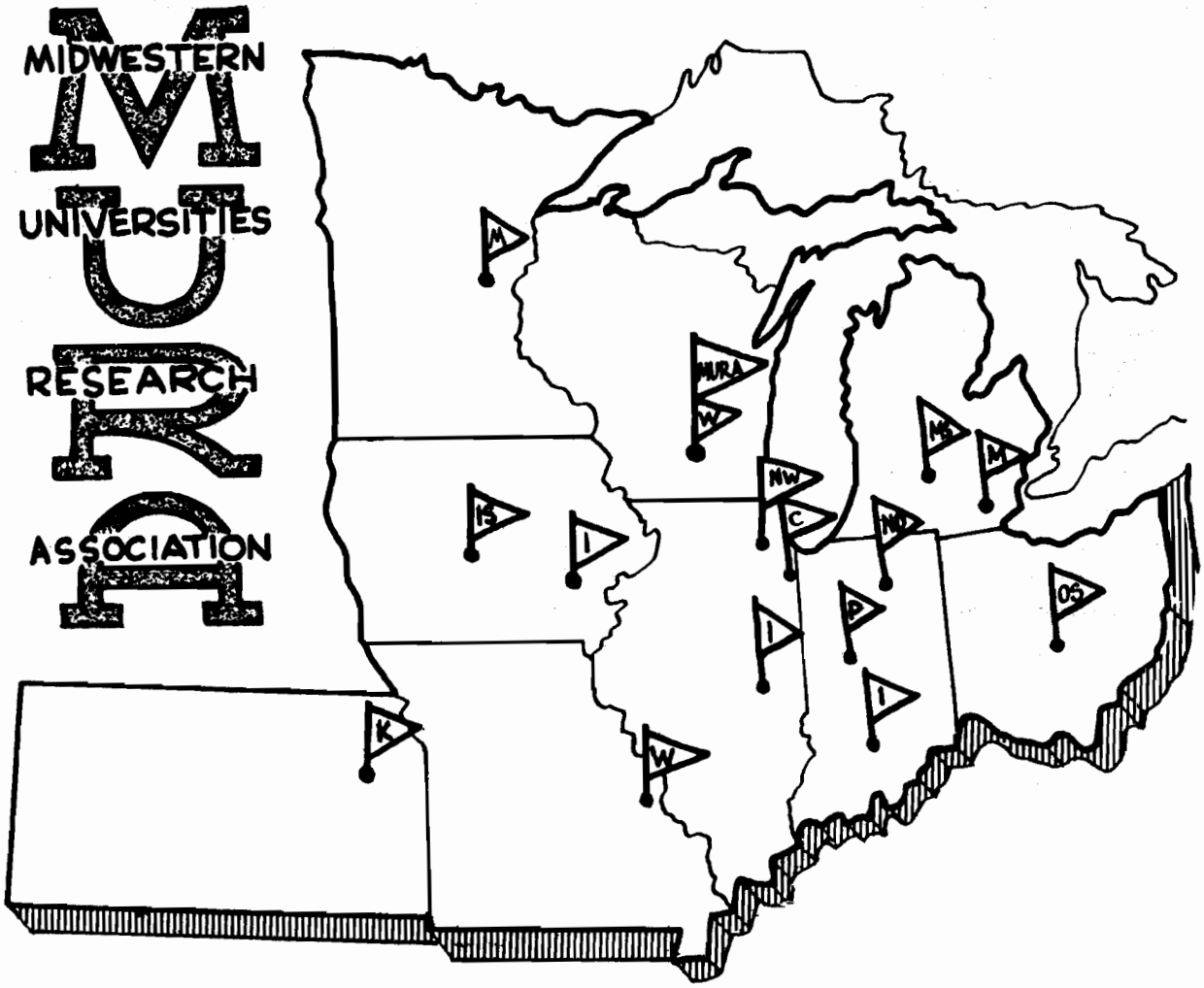

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D. E. Lyon, Jr.

