MZ-TH/92-31
July 1992

MZ-TH-92-31
Fermilab Library
0 1160 0018941 9

INSTITUT FÜR PHYSIK
JOHANNES GUTENBERG UNIVERSITÄT
MAINZ

ROUTE TO
NAME | LOCATION
RETURN TO FERMI LIBRARY

MULTIPOLE DECOMPOSITION OF KAON-PHOTOPRODUCTION OPERATORS

M.M. Tung
Institut für Physik
Johannes Gutenberg-Universität
Staudingerweg 7, Postfach 3980,
D-6500 Mainz, Germany

and

L.E. Wright
Department of Physics and Astronomy
Ohio University, Clippinger Laboratories
Athens, OH 45701, USA

Johannes Gutenberg (Kupferstich von Theret)

NOV 18 1992
LIBRARY

MZ-TH/92-31
July 1992

MULTIPOLE DECOMPOSITION OF KAON-PHOTOPRODUCTION OPERATORS

M. M. Tung

Institut für Physik
Johannes Gutenberg-Universität
Staudinger Weg 7, Postfach 3980
W-6500 Mainz, Germany

and

L. E. Wright

Department of Physics and Astronomy
Ohio University, Clippinger Laboratories
Athens, OH 45701, USA

ABSTRACT

We perform a full multipole decomposition of the kaon-photoproduction operator in the diagrammatic approach for the process $p(\gamma, K^+)\Lambda^0$. The multipole analysis is carried out by expanding the standard functions $\mathscr{F}_i(W, \cos\theta)$ in terms of partial amplitudes. To facilitate this procedure we have developed a general computational technique for calculating the corresponding multipoles directly from their Feynman amplitudes.

11.80.Et, 13.60.Le

1 Introduction

Four decades ago, Chew, Goldberger, Low and Nambu [1] proposed a method for calculating the relativistic dispersion relations for photopion production. The application of dispersion relation techniques to strong interactions will generally permit one to determine the meson-nucleon coupling constants from the experimental scattering cross section. However, for $K\Lambda$-photoproduction

$$\gamma + p \longrightarrow K^+ + \Lambda^0$$

this approach has been less successful. Instead, the full multipole decomposition of this process seems to be a promising method of establishing a mathematical correspondence between the coupling constants and experimental data. The purpose of understanding its reaction mechanism is to gain further knowledge of the fundamental kaon-hyperon-nucleon coupling constants. In addition, improvements in the basic kaon photoproduction operator can then be used in investigations of kaon photoproduction from nuclei as discussed by Bennhold and Wright [2].

In order to carry out this analysis the hadronic transition current $J^{5\mu}$ has to be contracted with the polarization four-vector ϵ_μ of the photon wave with distinct helicity λ. This will allow the expansion of the Lorentz invariant quantity $\epsilon_\mu J^{5\mu}$ in terms of four independent fundamental ($CGLN$) forms which are chosen such that their coefficients can be identified with the Feynman amplitudes of the process in consideration. Subsequently,

we can complete the multipole analysis by expanding the resulting functions $\mathscr{F}_i(W, \cos\theta)$ in terms of partial amplitudes referring to the relative orbital angular momentum and parity of the final state [3].

The $K\Lambda$-photoproduction reaction has contributions from several diagrams consisting of Born terms and resonances in the various channels [4, 5]. These diagrams lead to a set of amplitudes which originate from the exchange of particles or the existence of intermediate states. The direct s-channel comes from an exchange of a nucleon, while the u- and t-channel account for intermediate hyperonic and kaonic resonances, respectively. Thus, every set of amplitudes corresponding to Born terms or individual resonances gives rise to multipoles which are restricted by the selection rules of electromagnetic excitation. Finally, they can be summed up to obtain the total contribution to the production amplitude.

Our purpose here is to extent this formalism and develop a general computational scheme for obtaining the various multipoles directly from their corresponding Feynman amplitudes. In this framework the otherwise complex calculations are then reduced to the manipulation of operators, and will therefore facilitate this procedure considerably.

