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THE OPERATOR PRODUCT EXPANSION OF THE QCD PROPAGATORS

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Abstract We bring together for the first time the coefficients in covariant gauges of *all* the condensates of dimension four or less in the operator product expansion (OPE) of the quark, gluon and ghost propagators. It is stressed that contrary to general belief the condensates do not enter the OPE of the propagators in gauge-invariant combinations like $< m\bar{\psi}\psi >$ and $< G^2 >$. The results are presented in arbitrary dimension to lowest order in the light quark masses for the $SU(N_c)$ internal symmetry group. All terms which, through the equations of motion, may be viewed as being effectively of order α_s are included. The importance of the equations of motion if one is to fulfill the Slavnov-Taylor identities is demonstrated. We briefly consider the equivalent, but less complete, calculations in other gauges and give an overview of the status of the OPE of the QCD vertices. Finally we discuss what these nonperturbative structures tell us about the correct solutions of QCD and point out their significance for the Fourier acceleration technique as applied to lattice QCD.

To appear in *Mod. Phys. Lett. A – Brief Reviews*

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1. Introduction

That the operator product expansion (OPE) of gauge-invariant correlation functions yields important non-perturbative physics is evinced by the success of the QCD sum rules pioneered by Shifman, Vainshtein and Zakharov[1,2]. This has also lead to some models which in various ways attempt to include condensate effects[3]. In this article we will present the OPE of the (gauge-dependent) QCD propagators. Although this has attracted continued interest, it has been misunderstood right from the earliest works in this area[4]. The error has been to tacitly assume that only gauge-invariant condensates, like $< m\bar{\psi}\psi >$ and $< G^2 >$, enter the OPE of the propagators. (This is also assumed in the models of Ref. 3). In fact, as will be shown below, this is not the case and the non-perturbative effects in the OPE of these Greens' functions are far richer than assumed. Indeed the OPE of gauge-dependent quantities differs fundamentally from that of QCD sum rules.

Why should one be interested in the OPE of these gauge-dependent quantities? Next to nothing is known about the form of the non-perturbative solutions of QCD and that information that the OPE yields can give us valuable clues and constraints on these. Here we will primarily discuss the non-perturbative *propagators* given by the OPE. What is now known about the vertex structures will also however, be sketched. Furthermore, inside perturbation theory at least, there is some gauge-invariant information in the quark propagator, namely the pole mass[5] and it is clearly of interest to see what the OPE says about this quantity. Knowledge of the propagators can also be of use in lattice QCD, as will be discussed in the conclusions.

We give the OPE results for a general $SU(N_c)$ symmetry group in D-dimensions. This last means that the OPE results may be later used in conjunction with dimensional regularisation. Gauge-invariance implies that there are many different ways to formulate the theory. The results presented here will mostly have been obtained in the general Lorentz class. The use of a class of gauges with an explicit parameter helps one to see what is gauge invariant and what is not. One cannot here employ Fock-Schwinger gauge, $(x_\mu - x^0_\mu)A^\mu(x) = 0$, which is commonly used in QCD sum rules[6], since the perturbative gluon propagator in this gauge is neither tractable nor particularly well-defined[7]. In deriving this non-translationally invariant propagator one comes across various divergences which need to be regulated; experience in axial gauges teaches us that such regularisations need to be handled very carefully[8]. In those sum rules where no perturbative gluon propagators occur this gauge may be used.

We will present the leading terms in the OPE of the propagators. By this we mean that one considers the lowest dimensional condensates (dimension four or less) and works

to effective order α_s in the coupling. 'Effective' signifies here that we include terms which, although superficially of higher order in the coupling, are related via the equations of motion to terms of order α_s. This will be explained in detail below. The results are for light quarks only; terms of order m in the coefficients of gluonic and ghost condensates and those of order m^2 for fermionic condensates have been neglected.

The main conclusions from this review are as follows. The quark pole mass seems to become explicitly gauge-dependent when condensate effects are included, gauge-dependent combinations of condensates enter the OPE of the propagators and the non-perturbative corrections are *not* multiplicative corrections to the perturbative structures. In fact, it seems that when non-perturbative effects are included a sort of "Murphy's law" appears: any Lorentz structure that can enter does. This conclusion is of importance for people trying to solve the QCD Schwinger-Dyson (SD) equations and for model builders.

In Sect. 2 we discuss the basics of the OPE and give conventions. The consequences of gauge-invariance for the OPE of QCD sum rules are discussed. In the next three sections we respectively give the form of the quark, gluon and ghost propagators coming from the OPE. It is shown that the OPE of these propagators is distinguished by the appearance of gauge-dependent condensate combinations. In Sect. 6 our more limited knowledge of the OPE of the propagators in other gauges is reviewed. Finally in Sect. 7 we very briefly discuss the OPE of the vertices and present conclusions. We hope that this review will provide a compendium of results and a feel for the physical consequences of the non-perturbative effects described in part by the OPE.

2. The Operator Product Expansion

The OPE in covariant gauges comes under a variety of names (the plane wave method[9], co-ordinate space approach[10] and moments method[11]) but they may all be seen to be equivalent[12]. Here the scheme of Ref. 11 will be followed. We prefer this method for its simplicity. We stress however, that the results are reproduced in the other approaches. The method and the subtlety that certain combinations of condensates vanish as a consequence of the equations of motion[13] will be illustrated through the example of the fermionic condensates $<m\bar{\psi}\psi>$, $<\bar{\psi}i\partial\!\!\!/\psi>$ and $<\bar{\psi}gA\!\!\!/\psi>$. This is somewhat simpler than the case of gluonic condensates, where more operators appear.