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A. Subramanian

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D. E. Lyon, Jr.
University of Michigan, Ann Arbor, Michigan

and

A. Subramanian
Tata Institute of Fundamental Research, Bombay, India

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ABSTRACT

Design specifications for total absorption spectrometers using plastic scintillators and iron to detect and measure the energies of hadrons in cosmic radiation in the energy range 100-1000 BeV are given, based on results obtained with an experimental spectrometer set up and operated at Echo Lake, Colorado (elevation 10, 600 ft.) during the fall and winter of 1966-67. With a sample of 2000 events to study, it was concluded that the energy resolution with such a device is $\pm 20\%$, if the spectrometer is constructed using a total thickness of iron of at least 1000 gm/cm^2, and using plastic scintillator probes with a thickness of less than 2 gm/cm^2 to sample the nuclear cascade development, separated at most by 100 gm/cm^2 of absorber material and placed with this frequency throughout the entire thickness of the absorber.

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I. INTRODUCTION

It is well known that strongly interacting particles will dissipate their energies while passing through condensed material primarily through ionization through the production of π^o, η, etc. so that a stack of material of suitable thickness, e.g. iron, striated with layers of scintillation or gas counters to sample the resulting electromagnetic cascade, could be used to measure the energy of particles falling on it. The incident energy, E_t, would be

$$E_t = \iint \beta \, N(\beta, t) \, dt d\beta + U \, m(E_t). \qquad (1)$$

Here, $N(\beta, t)$ is the number of particles with specific ionization β at a thickness t, U is the mean energy loss in overcoming part of the binding energy of a nucleus of the absorber, and $m(E_t)$ is the mean number of nuclear encounters in the cascade. With such an arrangement the energy dissipated in disrupting nuclei, the second term in Equation (1), is not measured. When scintillators or Cherenkov counters are introduced as probes, the measured energy is

$$E_{sc} = \iint \epsilon(\beta) \, N(\beta, t) \, dt d\beta, \qquad (2)$$

where $\epsilon(\beta)$ is an efficiency factor for response to specific ionization, with $\epsilon(\beta) = 1$ at $\beta = \beta$ minimum. For a finite number of probes, n, separated by absorber thicknesses x_i, then, the energy of an individual hadron can be calculated as

$$E_c = f\,\beta \left[\frac{N_1}{2} x_1 + \sum_{i=2}^{n} \frac{N_{i-1} + N_i}{2} x_i + \Lambda N_n \right], \qquad (3)$$

where the N's are now the equivalent number of particles with specific ionization β, and f, taken to be 1.3, is a factor to account for unsampled energy losses in the absorber ($\simeq$ 10 percent for transition effects and to be explained later; $\simeq$ 20 percent for nuclear disintegrations and slow particle production[1]). Also, it is assumed that exponential absorption prevails at large thicknesses so the last term in Equation (3) represents an extrapolated area of the shower curve beyond the last probe:

$$\int_0^\infty e^{-x/\Lambda} N_n \, dx = \Lambda N_n .$$

Since the first suggestion of such a device by Grigorov et al.,[2] there has been a growing interest in the so-called ionization spectrometer to detect and measure the energies of hadrons in cosmic radiation, in the energy range 100 - 1000 BeV. Discussions have been given on some of the design features of these spectrometers,[2,3,4] and we consider here the parameters involved in their design when using scintillators and an iron absorber, and the possible sampling accuracies attainable. This study was made possible using data from an experimental spectrometer operated at Echo Lake, Colorado.

II. CHOICE OF MATERIAL AND PROBES

In terms of the cost and ease of construction, iron has been found to be the most practicable of the absorbers. It is

also desirable from the point of view of having a relatively high critical energy, ϵ_o, for electromagnetic cascades when using probes like scintillators or Cherenkov counters. The relatively high critical energy is desirable because of the so-called transition effect, which occurs because the absorption and regeneration properties of the probes are different from those of the absorber. That is, ϵ_o = 21 MeV, and the energy loss in the probes of electrons near minimum ionization is ~2 MeV/gm. This makes the absorptive effect of the probes on the cascade when iron and probes with $\lesssim$ 2 gm/cm^2 equivalent of plastic scintillator are used in the fabrication, not likely to be larger than a 10 percent underestimate at cascade maxima, as judged from the electromagnetic cascade energy spectra of electrons and positrons given by Pinkau.[5]