2 S-Matrix and Invariant Kinematics

The S-matrix element for the photoproduction process in first order can be defined as*

$$\begin{aligned} \mathscr{S}_{fi} &= \delta_{fi} - i \left\langle \Lambda K^+ \left| \int d^4x \;\; \phi^\dagger_{K^+} \;\; \bar{\psi}_\Lambda \, \epsilon_\mu \, \bar{\psi}_p \;\; J^{5\mu} \right| p\gamma \right\rangle \\ &= \delta_{fi} - i(2\pi)^4 \, \delta^{(4)}(p_K + p_\Lambda - p_p - p_\gamma) \left(\frac{M_p M_\Lambda}{4E_p E_\gamma E_K E_\Lambda} \right)^{\frac{1}{2}} \mathscr{M}_{fi} \;, \end{aligned} \tag{2.1}$$

where the covariant matrix element is defined by

$$\mathscr{M}_{fi} = \bar{u}_\Lambda \, \epsilon_\mu J^{5\mu} \, u_p \quad . \tag{2.2}$$

The explicit form of the covariant matrix element is derived by means of Feynman diagrams, where the exchange of the Σ hyperon, is included in the Born terms [4]. The hadronic transition current operator $J^{5\mu}$ incorporates the exchange of various heavy particles [5] and is a pseudo-vector due to the intrinsic negative parity of the kaon. The energy and momentum of the particles involved are labelled by obvious subscripts.

In the $K^+ - \Lambda$ center-of-momentum frame the four-momenta take the specific form

$$\begin{aligned} p_p &= (E_p \,, -\boldsymbol{k}) \qquad & p_\gamma &= (\, k \;,\; \boldsymbol{k}\,) \\ p_K &= (\,\omega \;,\; \boldsymbol{q}\,) \qquad & p_\Lambda &= (E_\Lambda \,, -\boldsymbol{q}\,) \end{aligned} \tag{2.3}$$

with the following relations

$$p_i = |\boldsymbol{p}_p| = |\boldsymbol{p}_\gamma| = k$$

*Note that by convention the normalization constants for bosons and for fermions differ. For the free boson wavefunction $\phi^*\phi = \frac{1}{2E}$, while for Dirac spinors $\bar{\psi}\psi = \frac{m}{E}$.

$$p_f = |\boldsymbol{p}_K| = |\boldsymbol{p}_\Lambda| = q \, . \tag{2.4}$$

We will denote the unit vectors in phase space by $\hat{\boldsymbol{k}} = \boldsymbol{k}/k$ and $\hat{\boldsymbol{q}} = \boldsymbol{q}/q$. An alternative, non-covariant choice for the two independent kinematical variables in this coordinate system are the total energy W and the scattering angle θ

$$\begin{aligned} W &= E_p + k = \omega + E_\Lambda \\ x &= \cos\theta = \hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{q}} \, . \end{aligned} \tag{2.5}$$

It is convenient to introduce the related Mandelstam variables in this system[†]

$$\begin{aligned} s &= W^2 \\ t &= M_K{}^2 - 2k\omega + 2kqx = M_\Lambda{}^2 + M_p{}^2 - 2E_\Lambda E_p + 2kqx \\ u &= M_\Lambda{}^2 - 2kE_\Lambda - 2kqx = M_p{}^2 + M_K{}^2 - 2E_p\omega + 2kqx \, . \end{aligned} \tag{2.6}$$

Inversion readily gives

$$\begin{aligned} W &= s^{\frac{1}{2}} \\ x &= \frac{1}{4kq}\left[t - u + \frac{1}{s}M_p{}^2(M_\Lambda{}^2 - M_K{}^2)\right] \, , \end{aligned} \tag{2.7}$$

where the following helpful relations have been used[‡]

$$E_p = \frac{W^2 + M_p{}^2}{2W}$$

[†]The metric tensor has negative signature. In general we will follow the conventions introduced by J.D. Bjorken and S.D. Drell, *Relativistic Quantum Mechanics*, (McGraw-Hill Book Company, New York, 1964).

[‡]All equations on this page have been derived by constructing scalar combinations (2.3) in the c.m. frame. Similar relations have been stated before in the context of pion-photoproduction [3].

$$
\begin{aligned}
k &= \frac{W^2 - M_p{}^2}{2W} \\
\omega &= \frac{W^2 - M_\Lambda{}^2 + M_K{}^2}{2W} \\
E_\Lambda &= \frac{W^2 + M_\Lambda{}^2 - M_K{}^2}{2W} \quad .
\end{aligned}
\tag{2.8}
$$

Some other useful identities are

$$
\begin{aligned}
q &= (E_\Lambda{}^2 - M_\Lambda{}^2)^{\frac{1}{2}} \\
(E_p + M_p)^{\frac{1}{2}} &= \frac{W + M_p}{(2W)^{\frac{1}{2}}} \\
(E_p - M_p)^{\frac{1}{2}} &= \frac{W - M_p}{(2W)^{\frac{1}{2}}} \quad .
\end{aligned}
\tag{2.9}
$$

