu} \tilde{p}_{1\nu} \\ &+ H_4(\tilde{P}_\mu \tilde{p}_{1\nu} + \tilde{p}_{1\mu} \tilde{P}_\nu) \\ &+ iH_5(\tilde{P}_\mu \tilde{p}_{1\nu} - \tilde{p}_{1\mu} \tilde{P}_\nu) \end{aligned} \tag{1}$$

where we have employed current conserved momenta variables: $\tilde{P}_\mu = P_\mu - (Pq)q_\mu/q^2$ and $\tilde{p}_{1\mu} = p_{1\mu} - (p_1 q)q_\mu/q^2$ (the proton's momentum is denoted by P and the momentum of the observed final hadron is p_1). The absorptive contribution proportional to H_5 has been slightly set aside. In the totally inclusive case, where no final hadron momentum is measured, one only has the familiar dispersive contributions proportional to H_1 and H_2 in eq(1). In that context the structure functions are then denoted by the more familiar names W_1 and W_2.

In Fig. 1 we depict the (2+1)–jet electroproduction process $e^-(k) + P(P) \to e^-(k') + \text{jet}1(p_1) + \text{jet}2(p_2) + \text{remnant}(p_r)$ where two jets and a remnant target jet are produced. The

(2+1)–jet production cross section and the associated lepton–hadron correlation effects can be calculated in the quark parton model using perturbatively calculated hard scattering cross sections. To $O(\alpha_s)$ in QCD one populates only the dispersive structure functions H_1, H_2, H_3 and H_4 as there are no loop contributions to that order. As mentioned above the $O(\alpha_s)$ contributions to the structure functions and the lepton–hadron correlation effects associated with them have been extensively studied in the literature [10–13,18–21].

The absorptive contribution to H_5 first come in at $O(\alpha_s^2)$ through the imaginary parts of the one-loop contributions. They have been investigated in [17,22,23]. Technically, the contribution to the structure function H_5 can be obtained in two ways. In [22] H_5 was computed by integrating over the intermediate state contributions whereas in [23] H_5 was computed by taking the imaginary part of the one–loop contributions. Although both approaches should yield the same final result due to unitarity there still remain some discrepancies between the results of [22] and [23].

We now turn our attention to the $O(\alpha_s^2)$ calculation on the four dispersive pieces H_1, H_2, H_3 and H_4. At $O(\alpha_s^2)$ we have the following processes that contribute to $H_{\mu\nu}$ (V denotes the off-shell gauge boson):

$$
\begin{array}{lll}
H_{\mu\nu}(\text{tree}): & V+q \rightarrow q+G+G \\
 & V+q \rightarrow q+\bar{q}+q \\
 & V+G \rightarrow q+\bar{q}+G \\
H_{\mu\nu}(\text{loop}): & V+q \rightarrow q+G \\
 & V+G \rightarrow q+\bar{q}
\end{array}
\qquad (2)
$$

and the corresponding antiquark processes with $q \leftrightarrow \bar{q}$. In Figs. 2 and 3 we show some of the contributing Feynman diagrams representing the tree graph and one–loop contributions of the parton subprocesses $V+q \rightarrow q+G+G$ and $V+q \rightarrow q+G$, respectively.

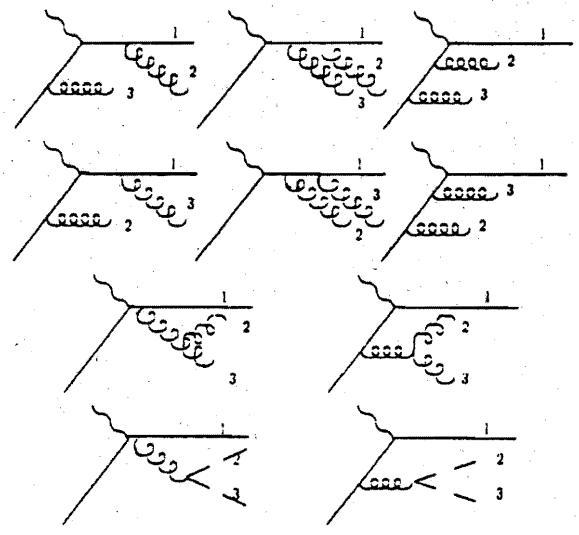

Fig. 2. *Tree graph contributions to the process* $V+q(t_0) \rightarrow q(t_1)+G(t_2)-G(t_3)$.

