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UNDERSTANDING SPONTANEOUS SYMMETRY BREAKING IN A NONCOMMUTATIVE GEOMETRY FRAMEWORK*

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* Talk given at the meeting on "Dynamics of Complex and Irregular Systems" held in Bielefeld, Dec. 16-20, 1991. Work performed in the framework of PROCOPE exchange project Mainz-Marseille-Luminy.

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UNDERSTANDING SPONTANEOUS SYMMETRY BREAKING IN A NONCOMMUTATIVE GEOMETRY FRAMEWORK*

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Abstract

A recently proposed construction of the standard model of electroweak interactions which is based on a noncommutative, graded algebra provides a novel geometric interpretation of spontaneous symmetry breaking: The theory is characterized by the presence of a constant background curvature which is invariant under all constant, generalized gauge transformations and which singles out a specific direction in the internal symmetry space. As that direction is given by the charge operator the residual symmetry of the theory must be the isotropy group of the charge operator, i.e. the well-known $U(1)_{e.m.}$ of electromagnetism. Thus, spontaneous symmetry breaking from the original $SU(2) \times U(1)$ to $U(1)_{e.m.}$ is a consequence of the noncommutative geometric structure of this approach.

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1. Introduction and Motivation

1.1 General Remarks and Survey

Recently, attempts to go beyond the conventional geometric framework of Yang-Mills theories making use of ideas of A. Connes' noncommutative geometry [1], have brought forth some promising results. To quote just a few of these, these novel approaches provide a certain unification of the bosonic sector of the standard model by making the Higgs fields part of a generalized connection and by fixing unambiguously their internal quantum numbers [2, 3]. As far as the fermionic sector is concerned, they are stringent enough to eliminate from the start most of the arbitrariness in the conventional construction of the standard model. It seems even that they imprint a certain pattern onto the mass matrices [3] and, hence, the generational mixing of quarks and leptons – features which are normaly alien to the usual Yang-Mills construction. Thus, it seems that noncommutative geometry steps in where the predictive power of standard gauge theory ends.

In this contribution I discuss the specific example of spontaneous symmetry breaking in the electroweak sector of the standard model. After a short reminder of its traditional construction, with special emphasis on the arbitrariness in introducing the fermionic matter fields, I describe a possible generalization of Yang-Mills theory based on a graded, noncommutative algebra that leads to precisely the minimal model, however, without recurrence to the empirical input on which the historical construction is based [2, 3]. I show that the generalized (super) connection, and, likewise, the corresponding generalized curvature, contain two parts, one of which is the well-known Yang-Mills connection, or curvature, while the second is an absolute element invariant under all constant, generalized gauge transformations [4, 5]. Regarding the latter, the constant curvature is seen to be proportional to the charge operator which singles out a specific direction in the internal symmetry space. It is then not surprising that the Lagrangian constructed from the generalized curvature can have no more symmetry than the isotropy group of this absolute element or background field, i.e. the well-known $U(1)$ symmetry of electromagnetism. Only when this background field is taken out, by hand, does one recover the full hidden symmetry $SU(2) \times U(1)$ on which the electroweak sector of the standard model is built. The geometric character of this description becomes clear, for instance, by the analogy to the study of atomic spectra in presence of a universal, constant magnetic background field: The observed, "effective" physics reveals no more than the axial symmetry due to rotations about the direction of that magnetic field. The full rotational symmetry is hidden and becomes evident only after taking out the background field.

1.2 Conventional Construction of the Minimal Model

The minimal model is built on two rather different pillars: Yang-Mills theory and empiricism of electromagnetic, weak and strong interactions [6, 7]. The first offers a (partial) unification of the "radiation" sector, i.e. the interactions of the gauge bosons, but not much more. The second is the essence of several decades of experimental experience.

In particular, there are but little theoretical constraints to guide us when introducing the fermionic matter fields into the model. Yet, the physics of quarks and leptons shows some remarkable regularities for which no deeper reason is apparent [8]. In other terms, the framework of pure Yang-Mills theories is too wide, and possesses too little structure yet to make the observed, and quite remarkable, regularities of fundamental fields and interactions follow from a unifying principle.

This state of affairs reminds one of an impressive installation that can be visited at the new Museum of Modern Art in Frankfurt [9]. At first sight it seems to consist of one large, empty and almost dark room. The large rectangular surface that one discerns at the backside of the room and that is barely illuminated, at first looks like a window or a picture. But when one approaches and tries to touch it one reaches into emptiness. One discovers another empty room that lies behind the opening and that is lit by an invisible source of light. There is a structure that one perceives after a while – provided one has ample time and patience – but cannot explain.