3 Multipole Decomposition

It is particularly useful to analyze the photoproduction amplitude in terms of four independent fundamental forms M_i

$$
\epsilon_\mu J^{5\mu} = \sum_{i=1}^{4} A_i(s,t,u) M_i \quad , \tag{3.1}
$$

where the coefficients A_i depend on the Mandelstam invariants and embody the various crossing combinations of the Feynman diagrams under consideration. The quantities M_i

have to be Lorentz and gauge invariant [1] and the most common linearly independent choice is

$$
\begin{aligned}
M_1 &= -\tfrac{1}{2}\gamma^5 F_{\mu\nu}\gamma^\mu\gamma^\nu \\
M_2 &= 2\gamma^5 F_{\mu\nu} p_p{}^\mu p_\Lambda{}^\nu \\
M_3 &= \gamma^5 F_{\mu\nu}\gamma^\mu p_p{}^\nu \\
M_4 &= \gamma^5 F_{\mu\nu}\gamma^\mu p_\Lambda{}^\nu
\end{aligned}
\tag{3.2}
$$

with

$$
F^{\mu\nu} = \epsilon^\mu p_\gamma{}^\nu - \epsilon^\nu p_\gamma{}^\mu \quad . \tag{3.3}
$$

Parity and angular momentum conservation provide a powerful means for classifying the possible transitions in the $K^+ - \Lambda$ center-of-momentum system. Thus, in order to expand $\mathscr{M}_{fi}$ in the form of the orbital angular momentum operator acting on an infinite series of Legendre polynomials it is convenient to define

$$
\mathscr{M}_{fi} = \frac{4\pi W}{(M_p M_\Lambda)^{\frac{1}{2}}} \sum_l (2l+1) \left[\hat{f}_{l+}(W)\hat{\Pi}_{l+} + \hat{f}_{l-}(W)\hat{\Pi}_{l-} \right] P_l(x) \quad , \tag{3.4}
$$

where the two projection operators $\hat{\Pi}_{l,\alpha}$, with $\alpha = \pm$, have been introduced to separate the two distinct cases of the possible total angular momentum coupling in the final state. Their action on the Legendre functions is readily given by

$$
(2l+1)\,\hat{\Pi}_{l,\alpha} = \left(l + \tfrac{1}{2} + \alpha\tfrac{1}{2}\right) - \alpha\, i\boldsymbol{\sigma}\cdot(\hat{\boldsymbol{q}} \times \hat{\boldsymbol{k}})\frac{d}{dx} \quad . \tag{3.5}
$$

Each factor $\hat{f}_{l,\alpha}$ contains two parts due to electric and magnetic transitions and satisfies the conventional selection rules

$$\begin{aligned} \hat{f}_{l+} &= -M_{l+}\,\boldsymbol{\epsilon}\cdot\hat{\boldsymbol{l}}_\gamma + iE_{l+1,-}\,\boldsymbol{\sigma}\cdot\hat{\boldsymbol{q}}\,\boldsymbol{\epsilon}\cdot(\hat{\boldsymbol{k}}\times\hat{\boldsymbol{l}}_\gamma) \\ \hat{f}_{l-} &= -M_{l-}\,\boldsymbol{\epsilon}\cdot\hat{\boldsymbol{l}}_\gamma - iE_{l-1,+}\,\boldsymbol{\sigma}\cdot\hat{\boldsymbol{q}}\,\boldsymbol{\epsilon}\cdot(\hat{\boldsymbol{k}}\times\hat{\boldsymbol{l}}_\gamma) \quad , \end{aligned} \tag{3.6}$$

where $\hat{\boldsymbol{l}}_\gamma$ specifies the relative orbital angular momentum of the initial state.

Substituting these expressions in eq. (3.4) and carrying out the operations implied is tedious, but does not pose major difficulties. For a more detailed exposition of the procedure we refer to a paper by Gourdin and Defour [6]. By expressing the matrix element eq. (2.2) in terms of two-component spinors representing the spin of the proton and the Λ-particle in the c.m. frame it is convenient to introduce the well-known functions $\mathscr{F}_i(W,\cos\theta)$, [1]. In the subsequent part of this work we will make use of the standard matrix notation as presented in ref. [3]. Thus, after assembling the Feynman amplitudes A_i and the multipoles into vectors,

$$\mathscr{A}(s,u) = \begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{pmatrix} \quad \text{and} \quad \mathscr{M}_l = \begin{pmatrix} E_{l+} \\ E_{l-} \\ M_{l+} \\ M_{l-} \end{pmatrix} \quad , \tag{3.7}$$

the following relation is obtained

$$\mathscr{M}_l(W) = \int_{-1}^{+1} dx\; D_l(x)\,C^{-1}(W)\,B(W,x)\,\mathscr{A}(W,x) \quad , \tag{3.8}$$