The non-perturbative propagator is defined as the difference between the full propagator and the perturbative part. In covariant gauges the full quark propagator may be written as

$$S(k) = S^{pert}(k) + S^{nonpert}(k) = S^{pert}(k) + \not{k}\widetilde{A}(k^2) + \widetilde{B}(k^2)\,. \tag{2.1}$$

From this we obtain the relationships[11]

$$\begin{aligned} \int_{k^2 \geq -\mu^2} i\frac{d^D k}{(2\pi)^D} m\widetilde{B}(k^2) &= -\frac{<m\bar{\psi}\psi>}{4N_c}\,, \\ \int_{k^2 \geq -\mu^2} i\frac{d^D k}{(2\pi)^D} k^2\widetilde{A}(k^2) &= -\frac{<\bar{\psi}i\not{\partial}\psi>}{4N_c}\,. \end{aligned} \tag{2.2}$$

Note that in Ref. 11 the latter relation was expressed as $\int \cdots = \frac{<m\bar{\psi}\psi>}{4N_c} + O(g)$. While this formally follows from the equation of motion, we will see below that it must be used cautiously. In Eq. 2.2 D is the spatial dimension, m is the quark mass and we have assumed an $SU(N_c)$ symmetry group. The integrals are cutoff at the renormalisation point, μ^2, because perturbative effects should dominate beyond this point in the full propagator. (Thus if one were to know all the perturbative corrections, the cutoff would be unnecessary).

In a similar way [14] we can find the relationship determining the mixed condensate

$$\int_{k^2,l^2 \geq -\mu^2} i\frac{d^D k d^D l}{(2\pi)^{2D}} \mathrm{tr}[\widetilde{\Gamma}^c_\mu(k,l)\gamma_\mu t^c] = -<\bar{\psi}g\not{A}\psi>\,. \tag{2.3}$$

OPE corrections are now calculated in the following way. The full Feynman diagram is written down and the equivalent perturbative one subtracted. The loop momenta are expanded in inverse powers of the external momenta (recall that the OPE is only valid in the deep Euclidean region, where this expansion makes sense) and moments are identified with condensates in the manner described above.

An example of the application of this method to QCD sum rules was given in Ref. 14. The coefficients of the condensates given by Eq.'s 2.2 and 2.3 in the vector two-point sum rule were found to leading order. It was seen that the condensates $<\bar{\psi}i\not{\partial}\psi>$ and $<\bar{\psi}g\not{A}\psi>$ entered in such a way as to form the condensate combination which vanishes through the quark equation of motion and to leave remaining just the usual gauge-invariant quark condensate, $< m\bar{\psi}\psi >$. This is a consequence of the gauge-invariance of the sum rule, which means that the only condensates which can enter are either a) gauge-invariants, b) such that they vanish via the equations of motion or c) because they are BRS-variations of other operators. Thus the coefficient of the mixed condensate does not need to be (and is usually not) calculated because knowledge of the coefficients of two of the three condensates suffices to perform the rearrangement into $< m\bar{\psi}\psi >$ and the equation of

motion condensate. The neglect of all the terms like $<\bar{\psi}i\partial\!\!\!/\psi>$ and $<\bar{\psi}gA\!\!\!/\psi>$ has led to incorrect condensate coefficients in the OPE of the propagators in the past[15].

Certain combinations of gluonic and ghost condensates also vanish in the way described above. This has been used in various sum rule calculations[13,16]. For completeness and because they will be needed below we now give all three equations of motion:

$$\bar{\psi}(i\partial\!\!\!/ + gA\!\!\!/ - m)\psi = 0\,, \tag{2.4a}$$

$$\frac{1}{2}(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)^2 + 3gf^{abc}\partial_\mu A^a_\nu A^b_\mu A^c_\nu + (gf^{abc}A^b_\mu A^c_\nu)^2 - \bar{\psi}gA\!\!\!/\psi - \bar{c}^a\Box c^a = 0\,, \tag{2.4b}$$

$$\bar{c}^a\Box c^a - gf^{abc}\partial_\mu\bar{c}^a A^b_\mu c^c = 0\,. \tag{2.4c}$$

It is clear that the vanishing of certain combinations of condensates through the equations of motion must be taken into account. Otherwise, for example, the sum rules would seem to become gauge-parameter dependent. Another method to remove the equation of motion contributions is to give the condensates a momentum which only gets taken to zero at the end of the calculation. This *non-zero momentum insertion* (NZI) method is described in Ref. 17.

There is a fundamental difference however, between the QCD propagators and sum rules, namely the former are gauge-dependent. This means that the condensates do not have to appear in particular combinations and so it is necessary to calculate *all* the coefficients of all condensates of a particular dimension. A remnant of gauge-invariance is here, as we shall see in Sect. 4, provided by BRS invariance and the Slavnov-Taylor identities (STI's). Otherwise there is no reason to assume that only the condensate combinations allowed in the sum rules must appear here. With these remarks, we are now in a position to consider the OPE of the propagators.