The bosonic sector of the standard model consists of the gauge bosons obtained in gauging the structure group $G = SU(2)_L \times U(1) \times SU(3)_c$, and a set of scalars which serve the purpose of hiding the original $SU(2)_L \times U(1)$ symmetry of the electroweak sector, through spontaneous symmetry breaking, in favour of the residual symmetry $H = U(1)_{e.m.}$. For this purpose the scalar multiplet (Higgs fields) must have an electrically neutral component along the homogeneous space G/H but, for the rest, could be anything. As is well known it is the empirical input [10]

$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2 \theta_W} = 1.003 \pm 0.004 \tag{1}$$

which constrains the weak isospin of the Higgs multiplet to be $t = \frac{1}{2}$. In the fermionic sector of the model the observed maximal parity violation in charged weak interactions tells us that the basic building blocks are fermion fields of definite chirality. In the case of one lepton family, for instance, these are a doublet of left-chiral fields (ν_f, f_L^-), and a singlet right-chiral field f_R^-. The eigenvalues of weak hypercharge are fixed by requiring that neutrinos decouple from photons and that charged leptons and quarks have pure vector (parity conserving) couplings to the photon. Fortunately, these assignments are in agreement with the conditions obtained by requiring chiral anomalies to be absent [11]. Finally, in a framework based on Lie groups only, there is no natural place for generational mixing without invoking new physics outside the standard model. Indeed, as the multiplets are then always fully reducible the gauge bosons cannot mediate between members of different multiplets and inter-generation transitions must be due to a mechanism outside the theoretical framework from which one started.

2. Graded Algebra and Electroweak Interactions

2.1 Construction of Generalized Connection and Curvature

The approach to the standard model making use of a noncommutative graded algebra that I shall use below is described in more detail in the contribution by R. Coquereaux to this meeting and I refer to his talk for details of the construction. The approach is based on an infinite dimensional graded algebra which, for the purposes of electroweak interactions alone, is given by $SU(2\,|\,1) \otimes \Lambda^*(M)$. Its elements are form-valued matrices whose supertrace vanishes

$$Str\, A \equiv tr\,(\Gamma A) = 0\ , \tag{2}$$

where $\Gamma = \mathrm{diag}\,(1, 1; -1)$. The algebra is equipped with a graded commutator

$$[A, B]_g := [A_0, B_0] + [A_0, B_1] + [A_1, B_0] + i\{A_1, B_1\}\,, \tag{3}$$

where the subscripts 0 and 1 denote even and odd parts of the matrices, respectively, and (besides the usual Cartan derivative d_C) a discrete exterior "matrix" derivative d_M which is defined by

$$d_M A := [\eta, A_0] + i\{\eta, A_1\}\,. \tag{4}$$

Here η is a constant, odd element of the algebra which is expressed in terms of odd generators of $SU(2\,|\,1)$ as follows [4, 5]

$$\eta = i\left(\Omega_+ + \Omega'_-\right)\ . \tag{5}$$

This discrete derivative is obtained from the corresponding derivative in an ambient space of 4×4 matrices where η is of the form

$$\eta_{4\times 4} = \left(\begin{array}{c|c} 0 & U \\ \hline U^\dagger & 0 \end{array}\right)$$

with $U \in U(2)$, and by projecting back to dimension 3 by means of the projection operator $p = \mathrm{diag}\,(1, 1, 1, 0)$ [3]. As shown in ref. [5] it can also be formulated directly in dimension 3 from the start. While d_M in dimension 4 is nilpotent, its projection (4) is not. Indeeed, with d denoting the combined exterior derivative (i.e. Cartan and matrix derivative) one has

$$d^2 A = d_M^2 A = i[\eta^2, A]\quad .$$

In dimension 4 the matrix η^2 is a multiple of the unit matrix. Its projection to dimension 3 is not.

The generalized connection has the form

$$\mathcal{A} = i\left(a\vec{W}\cdot\vec{I} + \frac{b}{2}W_8 Y\right) + \frac{i}{\sigma}\left(\overline{\phi^{(0)}}\Omega_+ + \overline{\phi^{(+)}}\Omega_- + \phi^{(0)}\Omega'_- + \phi^{(+)}\Omega'_+\right)\ , \tag{6}$$

where $\vec{W}$ and W_8 denote the 1-forms describing the gauge fields of $SU(2)$ and $U(1)$, respectively, while $(\phi^{(0)}, \phi^{(+)})$ denote the 0-forms describing the neutral and charged