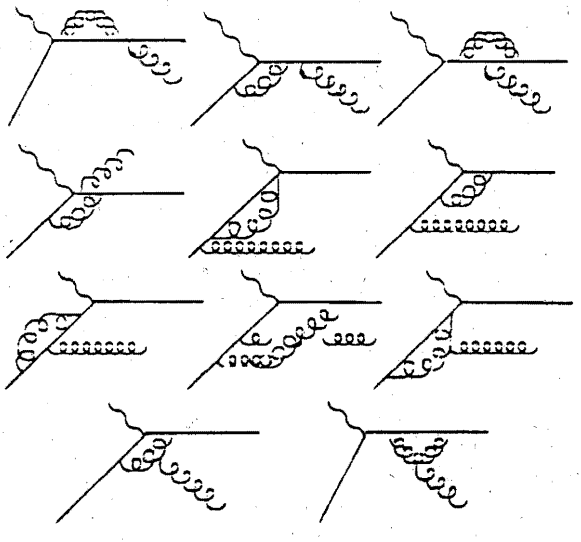

Fig. 3. *One-loop contributions to the process* $V+q(p_0) \rightarrow q(p_1)+G(p_2)$.

Note that the $O(\alpha_s^2)$ tree graph and one–loop matrix elements are required for the full hadron

tensor $H_{\mu\nu}$ and not only for the trace of the hadron tensor as in the first e^+e^-–calculations [4,5]. At Mainz we are in the fortunate situation that we can draw on a rich fundus of checked and rechecked $O(\alpha_s^2)$ matrix elements from corresponding $O(\alpha_s^2)$ calculations done in the crossed processes $e^+e^- \to$ 3–jets [8,26] and $p+\bar{p} \to V(\text{high } p_T)+X$ [24,25] *including lepton–hadron correlation effects*. The crossing of the $O(\alpha_s^2)$ matrix elements from the e^+e^-–case to DIS is straightforward for the tree graph contributions but requires a bit of care for the loop contributions [17,23].

The tree graph contributions are integrated over the unresolved phase space regions which are (2+1)–jet like [#2]. As a resolution criteria we use the usual invariant mass cut, i.e. we define the (2+1)–jet like region by $s_{ij} \leq y_c m^2$ where $s_{ij} = (p_i + p_j)^2$ is the invariant two-jet mass and m^2 is a typical mass scale of the process for which we choose the invariant mass squared of the hadronic final state W^2. The tree graph integrations are carried out up to $O(y_c^0)$ accuracy, i.e. the nonsingular pieces of the tree graph contributions will be of the form

$$A + B \ln y_c + C \ln^2 y_c \, .$$

Infrared (IR) as well as collinear (M) divergencies associated with the final partons are cancelled against corresponding IR/M divergencies of the one-loop contributions. The remaining collinear initial state divergencies are absorbed into the parton densities ("removed by renormalizing the parton densities") introducing a factorization scale dependence M^2 into the parton densities. The ultraviolet (UV) divergencies, finally, are removed by UV renormalization which introduces a renormalization scale dependence μ^2 into the strong coupling constant. We shall always use Q^2 for both scales, i.e. $M^2 = \mu^2 = Q^2$.