3. The Quark Propagator

The leading order terms in the OPE of the quark propagator follow from the diagrams of Fig. 1. They yield the following self-energy[11,14,18,19]:

$$\Sigma(p) = \frac{\pi\alpha_s}{2N_c^2 D p^4}\Big\{(N_c^2-1)\Big(D(D-1+\xi)\frac{p^2}{m} + 2(2-D)\xi\not{p}\Big) < m\bar{\psi}\psi >$$

$$+2\Big[\Big(D^2-5D+4-N_c^2(D-1)+(2-N_c^2)\xi\Big)\not{p} + (2-D)(D-1+\xi)m\Big] < \bar{\psi}g\not{A}\psi >$$

$$-\frac{D-2}{D+2}\not{p}\Big[4N_c(D+2)p^2 < A_\mu^a A_\mu^a > -2N_c D < (\partial_\mu A_\nu^a - \partial_\nu A_\mu^a)^2 >$$

$$-4N_c(D+2) < g f^{abc}\partial_\mu A_\nu^a A_\mu^b A_\nu^c > -2(D-4) < (g A_\mu^a A_\mu^a)^2 >$$

$$+N_c < (g d^{abc} A_\mu^b A_\mu^c)^2 > +2N_c(D+4) < (g f^{abc} A_\mu^b A_\nu^c)^2 >\Big]\Big\}. \tag{3.1}$$

Note that unsummed colour indices have been dropped. The quark equation of motion (2.4a) has been employed to eliminate $< \bar{\psi} i\not{\partial}\psi >$. There are no ghost condensates in leading order in α_s in the OPE of this propagator. We did not make use here of the gluon equation of motion, since it would introduce ghost condensates that otherwise do not enter this self-energy in leading order. The correct coefficient of the abelian condensate $< (\partial_\mu A_\nu^a - \partial_\nu A_\mu^a)^2 >$ has previously[18] been erroneously identified as the coefficient of the gauge-invariant $< G^2 >$. The coefficients of the condensates with three and four gluons are given here for the first time, and we see that the condensates do not combine to form $< G^2 >$.

Since gauge-dependent condensates appear, it should not be surprising that the dimension two, Lorentz-invariant condensate $< A^2 >$ has a non-vanishing coefficient in Eq. 3.1. (In QCD sum rules explicit calculation shows that its coefficient is zero, which is a consequence of gauge-invariance.) We cannot yet say what values these gauge-dependent condensates have.

If we were now to neglect terms superficially of order g and all purely gluonic condensates, we would regain the result of Pascual and de Rafael[19]

$$\Sigma(p) = \frac{(N_c^2-1)\pi\alpha_s < m\bar{\psi}\psi >}{2N_c^2 p^2}\left(\frac{2\xi(2-D)}{D}\frac{\not p}{p^2} + (D-1+\xi)\frac{1}{m}\right), \tag{3.2}$$

which is a generalisation of Politzer's original calculation[4] from Landau gauge to an arbitrary Lorentz gauge[3].

The authors of Ref. 19 saw that their result was gauge dependent and said that this precluded a physical interpretation for the mass term in Σ. It has been argued since by Elias, Scadron and collaborators in a series of papers that the pole mass is gauge invariant when fermionic condensates of dimension four and five are included[20]. To see this they must assume that the mass of the soft quarks is not the current one, but is itself equal to the pole mass. It is not clear to us if this is justified, or indeed what the numerical consequences of such an idea would be for QCD sum rules. It is however, then evident that the ξ-dependence of (3.2) vanishes at $\not p = m_{\text{Pole}}$. From (3.1) however, we see[14] that the mixed condensate terms $< \bar{\psi} g \not A \psi >$ do not, unlike the corresponding terms in QCD sum rules, simply vanish – even at the pole. The gluonic condensates also do not arrange themselves in a gauge-invariant way. It appears that there is a clear difference here between the OPE and perturbation theory, where even at two loops[5] the pole mass is gauge-invariant. One way out of this could be that higher order moments (involving condensates like $< \bar{\psi}(i\not\partial)^n\psi >$) yield, through repeated application of the equation of motion, terms that *at the pole* cancel the extra terms we see above. The other higher order in the coupling terms so generated would however, also then have to so vanish, which they will if the pole is gauge invariant. This requires further study. Certainly however, such a summation "to all orders in the masses" does not seem to be a consistent expansion for such evidently gauge dependent quantities as the propagator away from the pole[18], since terms which are only superficially of higher order in g are unjustifiably neglected. Leaving aside the question of the validity of the OPE at the pole mass found in Ref. 20, $m_{\text{Pole}} = 320\text{MeV}$, the formal gauge-dependence or independence of the pole may provide a distinction between perturbative QCD and the OPE.

In general from (3.1) we see that gauge-dependent admixtures of condensates appear. This, as we shall see general feature, means that it is impossible to give numerical values to the condensates entering the OPE of the propagators. In fact new, exotic condensates involving anticommutators of the vector potentials appear, which cannot contribute to $< G^2 >$ in any way. Nevertheless, when gauge-independent quantities are constructed from these propagators and the analogous vertices, only gauge-invariant condensate combinations can appear.

[3] These authors also corrected an overall factor

4. The Gluon Propagator

The OPE of the gluon propagator is the most complicated of the three. This is because gluon, ghost and quark condensates all enter here in leading order. It is also however, constrained by the relevant STI.