Higgs fields. The constants a and b are normalization factors, σ is a parameter with dimension of a mass. The 1-forms being multiplied by even matrices, the 0-forms by odd matrices, it is clear that the total grade of $\mathcal{A}$ is odd.
There are two equivalent ways of constructing the corresponding generalized curvature:
(i) One embeds the connection $\mathcal{A}$ in the space of 4×4 matrices, constructs the covariant derivative $\nabla_{3\times 3} = pd + \mathcal{A}$ and projects back to dimension 3. The curvature is then found to be [3]

$$\mathcal{F} = \widehat{\mathcal{F}} + p\cdot dp\cdot dp \quad , \tag{7}$$

where the first term is given by

$$\widehat{\mathcal{F}} = p\cdot(d\mathcal{A})\cdot p + \tfrac{1}{2}[\mathcal{A},\mathcal{A}]_g \quad , \tag{8}$$

while the second term is a constant matrix that can be expressed in terms of the matrix Γ and of generators of $SU(2\,|\,1)$, viz

$$p\cdot dp\cdot dp = -i\left(I_3 - \tfrac{1}{6}Y\right) + \tfrac{i}{3}\Gamma\,. \tag{9}$$

(ii) Alternatively, the curvature $\widehat{\mathcal{F}}$, eq. (8), is obtained by generalizing the structure equation in dimension 3, viz

$$\widehat{\mathcal{F}} = d\mathcal{A} + \tfrac{1}{2}[\mathcal{A},\mathcal{A}]_g \quad , \tag{10}$$

where d, the combined (Cartan and matrix) derivative is formulated in the space of 3×3 matrices from the start.

2.2 Generalized Gauge Transformations

Gauge transformations are defined in formal analogy to the case of conventional Yang-Mills theories as follows

$$\mathcal{A}' = \mathcal{A} + d\mathcal{E} + [\mathcal{A},\mathcal{E}]_g \quad . \tag{11}$$

Here, $\mathcal{E}$ is an element of $SU(2\,|\,1)\otimes\Lambda^*(M)$ whose total degree is zero. It may be written in terms of even and odd generators of the algebra, viz

$$\mathcal{E} = i\left(\alpha\vec{I}\cdot\vec{e} + \beta Y e_8\right) + i\gamma\left(\Omega_+\overline{\omega_1} + \Omega_-\overline{\omega_2} + \Omega'_-\omega_1 + \Omega'_+\omega_2\right)\,. \tag{12}$$

where $\vec{e}$ and e_8 are 0-forms, ω_1,ω_2 are 1-forms. In the sequel we shall only consider the *even* part of $\mathcal{E}$, eq. (12), which will be seen to be the direct generalization of gauge transformations in Yang-Mills theories. The odd part, when applied to gauge and matter fields of the theory, leads out of the original set of fields and must be considered separately [3].
Making use of the definition (4) one verifies that there is exactly one nontrivial constant background connection which is invariant under all gauge transformations (11) with constant coefficients $\vec{e}$ and e_8. It is given by

$$\mathcal{A}_0 = -\eta = -i\left(\Omega_+ + \Omega'_-\right) \quad . \tag{13}$$

The corresponding, constant curvature is calculated from eq. (10). It is

$$\widehat{\mathcal{F}}_0 = -i\eta^2 \quad .$$

This is an even element of the algebra and, via the graded commutator $\{\Omega_+, \Omega'_-\} = I_3 + \frac{1}{2}Y$, is seen to be

$$\widehat{\mathcal{F}}_0 = i\left(I_3 + \tfrac{1}{2}Y\right) \equiv iQ_{e.m.} \quad . \tag{14}$$

Thus, $\widehat{\mathcal{F}}_0$ is identical with the electric charge operator. It singles out a specific direction in the four-dimensional internal symmetry space, and in the eight-dimensional representation space of the graded algebra. As such, it represents an absolute element of the theory. Its role becomes clearer if we subtract the constant, invariant, connection from $\mathcal{A}$, and, likewise, take out the constant background curvature from the full generalized curvature, viz

$$\mathcal{A}_\theta := \mathcal{A} - \mathcal{A}_0, \qquad \mathcal{F}_\theta :\doteq \widehat{\mathcal{F}} - \widehat{\mathcal{F}}_0 . \tag{15}$$

Looking back at eq. (6) one notices that $\mathcal{A}_\theta$ is given by the same expression with the physical Higgs fields $\phi^{(i)}$ replaced by the (unshifted) fields $\theta^{(i)}$, the relation between physical and unshifted fields being

$$\theta^{(0)} = \phi^{(0)} + \sigma, \qquad \theta^{(+)} = \phi^{(+)} . \tag{16}$$

Working out the effect of the gauge transformations (11) on $\mathcal{A}_\theta$ and $\mathcal{F}_\theta$ shows that the latter transform exactly like ordinary Yang-Mills connection and curvature, respectively. Indeed, restricting to the even part in eq. (12), one finds

$$\mathcal{A}'_\theta = \mathcal{A}_\theta + d_C\mathcal{E} + [\mathcal{A}_\theta, \mathcal{E}] , \quad \mathcal{F}'_\theta = \mathcal{F}_\theta + [\mathcal{F}_\theta, \mathcal{E}]$$

with d_C the usual (Cartan) exterior derivative.