It is found that the cost of constructing probes, whether they be ion chambers, proportional counters, or scintillators, is about the same. In the case of large area scintillators or Cherenkov counters, the attainment of a uniform response for varying positions of the passage of ionizing particles requires careful design. Also, the linearity of their light output with the level of ionization has not been verified for the incidence of particles with high specific ionization. The linearity of response is good for gaseous detectors; however, their response is too slow if one wishes to use the spectrometer signal to trigger spark chambers. One could argue in favor of gaseous detectors because they have a faithful response for particles

with high specific ionization where scintillators or Cherenkov counters have little or none. At energies above 100 BeV, though, response to high specific ionization does not seem to be important since more than 70 percent of the incident energy is dissipated via particles with nearly minimum specific ionization.[1] This kind of response might even be undesirable because of the occurrence of large fluctuations in the sampled ionization at a particular depth in the absorber when there is nuclear disintegration close to a detector, causing fluctuations in the calculated energy. Scintillators smooth this kind of fluctuation because of their non-linear response to high specific ionization. Whatever one's choice may be for the probes, it is clear from economic considerations that one should use the minimum number of them commensurate with the requirements of accuracy in sampling the cascade.

III. THICKNESS OF ABSORBER

The total thickness of the absorber needed depends on the range of significant level of ionization for a given energy of the hadron. Recent calculations by Pinkau and Thompson[4] indicate considerable variations in this range depending upon the characteristics one assumes for the nuclear collisions of the hadrons. The range of this variation is 5 -20 nuclear interaction mean-free paths.

To study the growth and absorption of the electromagnetic cascade induced by high energy hadrons and thus settle the question of thickness of the absorber, use was made of data from

an experimental spectrometer operated during the fall and winter of 1966 at Echo Lake, Colorado, at an altitude of 10,600 feet. An iron stack with a thickness of 1070 gm/cm^2 and an area of 3' x 6' was used with plastic scintillators 3/4" thick as probes. These were placed at seven levels in the absorber and will hereafter be referred to as counters #1 through #7, counting from the top. Figure 1 shows the experimental arrangement. The response from each counter in the spectrometer was calibrated in terms of the energy loss suffered by relativistic muons passing through it. All triggers satisfying the condition

$$\sum_{i=2}^{6} N_i \gtrsim (30 \text{ minimum ionizing particles})$$

were recorded and the event energy calculated using Equation (3), with n = 5. The extrapolation beyond the fifth counter was taken as

$$200\ N_5 \ (\text{if } N_7 = 0,$$

or

$$\Lambda_i\ N_5 \ (\text{if } N_7 > 0$$
$$(\quad N_5 > N_7 \ .$$

Here Λ_i is calculated from $N_7 = N_5 e^{-533/\Lambda_i}$ and is an attempt to use the seventh counter's information on the number of minimum ionizing particles "leaking out" the bottom of the spectrometer. Also, the value $\Lambda = \Lambda_i$ was used only in those cases where

$$\frac{E_c\ (\Lambda) - E_c\ (\Lambda_i)}{E_c\ (\Lambda = 200)} < 5 \ .$$

Otherwise the value $\Lambda = 200$ gm/cm^2 was used. This seems arbitrary, but in the presence of fluctuations some cut-off criterion has to be employed. The reason for extrapolating beyond the fifth counter instead of the sixth is that the sixth counter's output was found to be erratic.

To investigate the average behavior of the electromagnetic cascade, events whose calculated energy, E_c, was greater than 50 BeV were grouped into bins of the order of 0.1 E_c wide. Figure 2 shows the average shower curves for some selected bins. The dominant features in these curves are a single growth of the shower up to a depth of $\simeq$ 100 gm/cm^2, followed by absorption as $e^{-x/200}$, where the values $\bar{\Lambda} \simeq 200$ show no energy dependence (Figure 3). It should be noted that while the average shower curves are smooth, the individual curves are not. Some examples are shown in Figure 4.