where $B(W,x)$, $C^{-1}(W)$ and $D_l(x)$ are 4×4 matrices analogous to those of pion-photoproduction and all $\mathscr{F}_i(W,\cos\theta)$ have already been substituted for. For the sake

of completeness their explicit form is given in the appendix. To establish a general formalism which is independent of the specific expressions of the amplitudes $\mathscr{A}$, we define the operators

$$\begin{aligned} \hat{\Omega}_0(l, W, \cdot) &= \frac{1}{2kq} \int_{-1}^{+1} dx \; D_l(x) C^{-1}(W) B(W, x) \left[\; \cdot \; \right] \\ \hat{\Omega}_i(l, W, \xi_i, \cdot) &= \frac{1}{2kq} \int_{-1}^{+1} dx \; \frac{D_l(x) C^{-1}(W) B(W, x)}{\xi_i - x} \left[\; \cdot \; \right] \quad i \neq 0, \end{aligned} \tag{3.9}$$

where the parameters ξ_i contain the typical kinematical quantities due to the propagators under consideration, and the angular dependence in the denominator has been separated out completely. In general, the properties of these operators are specified by their action on a set of unit vectors $\hat{e}_i$ and combinations $x^n \hat{e}_i$, with n any integer and $i = 1, \ldots, 4$. However, here we have only included $x\hat{e}_1$, which is the only additional term needed to calculate the multipoles arising from the Born graphs, due to an explicit dependence of A_1^{Born} on t. In the most general case of a more complicated angular dependence the amplitudes can be expanded in a power series of x. Thus, for our purposes $\{ \hat{e}_i \, , \, x\hat{e}_1 \,|\, i = 1, \ldots, 4 \}$ will suffice. The operator $\hat{\Omega}_0$ acting on these unit vectors $\hat{e}_i$ will again reproduce a linear combination of those with specific kinematic cofactors. But, in the case of $\hat{\Omega}_i$, $i \neq 0$, this procedure will lead to the more complicated set of projected basis vectors $\left\{ \hat{\omega}_k \,|\, k = 1, \ldots, 6 \right\}$ due to the additional propagator structure of the denominator. Symbolically:

$$\hat{\Omega}_0(\cdot) : \left\{ \quad \hat{e}_j \quad | j = 1, \ldots, 4 \right\} \quad \longmapsto \quad \left\{ \hat{e}_k \,|\, k = 1, \ldots, 4 \right\}$$

$$\forall_{i\neq 0}\ \hat{\Omega}_i(\,\cdot\,):\ \left\{\hat{e}_j\,,\ x\hat{e}_1\,|\,j=1,\ldots,4\right\}\quad\longmapsto\quad\left\{\hat{\omega}_k\,|\,k=1,\ldots,6\right\}\tag{3.10}$$

This projection will take a particularly simple form, if we choose

$$4\pi\ \hat{\omega}_1(W,\xi_i;l)\ =\ \frac{1}{2(l+1)}\begin{pmatrix} Q_l(\xi_i)\\ \frac{l+1}{l}\,Q_l(\xi_i)\\ Q_l(\xi_i)\\ -\frac{l+1}{l}\,Q_l(\xi_i)\end{pmatrix}$$

$$4\pi\ \hat{\omega}_2(W,\xi_i;l)\ =\ \frac{1}{2(l+1)}\begin{pmatrix} Q_{l+1}(\xi_i)\\ \frac{l+1}{l}\,Q_{l-1}(\xi_i)\\ Q_{l+1}(\xi_i)\\ -\frac{l+1}{l}\,Q_{l-1}(\xi_i)\end{pmatrix}$$

$$4\pi\ \hat{\omega}_3(W,\xi_i;l)\ =\ \frac{1}{2(l+1)}\begin{pmatrix} R_l(\xi_i)\\ \frac{l+1}{l}\,R_l(\xi_i)\\ R_l(\xi_i)\\ -\frac{l+1}{l}\,R_l(\xi_i)\end{pmatrix}$$

$$4\pi\ \hat{\omega}_4(W,\xi_i;l)\ =\ \frac{1}{2(l+1)}\begin{pmatrix} R_{l+1}(\xi_i)\\ \frac{l+1}{l}\,R_{l-1}(\xi_i)\\ R_{l+1}(\xi_i)\\ -\frac{l+1}{l}\,R_{l-1}(\xi_i)\end{pmatrix}$$

$$4\pi\ \hat{\omega}_5(W,\xi_i;l)\ =\ \frac{1}{2(l+1)}\begin{pmatrix} T_l(\xi_i)\\ -\left[\frac{l+1}{l}\right]^2\,T_l(\xi_i)\\ -\frac{1}{l}\quad T_l(\xi_i)\\ \frac{l+1}{l^2}\quad T_l(\xi_i)\end{pmatrix}$$