The OPE of the polarisation[11,17,21–23] is obtained from the diagrams of Fig. 2. The full result is

$$\Pi_{\mu\nu}(p) = \frac{8\pi N_c \alpha_s}{(N_c^2-1)Dp^2}\Big\{\Big[\frac{2(N_c^2-1)}{N_c^2} < m\bar{\psi}\psi > + \frac{1}{2}(D-3(1+\xi)) < \bar{\psi} g \not{A} \psi >$$

$$+ \frac{D-5-\xi}{2} p^2 < A_\mu^a A_\mu^a >$$

$$+ \frac{32-18D-6D^2+3\xi(D^2-3D-6)}{4(D-1)(D+2)} < (\partial_\mu A_\nu^a - \partial_\nu A_\mu^a)^2 >$$

$$- \frac{14D-20+2\xi(5-2D)}{2(D-1)} < g f^{abc} \partial_\mu A_\nu^a A_\mu^b A_\nu^c >$$

$$- \frac{D^2+10D-12+3\xi(3-2D)}{12(D-1)} < (g f^{abc} A_\mu^b A_\nu^c)^2 >$$

$$+ \frac{11D-D^2-33-3\xi}{24N_c(D+2)}$$

$$\times < 8(2(A_\mu^a A_\nu^a)^2 + (A_\mu^a A_\mu^a)^2) + N_c(2(g d^{abc} A_\mu^b A_\nu^c)^2 + (g d^{abc} A_\mu^b A_\mu^c)^2 > \Big] g_{\mu\nu}^{\perp}(p)$$

$$+\frac{D(N_c^2-1)}{2N_c^2}\Big(\frac{4-D}{D}g^{\perp}_{\mu\nu}(p)-\frac{p_\mu p_\nu}{p^2}\Big)<\bar{\psi}(i\partial\!\!\!/+gA\!\!\!/-m)\psi>$$

$$+\Big(\frac{1}{2}(-2+\xi)g^{\perp}_{\mu\nu}(p)+\frac{1}{2}(2-D)\frac{p_\mu p_\nu}{p^2}\Big)$$

$$\times<\bar{c}^a\Box c^a-\frac{1}{2}(\partial_\mu A^a_\nu-\partial_\nu A^a_\mu)^2-3gf^{abc}\partial_\mu A^a_\nu A^b_\mu A^c_\nu-(gf^{abc}A^b_\mu A^c_\nu)^2+\bar{\psi}gA\!\!\!/\psi>$$

$$+\Big(\frac{1}{2}g^{\perp}_{\mu\nu}(p)+\frac{1}{2}D\frac{p_\mu p_\nu}{p^2}\Big)<\bar{c}^a\Box c^a-gf^{abc}\partial_\mu\bar{c}^a A^b_\mu c^c>\Big\}. \tag{4.1}$$

This is clearly a complicated expression! However, we can straightforwardly see that the polarisation is transverse, i.e. the STI is fulfilled. This is because the combination of condensates in the longitudinal part of the polarisation vanishes as a consequence of the equations of motion and these terms (the last three) can be dropped in (4.1). The transverse part is however, gauge dependent. There is both an explicit gauge-parameter dependence and the condensates themselves appear in gauge-dependent ways.

Here we also note that the assumption that only gauge-invariant condensates enter the OPE of the propagators has been responsible for disagreements in the past. Most authors have calculated the coefficients of the operator $<(\partial_\mu A^a_\nu-\partial_\nu A^a_\mu)^2>$, but Larsson[15] calculated the coefficients of $<(gf^{abc}A^b_\mu A^c_\nu)^2>$ in the quark and gluon propagators. Both sets of authors assumed that they had calculated the coefficients of $<G^2>$, and contradictory results were published. The full equations (3.1) and (4.1) explain these discrepancies. (Note however, that some factors in Ref. 15 are also incorrect.)

5. The Ghost Propagator

The effective ghost propagator acquires the following non-perturbative corrections[11,23] to its self-energy from the diagrams of Fig. 3

$$\Pi(p^2) = \frac{4\pi N_c \alpha_s}{(N_c^2-1)Dp^2}\Big\{\Big(1-D+\frac{1}{2}\xi(3D-5)\Big) < \bar{c}^a \Box c^a > + p^2 < A_\mu^a A_\mu^a >$$

$$+ \frac{2-D}{2(D+2)} < (\partial_\mu A_\nu^a - \partial_\nu A_\mu^a)^2 > - \frac{1}{2} < g f^{abc} \partial_\mu A_\nu^a A_\mu^b A_\nu^c >$$

$$+ \frac{1}{4N_c(D+2)} < 8(2(A_\mu^a A_\nu^a)^2 + (A_\mu^a A_\mu^a)^2) + N_c(2(g d^{abc} A_\mu^b A_\nu^c)^2 + (g d^{abc} A_\mu^b A_\mu^c)^2 >$$

$$- \frac{1}{2}\xi(D+1) < \bar{c}^a \Box c^a + g f^{abc} \partial_\mu \bar{c}^a A_\mu^b c^c > \Big\}. \tag{5.1}$$

Again the corrections are gauge-dependent. There are no quark condensate contributions to the ghost self-energy in this order in the coupling. It should be stressed that there is no reason to consider these corrections less important than those in the other propagators. Although it is often assumed that unphysical degrees of freedom like ghosts do not receive non-perturbative corrections[24], we see that this is not the case. This should not be surprising, since the fermions in the Lagrangian and the three non-longitudinal gluons are not the physical degrees of freedom in the covariant formulation of QCD.

6. Non-covariant Gauges

The Fock-Schwinger gauge has the apparent advantage over the covariant gauges that the vector potential may be written in terms of the field strengths. For the choice $x_0 = 0$ for example, one has: $A_\mu = \int_0^1 \alpha d\alpha G_{\rho\mu}(\alpha x) x^\rho$. This makes for a very simple form of the gluonic condensate corrections to the quark propagator[6]

$$\Sigma = \frac{\alpha_s \pi m}{3p^6} < G^2 >, \quad (N_c = 3, D = 4). \tag{6.1}$$

The most direct problem with this gauge[7], as stated in the introduction, is perturbative[4]. No gluon propagator is known and it is by no means obvious if ghosts really decouple. It is thus not possible to write the equivalents of Eq.'s 3.1, 4.1 and 5.1 in this gauge. (The use of two different gauges, one for perturbation theory and one for the condensates has

[4] Non-perturbative problems are briefly discussed in Ref.25

been argued[26] to give gauge-invariant results on-shell, but off-shell this is certainly not the case.)