From the smoothness of the average shower curves and their characteristic energy independent absorption, one would presume that a spectrometer with a thickness of approximately 700 gm/cm^2 would be adequate. However, about 25 percent of the events in each energy bin show a shower development where a substantial fraction of the dissipation occurs at depths greater than 700 gm/cm^2, and in these cases $\langle\Lambda_i\rangle$ for each energy bin does not agree with $\Lambda = 200$. Also, the dispersion, $\Delta\Lambda$, for each energy bin is of the order of $\langle\Lambda_i\rangle$. Cowan and Moe also find such fluctuations.[7] Thus, the assumption of experimental absorption beyond the fifth counter is not good for individual

events and calls for a total spectrometer thickness of at least 1000 gm/cm^2 with adequate striation throughout.

Regarding the lateral spread of showers in the absorber, Ranft[8] has calculated the radial spreading of particles using Monte Carlo techniques and the transverse momentum distribution of particles in the cascade. Radial spreading of the order of 10 cm can be expected in a distance of about one meter in iron for particles with energies ~ 3 BeV. Hence, this does not pose a serious problem regarding the lateral dimension of the spectrometer required to contain the nuclear cascade.

IV. NUMBER AND SPACING OF PROBES

The estimation of the energy of a given event involves the integration of the ionization curve, and with a discrete set of probes a formula such as Equation (3) has to be used. The error in integration depends on the shape of the shower curve, and to first order depends on the curvature at various points. The difference, due to a finite spacing of probes, between the area under a smooth shower curve of the form $Ax^{\alpha} e^{-x/\Lambda}$ and that determined from the trapezoidal integration of Equation (3) was calculated for various probe structures and values for A, α, and Λ. The results are given in Table I and are only around 5 percent for the various probe structures with ≥ 5 probes, making it clear that it is not necessary to try to optimize the position of the probes for best sampling on the average. With this information and that on the fluctua-

tions in shower development described in Section II, it seems that an equal spacing of the probes is best. In support of this we cite that, as stated previously, while average shower curves are smooth and can be fitted to the form $Ax^{\alpha}\ e^{-x/\Lambda}$, individual samplings show a shower development that is far from smooth, the sources of such irregular development being fluctuations in the level of the first collision, nuclear interactions close to a detector giving rise to an unusually large signal in that detector, and the statistical nature of the development itself.

Thus, we are not interested in finding out how well an observed event fits to a smooth function, but in estimating to what accuracy the trapezoidal integration rule of Equation (3) determines the area under a particular shower curve. To this end a 5 x 5 matrix, A, was formed of the average square deviations of the counts in individual counters from the linear interpolation of two counters on either side. That is,

$$A_{ij} = \overline{\Delta N_i \Delta N_j} \ ,$$

where ΔN_2, for example, is the deviation of the count in counter #2 from the linear interpolation of counters #1 and #3. The matrix is 5 x 5 because counter #6 has no adjacent counter deeper in the spectrometer and close enough to it to enable one to calculate such a deviation. The matrix was formed for each of the various energy bins and with a total sample of 2,000 events. Counter #6 was used in this analysis because event energies calculated ignoring counter #6, where the response from

counter #6 was erratic and large, fell into energy bins below 50 BeV. That the remaining events were usable is supported by the resulting smoothness of the average shower curves (Figure 2). These were not smooth when energies were calculated including the response from counter #6.

If one then forms the product

$$\Delta E_c^2 = \left(\frac{\partial E_c}{\partial N}\right)^T \cdot A \cdot \left(\frac{\partial E_c}{\partial N}\right) , \qquad (5)$$

with $\frac{\partial E_c}{\partial N_2} = \frac{\partial E_c}{\partial N_4} = 0$, the assertion is that this ΔE_c^2 represents the average square sampling error between a spectrometer with counters only at positions 1, 3, and 5, and a spectrometer with counters at positions 1 through 6. This was confirmed by a direct comparison of the matrix method with the calculation

$$\Delta E_c^2 = \overline{(E_{1,3,5} - E_{1,2,\ldots,6})^2} .$$

The two methods agree to within a few percent. With this calculation, then, it was found that $\Delta E_c/E_c \sim .4$, independent of energy, showing that there is a significant gain in going from a three to a six counter spectrometer.