2.3 The Lagrangian

As described in the contribution of R. Coquereaux to this meeting and in refs. [2, 5] the lagrangian is obtained from the trace of the scalar product of the curvature with its conjugate,

$$\mathcal{L} = -\tfrac{\sigma^4}{g^2}\mathrm{tr}\,\langle \mathcal{F}^\dagger \mathcal{F}\rangle , \tag{17}$$

the scalar product being the natural scalar product of k-forms, $k = 1, 2, 3$ over M^4. Working this out [5] one obtains the bosonic sector of the Lagrangian of the standard model of electroweak interactions, with the Higgs potential given by

$$V = \tfrac{2}{g^2}\left(\overline{\theta^{(0)}}\theta^{(0)} + \overline{\theta^{(+)}}\theta^{(+)} - \sigma^2\right)^2 + const . \tag{18}$$

The Higgs potential (18) has a local maximum at the origin and a circle of degenerate minima with radius σ/g.[1] Note that the generalized connection (6) contains the *physical*

[1]The connection is defined such that it contains the coupling constant g as a factor. This means that the fields must be rescaled by that factor at the end. The factor g^2 in the denominator in eq. (18) is then cancelled.

Higgs fields which are related to the unshifted fields by eqs. (16).

3. Interpretation and Outlook

3.1 Spontaneous Symmetry Breaking

The construction of the standard model sketched above leads to the correct form of the lagrangian: The symmetry $SU(2) \times U(1)$ is hidden, it is spontaneously broken down to the expected residual symmetry $U(1)_{e.m.}$. The cause for the spontaneous breaking of the original symmetry must be sought in the noncommutative algebraic structure encoded in eqs. (2 – 5). This may be seen more clearly by replacing the matrix η of eq. (5) by

$$\tilde{\eta} := i\rho \left(\Omega_+ + \Omega'_-\right)$$

and by letting the factor ρ run from $\rho = 0$ to $\rho = 1$. In the former case the effect of the discrete derivative (4) is zero. In the latter we return to the full noncommutative structure. With η replaced by $\tilde{\eta}$ the invariant connection (13) is multiplied by ρ, the constant background field (14) is multiplied by ρ^2, and the constant σ^2 in the brackets on the right-hand side of eq. (18) is replaced by $(\sigma\rho)^2$. Thus, if we set the parameter ρ equal to zero, i.e. if we "take out" the new, noncommutative structure and return to the standard framework of Yang-Mills theory, the invariant background field and all effects of spontaneous symmetry breaking disappear. We conclude that the noncommutative algebraic extension of the theory implies, as a necessary and unavoidable consequence spontaneous breakdown of the gauge symmetry that would be there in the pure Yang-Mills theory. The interpretation is a geometric one because symmetry breakdown is traced back to the presence of the invariant background field $\widehat{\mathcal{F}}_0$ which is characteristic for the noncommutativity of the underlying algebra. As the theory contains this field as an absolute element it is not surprising that the final lagrangian has no more symmetry than the isotropy group of this field, i.e. the $U(1)$ of electromagnetism. This is why I made the comparison to atomic physics in the introduction (sect. *1.1*). If our universe contained a constant, magnetic background field, atomic spectroscopists would deduce from their data axial symmetry of the underlying hamiltonian, corresponding to rotations about the direction of the field. The full rotational $SO(3)$ symmetry would be discovered only after subtracting by hand the effects of that background field.

3.2 Outlook

The lagrangian that we obtain in this extended gauge theory is precisely the one of the standard model. This is gratifying because we obtain the right degrees of freedom and the right number of fields. This is different from earlier work which attempted to gauge the graded Lie algebra $SU(2\,|\,1)$ [12] but ran into difficulties that could not be solved as yet [13]. Quantization, in our scheme, proceeds in the standard way, and is identical to the case of the usual standard model.

On the other hand, this approach raises new questions. Our construction is a classical one. The graded algebraic structure does not manifest itself in new invariance properties

that could have a chance of surviving the quantization procedure. It is likely that the relevance of the graded structure for the standard model is justified by the fermionic matter fields, cf. ref. [3]. Indeed, as shown there quarks and leptons fit very naturally into the algebraic framework of the theory and much of the apparent arbitrariness in the usual procedure of introducing matter fields into the standard model is eliminated. For instance, maximal parity violation in charged weak interactions is natural because left-chiral fields and right-chiral fields in representations of $SU(2 \mid 1)$ fall into different multiplets of weak isospin with different dimensions. The same difference of dimensions is relevant for the specific properties of the bosonic sector discussed in this contribution. Thus, for the first time, there is a link between the phenomena of parity violation and of spontaneous symmetry breaking.

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