In axial gauges there are also many subtleties in perturbation theory. Generally it is necessary to regularise these gauges[27], and it is still unclear if this can be done in such a way as to retain the attractive features of these formulations of QCD[28]. In Ref. 29 an attempt was made to consider OPE corrections to the propagators in axial gauges and, to a lesser extent, in Coulomb gauge. The results are not complete because only terms that were superficially of leading order in the coupling were retained. The results do give us some useful information however. Firstly, the quark condensate contributions to the quark propagator in axial gauges was

$$\Sigma(p) = \frac{\pi\alpha_s(N_c^2-1)<m\bar{\psi}\psi>}{2N_c^2p^2}\left\{\frac{1}{m}\left(2+\frac{\eta^2p^2}{(p\cdot\eta)^2}\right)-\frac{2\not{p}}{p^2}+\not{\eta}\left(\frac{4p\cdot\eta}{p^2\eta^2}+\frac{2}{p\cdot\eta}-\frac{\eta^2p^2}{(p\cdot\eta)^3}\right)\right\}, \quad (6.2)$$

which yields the same gauge-invariant pole mass as one finds from Eq. 3.2 above. But recall that terms like $<\bar{\psi}g\not{A}\psi>$ were neglected, so this is incomplete. Secondly, the quark condensate contribution to the gluon polarisation was (the equation of motion has been implicitly used and again the $<\bar{\psi}g\not{A}\psi>$ terms have been dropped)

$$\Pi_{\mu\nu}(p) = \frac{8\pi\alpha_s<m\bar{\psi}\psi>}{N_cp^2}\left(g_{\mu\nu}^{\perp(1)}(p)-g_{\mu\nu}^{\perp(2)}(p)\right),$$

$$g_{\mu\nu}^{\perp(1)}(p) := g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}, \quad g_{\mu\nu}^{\perp(2)}(p) := \frac{g_{\mu\nu}(p\cdot\eta)^2}{p^2\eta^2}-\frac{(p_\mu\eta_\nu+p_\nu\eta_\mu)p\cdot\eta}{p^2\eta^2}+\frac{\eta_\mu\eta_\nu}{\eta^2}. \quad (6.3)$$

This fulfills the STI and presumably the terms dropped from the equation of motion will not change this. The significant feature to note is that in axial gauges (with the extra vector, η_μ available) two tensor structures are compatible with the STI and that both of these appear; i.e. new structures other than the perturbative one enter. We will return to this in the conclusions.

The question of whether ghost and longitudinal gluon condensate decouple or not cannot be answered from the work of Ref. 29, since the equations of motion were not taken into account.

7. Discussion

The most glaring difference between the OPE of the propagators and that of QCD sum rules is that the condensates enter here in gauge-dependent combinations whose numerical values are unknown. As has been seen, the coefficient of $<(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)^2>$ is not the coefficient of $<G^2>$! Thus the information coming from the OPE about the solutions of QCD is more subtle than is generally believed[30].

That the condensates do combine correctly for gauge-invariant quantities, was seen both in the QCD sum rule calculations refered to in Sect. 2 and in calculations of the effective potential of QCD[31].

The OPE in axial gauges revealed that new Lorentz structures appear, (6.3). The OPE of the vertices in covariant gauges displays this phenomenon to a far greater extent. This was first seen in Ref. 32, where the OPE of the QCD three-point functions with fermionic condensates was investigated. It became clear that the OPE effects did *not* merely yield a new multiplicative factor in front of the Lorentz structure of the perturbative vertex. Non-perturbative effects in standard model three- and four-point functions have also been investigated[33]. In none of these approaches were condensates that are only superficially of higher order in the coupling retained. Such full calculations of the OPE of the three-point functions are now being carried out[34]. Indeed there is already evidence that use of the equations of motion alone does not suffice to satisfy the STI for the three-point gluon vertex and that at least some of the exotic condensates involving anticommutators, $\sim d^{abc}$, need to be set to zero to fulfill this identity[35]. If this proves to generally be the case it will lead to a simplification of the above results.

The results presented here should provide useful information about the complex structure of the solutions of the QCD SD-equations. This was already noticed by the authors of Ref. 36, whose results we find very promising although they still face various difficulties: e.g. the values for both condensates, $<\bar{\psi}\psi>$ and $<G^2>$, have the wrong sign. This certainly is a consequence of the fact that the most general ansatz has not been used. In particular the ghost Greens functions were approximated by the perturbative ones. Here we expect that the OPE of propagators and vertices may help to restrict the ansätze for the solutions. It should however be kept in mind that the OPE results are only valid in the deep Euclidean region.

The OPE leads to a picture of propagators which, while obeying the STI, have running, gauge-dependent masses. This has consequences for the Fourier acceleration technique[37] in lattice QCD. Here an ansatz for the two- point functions of the theory is used to combat critical slowing down. Generally it has been assumed that the effective propagators are

well described by free ones with constant mass terms[38]. In non-confining theories Fourier acceleration has been successful. It is not surprising that this has not been so much the case for QCD simulations[39]. Not only does this ansatz, which is reminiscent of the Nambu Jona-Lasinio (NJL) model, not emerge from the OPE, but it is not intuitively favoured since such propagators, like the NJL model itself, do not say anything about confinement. It would be of great interest to use a running mass ansatz (to be numerically optimised) in the Fourier acceleration of lattice QCD.