$$4\pi\ \hat{\omega}_6(W,\xi_i;l)\ =\ \frac{1}{2(l+1)}\begin{pmatrix} T_{l+1}(\xi_i)\\ -\frac{l+1}{l-1}\,T_{l-1}(\xi_i)\\ 0\\ 0\end{pmatrix},\tag{3.11}$$

where Legendre functions of the second kind as well as combinations of these have been

used [5, 7, 8]

$$
\begin{aligned}
Q_l(y) &= \tfrac{1}{2}\int_{-1}^{+1} dx\, \frac{P_l(x)}{y-x} \\
R_l(y) &= \tfrac{1}{2}\int_{-1}^{+1} dx\, P_l(x)\frac{x}{y-x} = \frac{l}{2l+1}\,Q_{l-1}(y) + \frac{l+1}{2l+1}\,Q_{l+1}(y) \\
T_l(y) &= \frac{l}{2l+1}\Big[Q_{l-1}(y) - Q_{l+1}(y)\Big]
\end{aligned}
\tag{3.12}
$$

The six projected vectors $\hat{\omega}_i$ arise exclusively from the term $D_l(x)\,(\xi_i - x)^{-1}$ in eq. (3.9). Upon carrying out the complete operation $\hat{\Omega}_i(\,\cdot\,)$, with $i = 0, \ldots, 4$, we also expect a contribution from $C^{-1}(W)\,B(W,x)$, which contains the masses and energies of the proton and Λ-particle. Using the shorthand notation

$$
\begin{aligned}
P^{\pm} &= (\,E_p \pm M_p\,)^{\frac{1}{2}} \\
\Lambda^{\pm} &= (\,E_\Lambda \pm M_\Lambda\,)^{\frac{1}{2}} \\
\Lambda^{\pm}_W &= \left(\frac{E_\Lambda \pm M_\Lambda}{2W}\right)^{\frac{1}{2}}
\end{aligned}
\tag{3.13}
$$

and exploiting the orthogonality of the Legendre polynomials gives in a straightforward manner the following chart, which explicitly displays the transformation properties of the operators $\hat{\Omega}_i(\,\cdot\,)$. With its help we can at once evaluate the multipoles $\mathscr{M}_l(W)$ as defined

in eq. (3.8).

$$\hat{\Omega}_0\,\hat{e}_1 \;=\; -\frac{1}{8\pi}\,q^{-1}\begin{pmatrix} \Lambda_W^+\delta_{l0} \\ 0 \\ 0 \\ \Lambda_W^-\delta_{l1} \end{pmatrix}$$

$$\hat{\Omega}_0\,\hat{e}_2 \;=\; \frac{1}{24\pi}\begin{pmatrix} \Lambda^-P^+\delta_{l0} + \frac{1}{2}\Lambda^+P^-\delta_{l1} \\ -\Lambda^-P^+\delta_{l2} \\ -\frac{1}{2}\Lambda^+P^-\delta_{l1} \\ \Lambda^+P^-\delta_{l1} \end{pmatrix}$$

$$\hat{\Omega}_0\,\hat{e}_3 \;=\; \frac{1}{16\pi}\,q^{-1}\begin{pmatrix} \Lambda^+P^+\delta_{l0} \\ 0 \\ 0 \\ -\Lambda^-P^-\delta_{l1} \end{pmatrix}$$

$$\hat{\Omega}_0\,\hat{e}_4 \;=\; \frac{1}{16\pi}W^{-1}\begin{pmatrix} (\frac{1}{3}\Lambda^- P^- - \Lambda^+ P^+ \xi_1)\delta_{l0} + \frac{1}{6}\Lambda^+ P^+ \delta_{l1} \\ -\Lambda^+ P^+ \xi_1 \delta_{l0} + \frac{1}{6}\Lambda^+ P^+ \delta_{l1} \\ -\Lambda^+ P^+ \xi_1 \delta_{l0} + \frac{1}{6}\Lambda^+ P^+ \delta_{l1} \\ \Lambda^+ P^+ \xi_1 \delta_{l0} + (\Lambda^- P^- \xi_1 - \frac{1}{6}\Lambda^+ P^+)\delta_{l1} \end{pmatrix}$$

$$+ \quad \frac{1}{24\pi}\begin{pmatrix} -\frac{1}{2}\Lambda_W^+ \delta_{l1} - \Lambda_W^- \delta_{l0} \\ \Lambda_W^- \delta_{l2} \\ \frac{1}{2}\Lambda_W^+ \delta_{l1} \\ -\Lambda_W^+ \delta_{l1} \end{pmatrix} \tag{3.14}$$