If one performs the calculations of Equation (5) with all $\partial E_c/\partial N$ nonzero, $\Delta E_c^2/16$ represents the average square sampling error between the six-counter spectrometer in this experiment and a twelve-counter spectrometer where the probe spacing is halved. The factor 16 represents the differences between forming the A matrix with six and twelve counters, since the ΔN's represent sagittas in the linear interpolation procedure

and would reduce by a factor of 4 when the probe spacing is halved (Figure 5). The result here was that $\Delta E_c/E_c \sim 0.03$, again independent of energy. We note that if the $\langle \Delta N^2 \rangle$ rms are not small but are rather representative of sharp spikes, then the estimate of sampling error is $\Delta E_c^2/4$, and here $\Delta E_c/E_c \sim 0.07$, independent of energy.

With these results and those of Section III, one sees that the best sampling of individual shower development is obtained in a spectrometer of at least 1000 gm/cm^2 thick with probes spaced equally throughout, separated by $\lesssim$ 100 gm/cm^2. This result agrees with that of V. S. Murzin.[1]

V. CONCLUSION

If one constructs a total absorption spectrometer from iron 1000 gm/cm^2 thick with ten plastic scintillator probes equally spaced, the sampling of the development of individual showers in the 100 - 1000 BeV range can be done so that the error in the estimation of the area under the shower curve contributes only ~3 percent to the uncertainty in the energy. This means that the error in E_c depends mainly on the division of energy dissipation of hadrons between "visible" and "invisible" energy losses and on fluctuations in this division. No direct comparison of calculated energies to true energies to obtain the $P(E_t, E_c)$ distribution could be made with the apparatus at Echo Lake. However, it is estimated that this distribution has a peak that is shifted to lower energy by .2 E_t and that the full width at half maximum of this distribution is of

the order of 20 percent.[1] The accuracy of determination of the energy of hadrons with such a device is, thus, ~20 percent. Murzin[1] has claimed that with ionization chambers as probes there is a systematic underestimate of the order of 8 percent and a spread of the distribution of E_c for a given E_t of the order of 10 percent. At the present time there has been no direct calibration of a spectrometer in the energy range 100 - 1000 BeV.

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*Work supported by the National Science Foundation Contract NSF GP-5811 and the U. S. Office of Naval Research Contract NONR-1224 (23).

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TABLE I

Error in Integrating Smooth Shower Curve Using Formula (3)

Shower Curve Parameters			Underestimate of Energy in % by Probe Arrangements*				
A	α	Λ	I	II	III	IV	V
42	0.58	143	8	19	3.9	5.4	.06
2.72	1	200	16	18	0.1	5	3
49.5	0.33	418	13	17	3.3	7.1	5.6

* I - Probes at 68, 133, 266 and 399 gm/cm^2

II - Probes at 133, 266 and 399 gm/cm^2

III - Probes at 68, 133, 266, 399, 532, 665, 1070 gm/cm^2

IV - Probes at 68, 133, 266, 399, 532, 665 gm/cm^2

V - 12 Probes equally spaced with thickness of layer 65 gm/cm^2

FIGURE CAPTIONS

1. Experimental arrangement for M.U.R.A. Cosmic Ray experiment to detect massive particles in cosmic rays.
2. Average shower curves for some selected energy bins.
3. Behavior of characteristic absorption length for average shower curves.
4. Individual event shower development in the total absorption spectrometer. The curves are drawn only to guide the eye. The X's are the actual data points.
5. Relation between average deviations in a six and a twelve counter spectrometer.