Let us now pause and dwell in more detail on that one feature of jet production in DIS that is characteristic for DIS and which occurs in connection with the initial state singularities. Consider first a final state singularity as e.g. drawn in Fig. 4(a) which occurs when $s_{ij} = s_{12} = (t_1 + t_2)^2 \to 0_+$, i.e. when the intermediate fermion goes on its mass shell from the time–like region. The IR/M integration is best done in the "time-like" $(1,2)$ CMS. The IR/M integrations are not very difficult and go as follows. An analysis of the matrix elements in the limit $s_{12} \to 0$ shows that one can factor the Born term and remains with a simple double integration. The only piece which is potentially difficult and does not factorize the Born term is of the form

$$(\text{non-Born-term}) \times \int_0^\pi d\varphi \sin^{-2\varepsilon}\varphi (1 - 2(1-\varepsilon)\cos^2\varphi) \qquad (3)$$

involving an azimuthal integration over the azimuth φ in the $(1,2)$ system with the $n = (4 - 2\varepsilon)$–dimensional integration measure $\sin^{-2\varepsilon}\varphi$. However, the azimuthal averaging in eq(3) can be seen to give a zero result. Remember that in the e^+e^-–case one has only such "simple" final state singularities.

In the case of an initial state singularity as e.g. drawn in Fig. 4(b) one would be interested in the limit $s_{ij} = s_{03} = (t_3 - t_0)^2 \to 0_-$, i.e. when the intermediate fermion goes on mass shell from the space–like region. The corresponding IR/M integration can no longer be done in the $(0,3)$ system as it is space–like. One has to perform the integrations in another time–like system as e.g. again the $(1,2)$ CMS. However, in this case the integration limits in the azimuthal integration eq(3) are no longer $[0,\pi]$ but some general

[#2] (3+1)–jet cross sections outside of the singular phase space regions have been studied in great detail in [19].

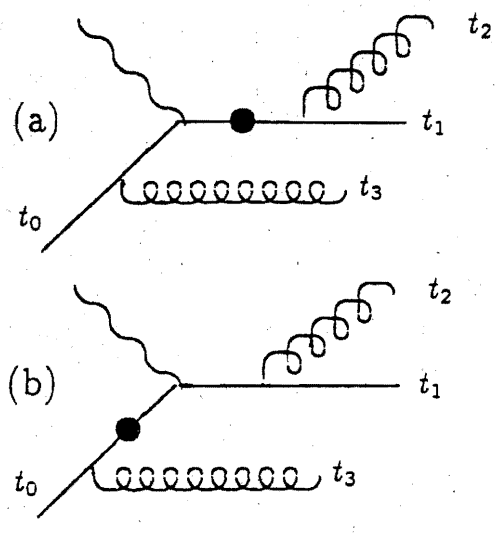

Fig. 4. *Final (a) and initial (b) state singularities. The dot on the intermediate state fermion shows which intermediate state is taken to its mass shell limit.*

limits $[\varphi_1, \varphi_2]$ where the integration limits depend in an intricate way on the boundaries of the jet resolution criteria. One thus remains with non–Born–term like contributions which, in addition, can no longer be integrated analytically by present methods.

To get any further the integrations have to be done in a somewhat back–handed way involving both analytical and numerical integrations. In the first step one analytically integrates over the <u>whole</u> phase–space region including the singular (2+1)–jet region. In the second step one numerically integrates over the <u>complement</u> of the (2+1)–jet region which is IR/M safe since there are no singular regions in the complement. The difference of the two contributions is the desired (2+1)–jet tree graph contribution.

After this excursion into the intricacies of the IR/M tree graph integrations in the DIS case we are now ready to present some numerical results.