And similarly for the case $i \neq 0$

$$\hat{\Omega}_i\,\hat{e}_1 \;=\; -\Lambda_W^+ q^{-1}\,\hat{\omega}_1 \;+\; \Lambda_W^- q^{-1}\,\hat{\omega}_2$$

$$\hat{\Omega}_i\,\hat{e}_2 \;=\; \Lambda^+ P^-\,\hat{\omega}_5 \;+\; \Lambda^- P^+\,\hat{\omega}_6$$

$$\hat{\Omega}_i\,\hat{e}_3 \;=\; \tfrac{1}{2}\Lambda^+ P^+ q^{-1}\,\hat{\omega}_1 \;+\; \tfrac{1}{2}\Lambda^- P^- q^{-1}\,\hat{\omega}_2$$

$$
\left\{
\begin{aligned}
\hat{\Omega}_1\,\hat{e}_4 &= -\Lambda_W^+\,\hat{\omega}_5 - \Lambda_W^-\,\hat{\omega}_6 + \frac{1}{16\pi}W^{-1}\begin{pmatrix} -\Lambda^+P^+\delta_{l0} \\ 0 \\ 0 \\ \Lambda^-P^-\delta_{l1} \end{pmatrix} \\
\hat{\Omega}_i\,\hat{e}_4 &= -\frac{\xi_1}{2W}\Lambda^+P^+\,\hat{\omega}_1 - \frac{\xi_1}{2W}\Lambda^-P^-\,\hat{\omega}_2 \\
&\quad + \frac{1}{2W}\Lambda^+P^+\,\hat{\omega}_3 + \frac{1}{2W}\Lambda^-P^-\,\hat{\omega}_4 \\
&\quad - \Lambda_W^+\,\hat{\omega}_5 - \Lambda_W^-\,\hat{\omega}_6
\end{aligned}
\right\} \text{ also } i \neq 1
$$

$$
\hat{\Omega}_i(x\hat{e}_1) = -\Lambda_W^+ q^{-1}\,\hat{\omega}_3 + \Lambda_W^- q^{-1}\,\hat{\omega}_4 \tag{3.15}
$$

4 Applications

After having developed the framework of multipole expansion in the previous section we are now in the position to economically calculate the multipole amplitudes arising from the various resonances. To demonstrate our method we carry out the multipole decomposition of the prominent spin $\frac{1}{2}$ resonances in the s- and u-channels as given in refs. [5] and [9]. We have also worked out the multipole amplitudes resulting from the

Born terms [10], but these disagree with the results of ref. [5, 8].[§]

The evaluation of the multipoles originating from the spin $\frac{1}{2}$ resonances in the s-channel is easy to carry out. The Feynman amplitudes are given by

$$\begin{aligned} \mathscr{A}^{\frac{1}{2}\pm} &= \frac{G_{N^*}e}{s - M_{N^*}^2 + iM_{N^*}\Gamma_{N^*}} \frac{M_{N^*} - M_p}{M_{N^*} \mp M_p} \hat{e}_1 \\ &\pm \frac{2G_{N^*}e}{s - M_{N^*}^2 + iM_{N^*}\Gamma_{N^*}} \frac{1}{M_{N^*} + M_p} \hat{e}_3 \end{aligned} \tag{4.1}$$

Relation eq. (3.8) is expressed in operator notation as

$$\mathscr{M}_l^{\frac{1}{2}\pm}(W) = 2kq\, \hat{\Omega}_0\, \tilde{A}^{\frac{1}{2}\pm} \tag{4.2}$$

or equivalently

$$\mathscr{M}_l^{\frac{1}{2}\pm} = \frac{2kq\, G_{N^*}\, e}{W^2 - {M_{N^*}}^2 + iM_{N^*}\Gamma_{N^*}} \left[\frac{M_{N^*} - M_p}{M_{N^*} \mp M_p} \hat{\Omega}_0\, \hat{e}_1 \pm \frac{2}{M_{N^*} + M_p} \hat{\Omega}_0\, \hat{e}_3 \right] \tag{4.3}$$

To perform the actual computation the chart in the preceeding section provides the explicit solutions for $\hat{\Omega}_0\, \hat{e}_1$ and $\hat{\Omega}_0\, \hat{e}_3$. The result has a simple form, since the generating