In the above we have discussed the usual formulation of QCD. A gauge-invariant version of the theory has been suggested by Cornwall[39], and in Ref. 40 the OPE of the effective gluon propagator was investigated. It appeared that the propagator was gauge-invariant even when mixing with equation of motion condensates was taken into account, but the three and four gluon operators were not calculated. Further work is necessary both here and to see how quark condensates enter in this scheme[41]. The OPE of supersymmetric QCD has been considered in Ref.'s 42 and 43. However, it was assumed there that only gauge-invariant condensates enter the OPE. Finally we note that the renormalisation group improvement of the results reviewed here is highly desirable.

Acknowledgements ML thanks Martin Schaden, Andreas Streibl and Jörg Ahlbach for many discussions, Vic Elias and Tom Steele for discussions and hospitality at the University of Western Ontario, Prof. V. Spiridonov for correspondence and the Graduate College of the University of Mainz for financial support. MO thanks Prof. M. Scadron for discussions and Prof. E. Werner for his support.

Figure Captions

Fig. 1 Diagrams with leading OPE contributions to the quark propagator. Non-perturbative propagators and vertices are represented by dashed circles.

Fig. 2 The leading OPE corrections to the gluon propagator.

Fig. 3 The leading OPE corrections to the ghost propagator.

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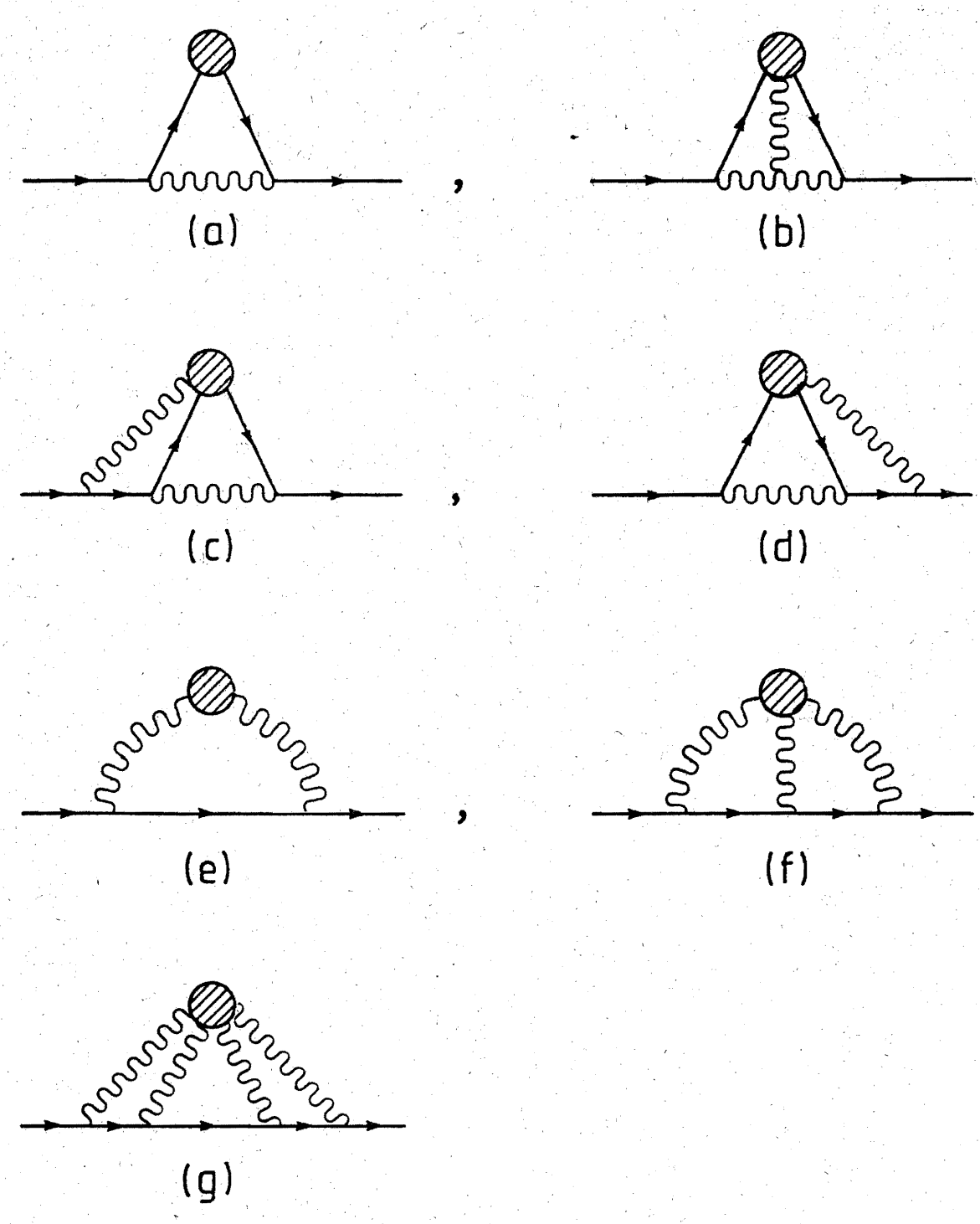
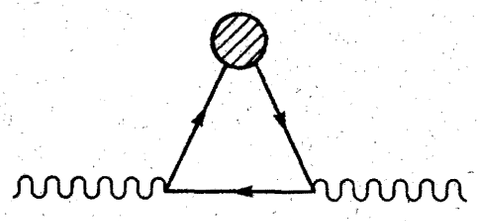