Let us first write down the full five–fold differential cross section distribution. One has [18]

$$\begin{aligned}\frac{d\sigma^{pc}}{dx d\eta dz dy d\chi/2\pi} = [1+(1-y)^2] \quad & \hat{H}_U \\ +[2(1-y)] \quad & \hat{H}_L \\ +[2(1-y)\cos 2\chi] \quad & \hat{H}_T \\ +[(2-y)\sqrt{1-y}\cos\chi] \quad & \hat{H}_I \\ +[y(1-y)\sin\chi] \quad & \hat{H}_5 \end{aligned} \tag{4}$$

where we use the standard set of variables $x = Q^2/2Pq$, $x_p = Q^2/2p_0q$, $y = p_0q/p_0k$, $z = p_0p_1/p_0q$ with $q = k-k'$, $Q^2 = -q^2$ and $p_0 = \eta P$, where η is the proton's momentum fraction carried by the parton that initiates the hard scattering process; $s = (P+k)^2$ denotes the overall C.M. energy.

The $\hat{H}_i$, $(i = U, L, T, I, 5)$ are the five helicity structure functions that describe p.c. one–photon–exchange (2+1)–jet production (U: unpolarized transverse; L: longitudinal; T: transverse interference; I: transverse–longitudinal interference; 5: imaginary part of the transverse–longitudinal interference part). The five helicity structure functions $\hat{H}_i$ can be linearily related to the covariant structure functions defined in eq(1) [23]. After azimuthal averaging we remain with the unpolarized transverse and longitudinal contributions $\hat{H}_U$ and $\hat{H}_L$. One has

$$\begin{aligned}\frac{d\sigma^{pc}}{dx d\eta dz dy} &= [1+(1-y)^2]\hat{H}_U \\ &+ [2(1-y)]\hat{H}_L \end{aligned} \tag{5}$$

$$\begin{aligned} &= \tfrac{1}{4}[1+(1-y)^2]H_g \\ &+ \tfrac{1}{2}[6-6y+y^2]H_L \end{aligned} \tag{6}$$

where we have introduced the "metric" structure function $H_g = -g^{\mu\nu}H_{\mu\nu} = 4\hat{H}_U - 2\hat{H}_L$ in eq(6) #3. The $O(\alpha_s^2)$ (2+1) jet production rate

#3 In [16] we have used the notation $H^{UT} := H^g$ for

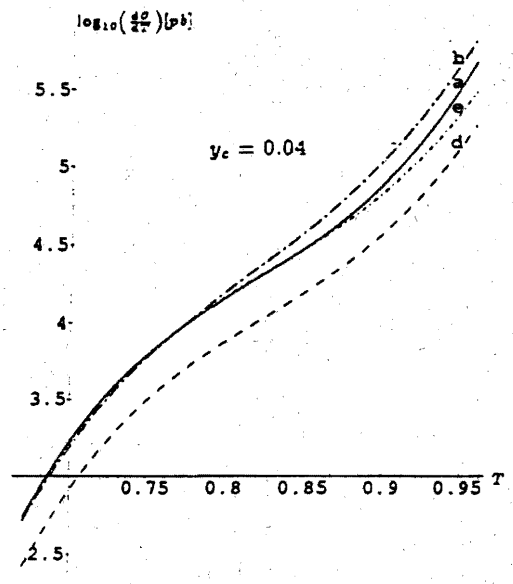

Fig. 5. *Thrust distributions: $O(\alpha_s)$ (a) and $O(\alpha_s^2)$ (b) metric contributions and $O(\alpha_s)$ (d) and $O(\alpha_s^2)$ (e) longitudinal contributions.*

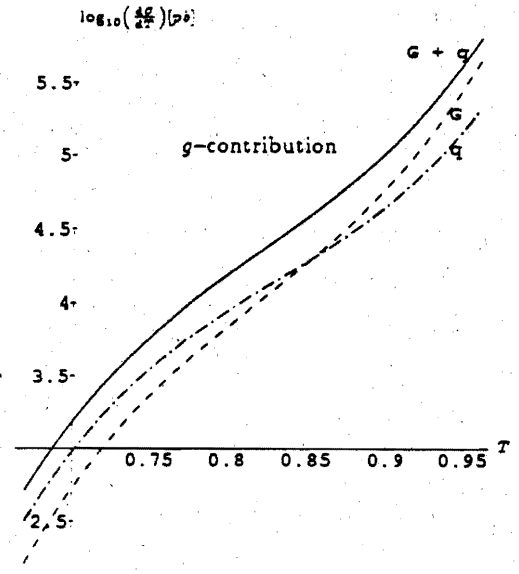

Fig. 6. *$O(\alpha_s^2)$ metric distributions separated into quark (q) and gluon (G) initiated contributions.*

corresponding to H_g and H_L have been calculated in [14,15] and [16,17], respectively.