[§]In these references the specific decomposition

$$\mathscr{F} = \sigma \cdot \epsilon\, \mathscr{F}_1 + i\sigma \cdot \hat{q}\, \sigma \cdot (\hat{k} \times \epsilon)\, \mathscr{F}_2 + \sigma \cdot \hat{k}\, \hat{q} \cdot \epsilon\, \mathscr{F}_3 + \sigma \cdot \hat{q}\, \hat{q} \cdot \epsilon\, \mathscr{F}_4$$

leads to an incompatible identification of the Feynman amplitudes unless the sign of the second column in the matrix D_l of ref. [3] is changed.

amplitudes $\mathscr{A}^{\frac{1}{2}\pm}$ are independent of x.

$$\mathscr{M}_l^{\frac{1}{2}\pm} = \frac{k}{4\pi} \frac{G_{N^*}\, e}{W^2 - M_{N^*}^2 + iM_{N^*}\Gamma_{N^*}} \times$$

$$\begin{pmatrix} \left[-\dfrac{M_{N^*} - M_p}{M_{N^*} \mp M_p} \Lambda_W^+ \pm \dfrac{\Lambda^+ P^+}{M_{N^*} + M_p} \right] \delta_{l0} \\ 0 \\ 0 \\ \left[-\dfrac{M_{N^*} - M_p}{M_{N^*} \mp M_p} \Lambda_W^- \mp \dfrac{\Lambda^- P^-}{M_{N^*} + M_p} \right] \delta_{l1} \end{pmatrix} \tag{4.4}$$

For the operator algebra in the u-channel it is necessary to introduce the parameter

$$\xi_5 = \frac{M_\Lambda^2 - M_{Y^*}^2 + iM_{Y^*}\Gamma_{Y^*} - 2kE_\Lambda}{2kq}, \tag{4.5}$$

which corresponds to a new operator $\hat{\Omega}_5$ according to eq. (3.9). The Feynman amplitudes from refs. [5] and [9] are conveniently written as

$$\begin{aligned} \mathscr{A}^{Y(\frac{1}{2}\pm)} = & \frac{1}{2kq} G_{Y^*}\, e \frac{1}{\xi_5 - x} \frac{M_{Y^*} - M_\Lambda}{M_{Y^*} \mp M_\Lambda} \hat{e}_1 \\ & \pm \frac{1}{kq} G_{Y^*}\, e \frac{1}{\xi_5 - x} \frac{1}{M_{Y^*} + M_\Lambda} \hat{e}_4 \end{aligned} \tag{4.6}$$

so that similarly

$$\begin{aligned} \mathscr{M}_l^{Y(\frac{1}{2}\pm)}(W) = & \left[G_{Y^*}\, e \frac{M_{Y^*} - M_\Lambda}{M_{Y^*} \mp M_\Lambda} \right] \hat{\Omega}_5 \hat{e}_1 \\ & \pm \left[2G_{Y^*}\, e \frac{1}{M_{Y^*} + M_\Lambda} \right] \hat{\Omega}_5 \hat{e}_4 \end{aligned} \tag{4.7}$$

Again we can replace $\hat{\Omega}_5\,\hat{e}_1$ and $\hat{\Omega}_5\,\hat{e}_4$ by linear combinations of $\hat{\omega}_1,\ldots,\hat{\omega}_6$. The procedure will be more transparent after introducing the matrix

$$\mathscr{Q}_l(W,\xi_i) \;=\; \Big(\,\hat{\omega}_1(W,\xi_i;l)\;\ldots\;\hat{\omega}_6(W,\xi_i;l)\,\Big) \tag{4.8}$$

and collecting the different coefficients of $\hat{\omega}_i$. Now the result reads in compact form

$$\mathscr{M}_l^{Y(\frac{1}{2}\pm)}(W) \;=\; \mathscr{Q}_l(\xi_5)\,\mathscr{H}_5^{\pm}\;, \tag{4.9}$$

where

$$\mathscr{H}_5^{\pm} \;=\; \frac{G_{Y^*}\,e}{M_{Y^*}+M_\Lambda}\begin{pmatrix} -\dfrac{M_{Y^*}^2-M_\Lambda^2}{M_{Y^*}\mp M_\Lambda}\,\Lambda_W^+ q^{-1} \mp \dfrac{\xi_1}{W}\,\Lambda^+P^+ \\[2ex] \dfrac{M_{Y^*}^2-M_\Lambda^2}{M_{Y^*}\mp M_\Lambda}\,\Lambda_W^- q^{-1} \mp \dfrac{\xi_1}{W}\,\Lambda^-P^- \\[2ex] \pm\,\dfrac{\Lambda^+P^+}{W} \\[2ex] \pm\,\dfrac{\Lambda^-P^-}{W} \\[2ex] \mp 2\Lambda_W^+ \\[2ex] \mp 2\Lambda_W^- \end{pmatrix} \tag{4.10}$$

Acknowledgements

This work was supported in part by a grant of the "Graduiertenkolleg Teilchenphysik" of the FRG and the U.S. Department of Energy under Contract No. FG02-87ER40370.