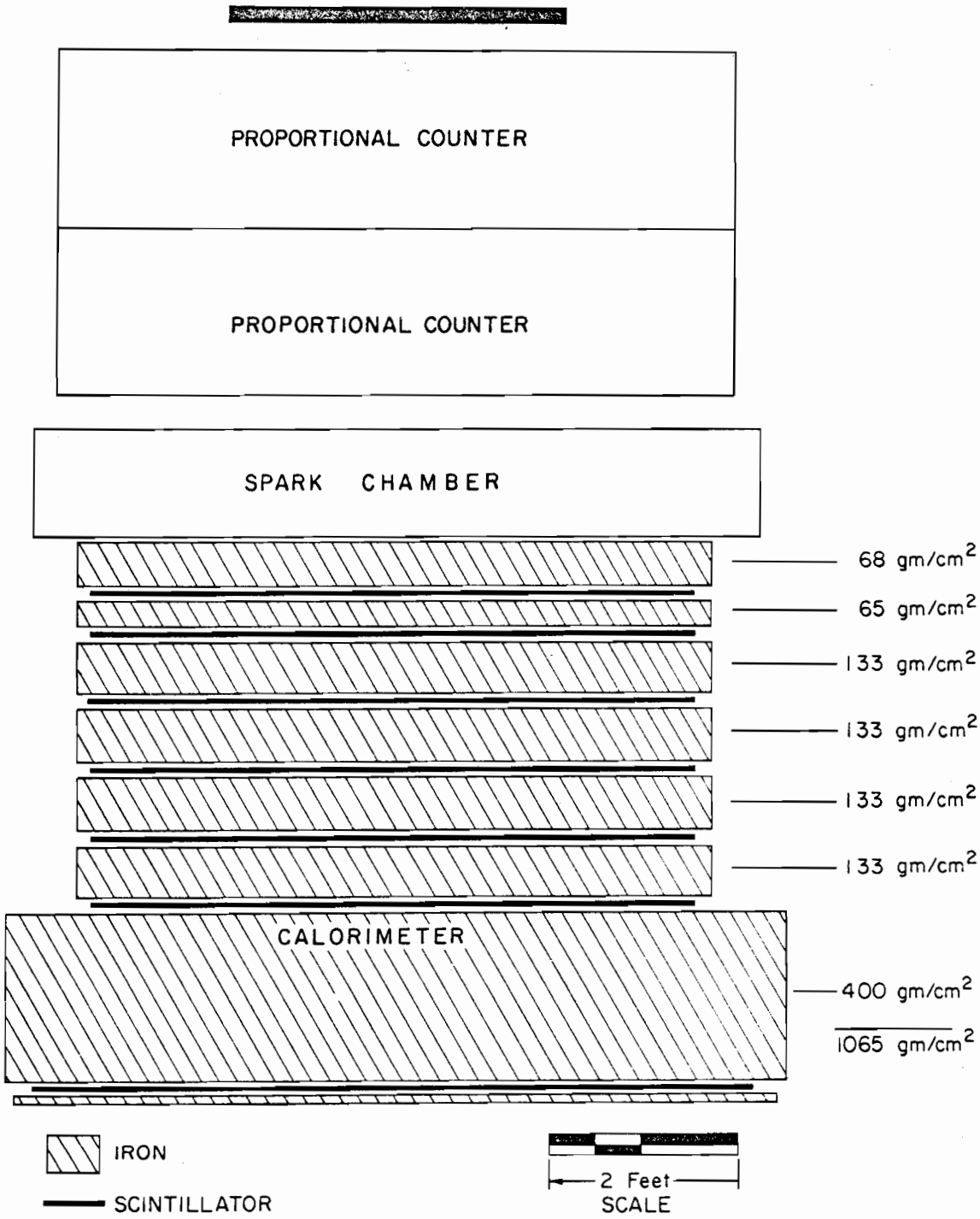

PROPORTIONAL COUNTER
PROPORTIONAL COUNTER
SPARK CHAMBER
68 gm/cm²
65 gm/cm²
133 gm/cm²
133 gm/cm²
133 gm/cm²
133 gm/cm²
CALORIMETER
400 gm/cm²
1065 gm/cm²
IRON
SCINTILLATOR
2 Feet
SCALE

FIG 1.