Let us now turn to our numerical results. In Fig. 5 we show the differential thrust distribution for (2+1) jet production #4. We have taken $y_c = 0.04$ for the cut variable, $m^2 = W^2$ and have limited the invariant jet mass by $4\,GeV^2$ from below. The lower limit of the Q^2 integrations is $4\,GeV^2$. In order to exhibit the strength of the longitudinal contribution H_L we separately plot the contribution of the metric (upper two curves (a) and (b)) and longitudinal (lower two curves (d) and (e)) structure functions H_g and H_L. The $O(\alpha_s^2)$ corrections to H_L are quite large over the whole thrust range and amount to

the metric structure function. We prefer the present notation replacing H^{UT} in [16] by H_g.

#4 For the definition of the thrust variable and a discussion of $O(\alpha_s)$ thrust distributions see [2].

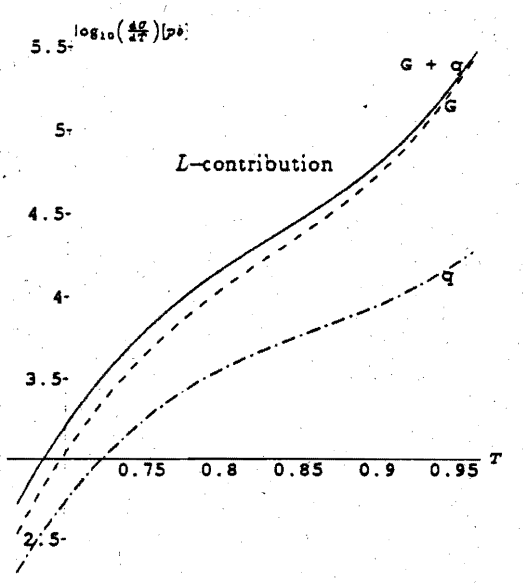

Fig. 7. *$O(\alpha_s^2)$ longitudinal distributions separated into quark (q) and gluon (G) initiated contributions.*

$\approx 50\%$. The $O(\alpha_s^2)$ corrections to H_g are smaller and are largest for large thrust values ($\approx 20\%$). The longitudinal contribution is smaller than the metric contribution for large thrust values by about 50%. At smaller thrust values the $O(\alpha_s^2)$ corrected longitudinal and metric contributions are of equal size. All in all, the longitudinal contribution is somewhat smaller than the metric contribution but is certainly not negligible.

In Figs. 6 and 7 we break down the $O(\alpha_s^2)$ quark and gluon contributions to the metric and longitudinal contributions. Whereas the quark and gluon contributions to H_g are approximately equal (Fig. 6) the gluon contribution to H_L is clearly dominant.

3. Summary

In this report we have given a qualitative account and some numerical results on the $O(\alpha_s^2)$ calculation of the deep inelastic leptoproduction of (2+1) jets emphasizing the importance of the longitudinal contribution. The present investigation stands only at the beginning of what will be an extended experimental and theoretical effort to study high Q^2 jet physics at HERA. What remains to be done on the theoretical side are the inclusion of parity–violating effects that arise e.g. in the charged current interactions. One needs to do more numerical studies on the sensitivity of the results on the choice of renormalization and factorization scales with an eye on optimizing these choices. Further one wants to investigate the dependence of the results on various available parametrizations of parton distributions and on different jet definition schemes. Finally one wants to extend the present calculation to compute the high Q^2 single jet inclusive cross section at $O(\alpha_s^2)$.

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