Appendix

If we adopt the standard matrix notation as previously introduced in the literature for pion photoproduction [3], the matrics involved in the multipole definition eq. (3.8) have the form

$$B(s,u) = \begin{pmatrix} -1 & 0 & \frac{W+M_p}{2} & -\frac{u-M_\Lambda{}^2}{2(W-M_p)} \\ 1 & 0 & \frac{W-M_p}{2} & -\frac{u-M_\Lambda{}^2}{2(W+M_p)} \\ 0 & W-M_p & 0 & -1 \\ 0 & W+M_p & 0 & -1 \end{pmatrix} \tag{A.1}$$

The result for $D_l(x)$ is constrained by the following relations

$$\begin{aligned} \int_{-1}^{+1} dx\; D_l(x)\, G_m(x) &= \delta_{lm}\, I \\ \text{and} \qquad \int_{-1}^{+1} dx\; \left[D_l\,,\, G_m \right] &= 0 \;, \end{aligned} \tag{A.2}$$

where

$$G_l(x) = \begin{pmatrix} P'_{l+1} & P'_{l-1} & l\, P'_{l+1} & (l+1)P'_{l-1} \\ 0 & 0 & -(l+1)P'_l & -l\, P'_l \\ P''_{l+1} & P''_{l-1} & -P''_{l+1} & P''_{l-1} \\ -P''_l & -P''_l & P''_l & -P''_l \end{pmatrix}. \tag{A.3}$$

After exploiting the following mathematical properties of the Legendre function

$$\begin{aligned} \int_{-1}^{+1} dx\; P'_k(x)\, P_l(x) &= 2\,\Theta(k,l) \\ \int_{-1}^{+1} dx\; P''_k(x)\, P_l(x) &= \left[k(k+1) - l(l+1) \right] \Theta(k-1,l) \;, \end{aligned} \tag{A.4}$$

$$\text{where} \qquad \Theta(m,n) \;=\; \begin{cases} 1 & m,n \in N_0\colon\; m > n \\ & \wedge \quad m+n \quad \text{odd} \\ \\ 0 & \text{otherwise} \end{cases}$$

the unique solution is found to be

$$D_l(x) \;=$$

$$\begin{pmatrix} \frac{1}{2(l+1)}\left\{ P_l \quad P_{l+1} \quad \frac{l}{2l+1}(P_{l-1}-P_{l+1}) \quad \frac{l+1}{2l+3}(P_l - P_{l+2}) \right\} \\ \\ \frac{1}{2l}\left\{ P_l \quad P_{l-1} \quad \frac{l+1}{2l+1}(P_{l+1}-P_{l-1}) \quad \frac{l}{2l-1}(P_l - P_{l-2}) \right\} \\ \\ \frac{1}{2(l+1)}\left\{ P_l \quad P_{l+1} \quad \frac{1}{2l+1}(P_{l+1}-P_{l-1}) \quad 0 \right\} \\ \\ \frac{1}{2l}\left\{ -P_l \quad -P_{l-1} \quad \frac{1}{2l+1}(P_{l-1}-P_{l+1}) \quad 0 \right\} \end{pmatrix} . \tag{A.5}$$

References

[1] G.F. Chew, M.L. Goldberger, F.E. Low and Y. Nambu, *Phys. Rev.* **106** (1957) 1345

[2] C. Bennhold and L.W. Wright, *Phys. Rev. C* **39** (1989) 927

[3] F.A. Berends, A. Donnachie and D.L. Weaver, *Nucl. Phys.* **B4** (1968) 1

[4] R.A. Adelseck, M.S. Thesis, Ohio University, 1985 (unpublished)

[5] R.A. Adelseck, C. Bennhold and L.E. Wright, *Phys. Rev. C* **32** (1985) 1681

[6] M. Gourdin and J. Dufour, *Nuovo Cimento* **27** (1963) 1410

[7] I.S. Gradshteyn and I.M. Ryzhik, *Table of Integrals, Series, and Products* (Academic Press, New York, 1980) 821

[8] C. Bennhold, *Dirac Wave Functions In Hypernuclear Formation Through Kaon Photoproduction*, Ph.D. Thesis, Ohio University, 1987

[9] R.A. Adelseck, *Final-State Hyperon-Nucleon Interaction In Kaon Photoproduction From The Deuteron*, Ph.D. Thesis, Ohio University, 1988

[10] M.M. Tung, M.S. Thesis, Ohio University, 1990 (unpublished)