Fig. 1



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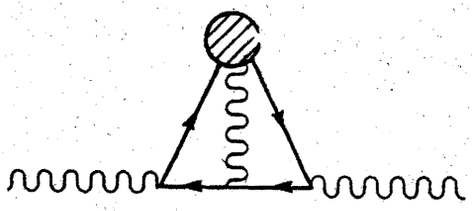

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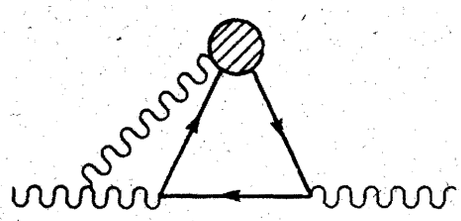

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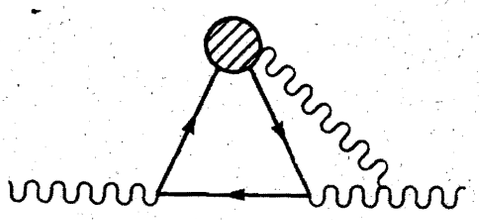

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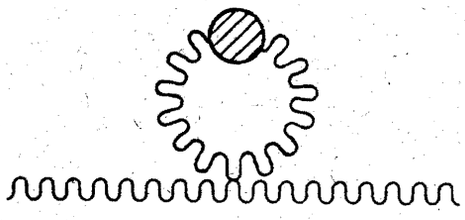

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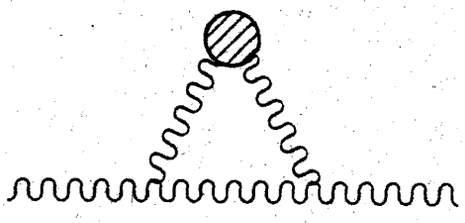

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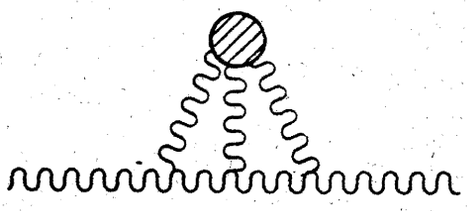

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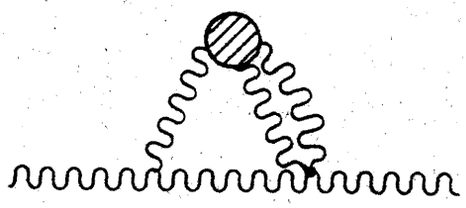

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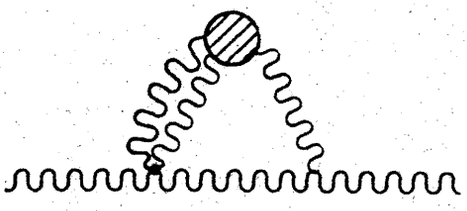

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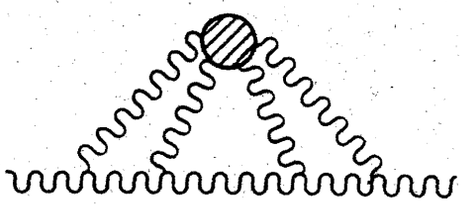

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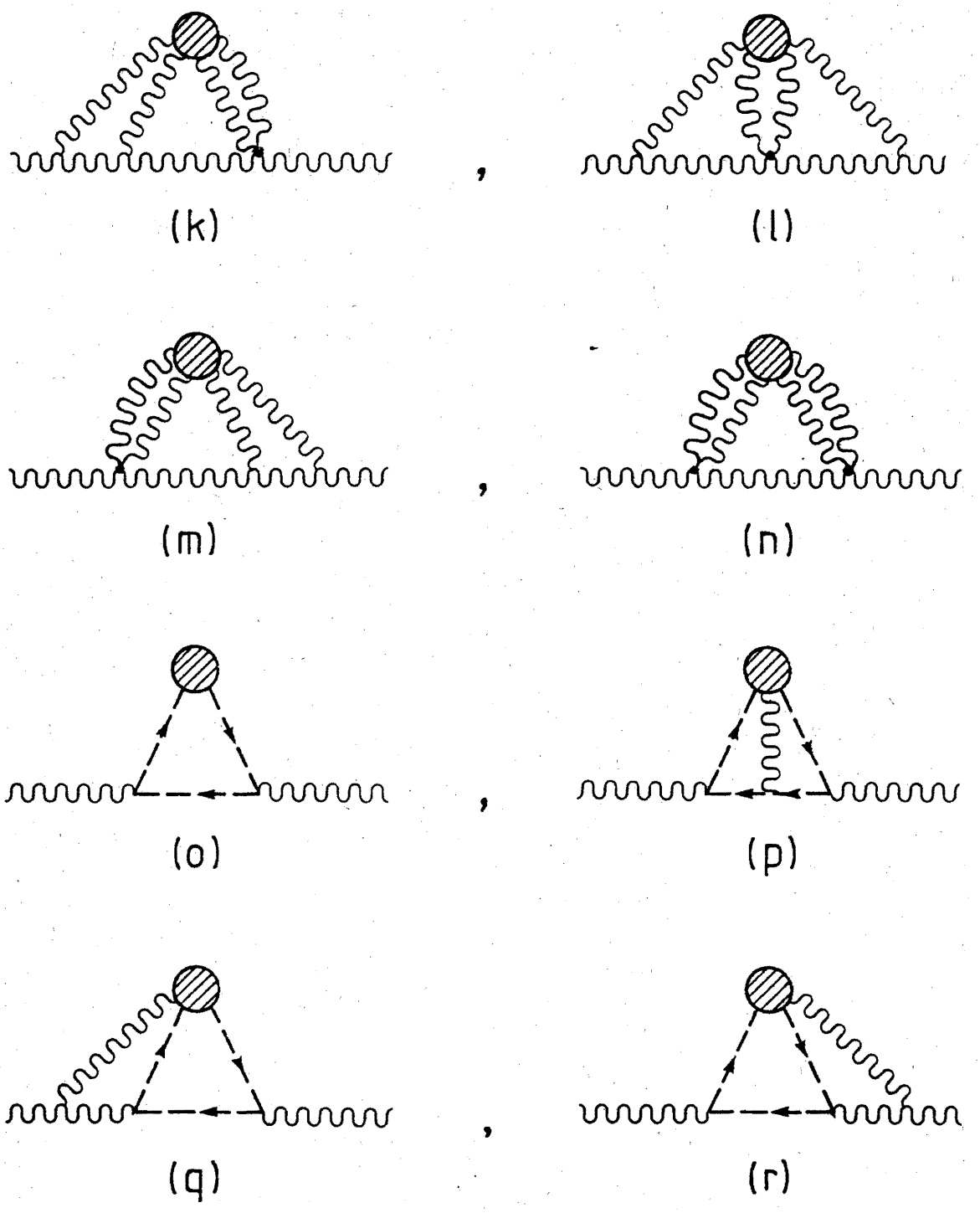

Fig. 2

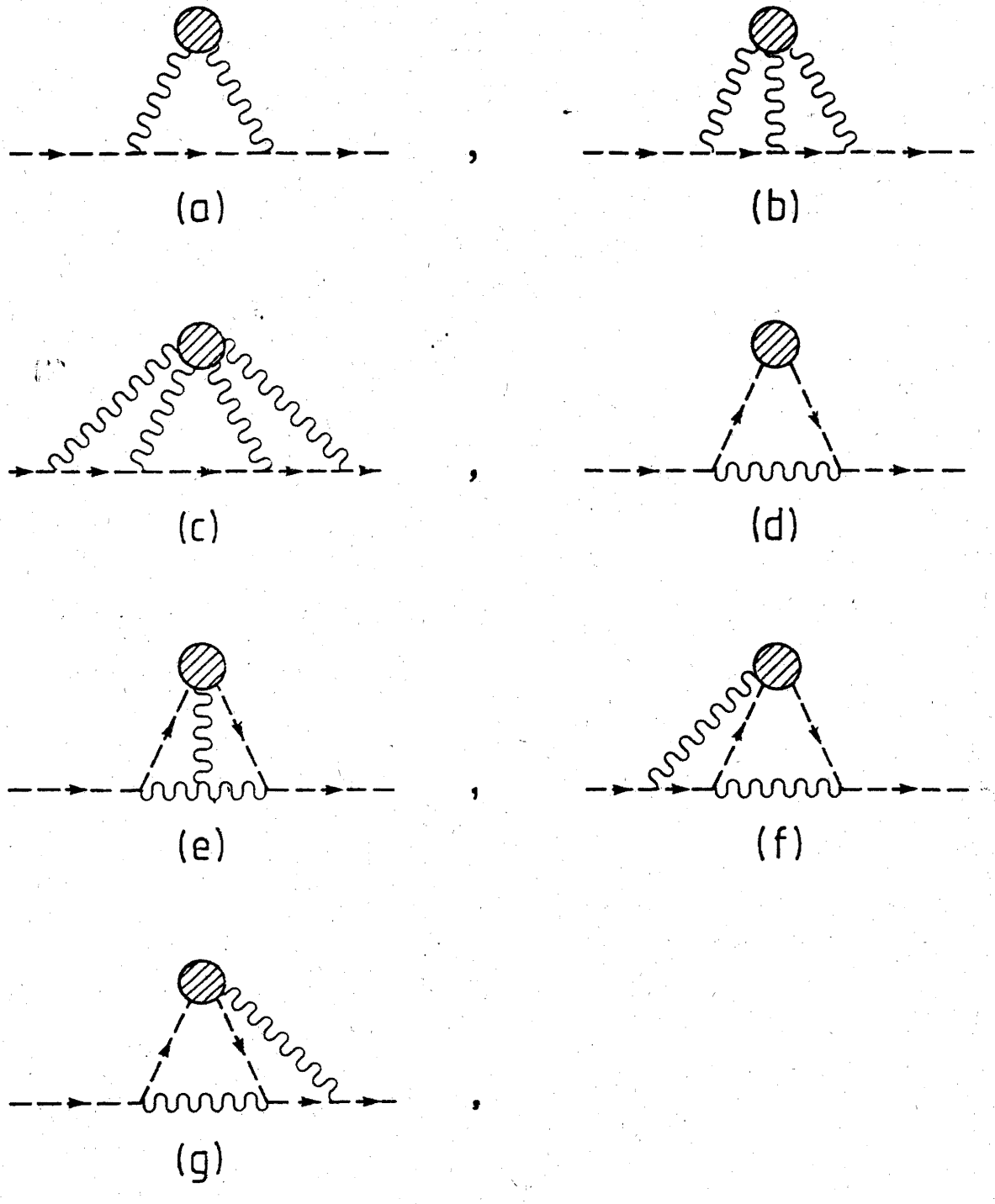

Fig. 3