FIG 2.

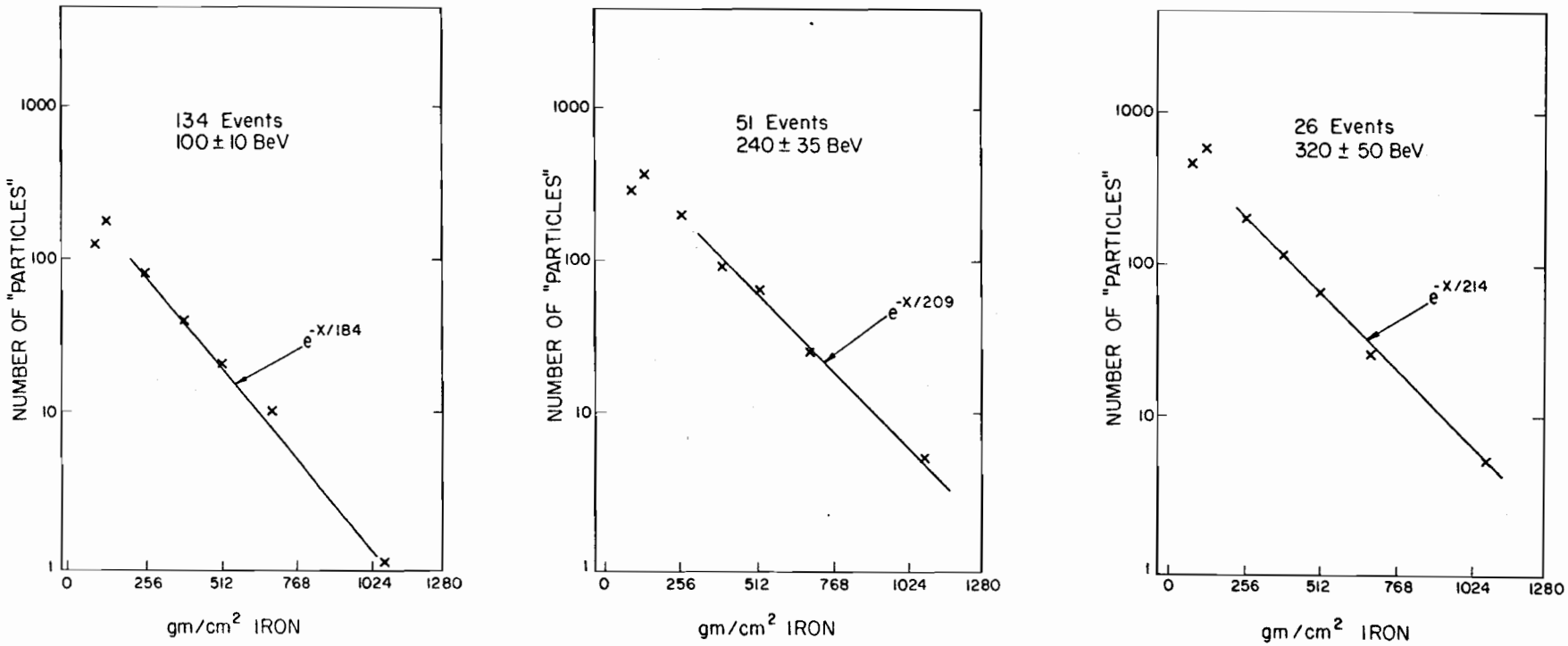
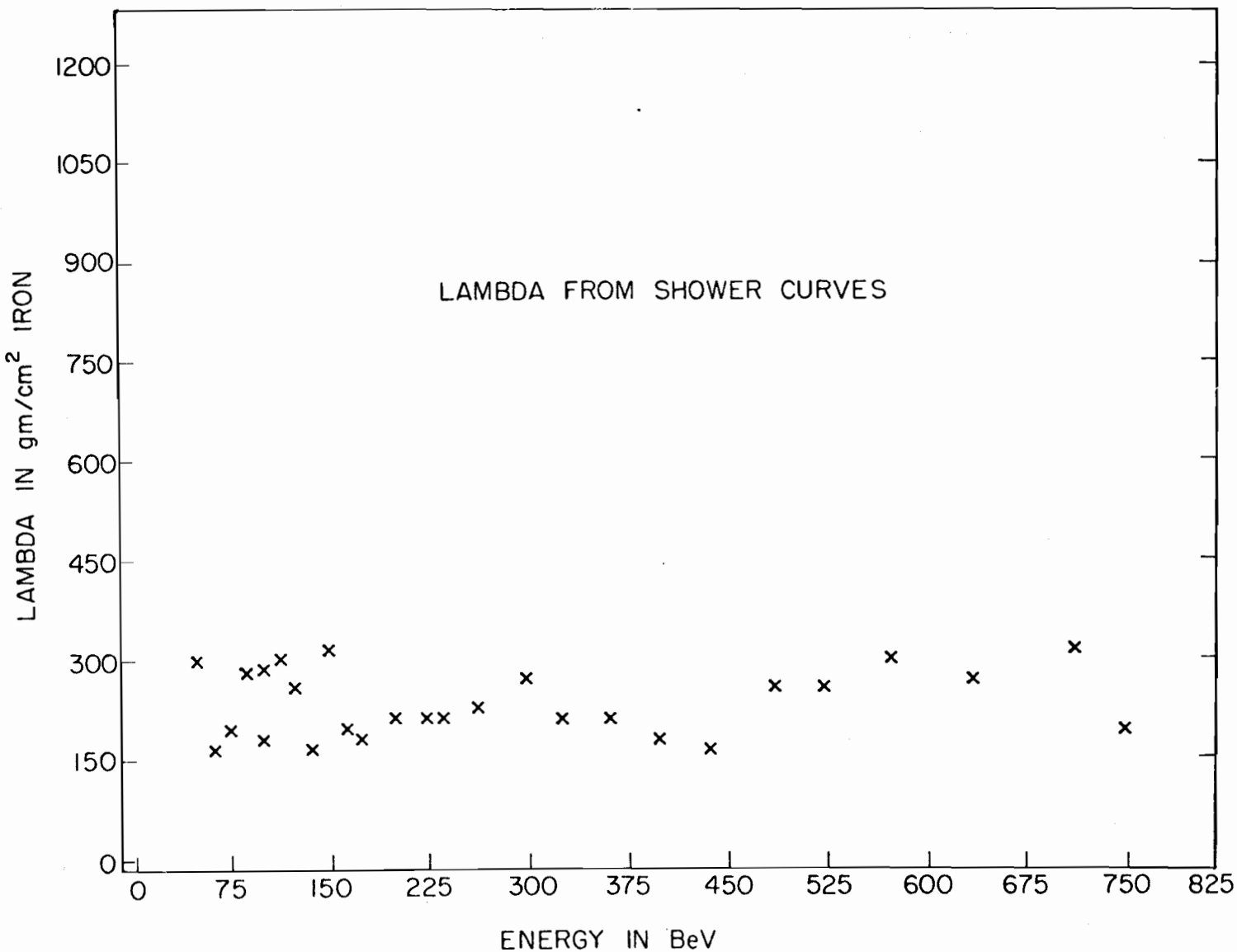

134 Events
100 ± 10 BeV
$e^{-X/184}$
51 Events
240 ± 35 BeV
$e^{-X/209}$
26 Events
320 ± 50 BeV
$e^{-X/214}$
NUMBER OF "PARTICLES"
1000
100
10
1
0
256
512
768
1024
1280
gm/cm² IRON

Fig. 3.


LAMBDA FROM SHOWER CURVES
LAMBDA IN gm/cm² IRON
1200
1050
900
750
600
450
300
150
0
0
75
150
225
300
375
450
525
600
675
750
825
ENERGY IN BeV

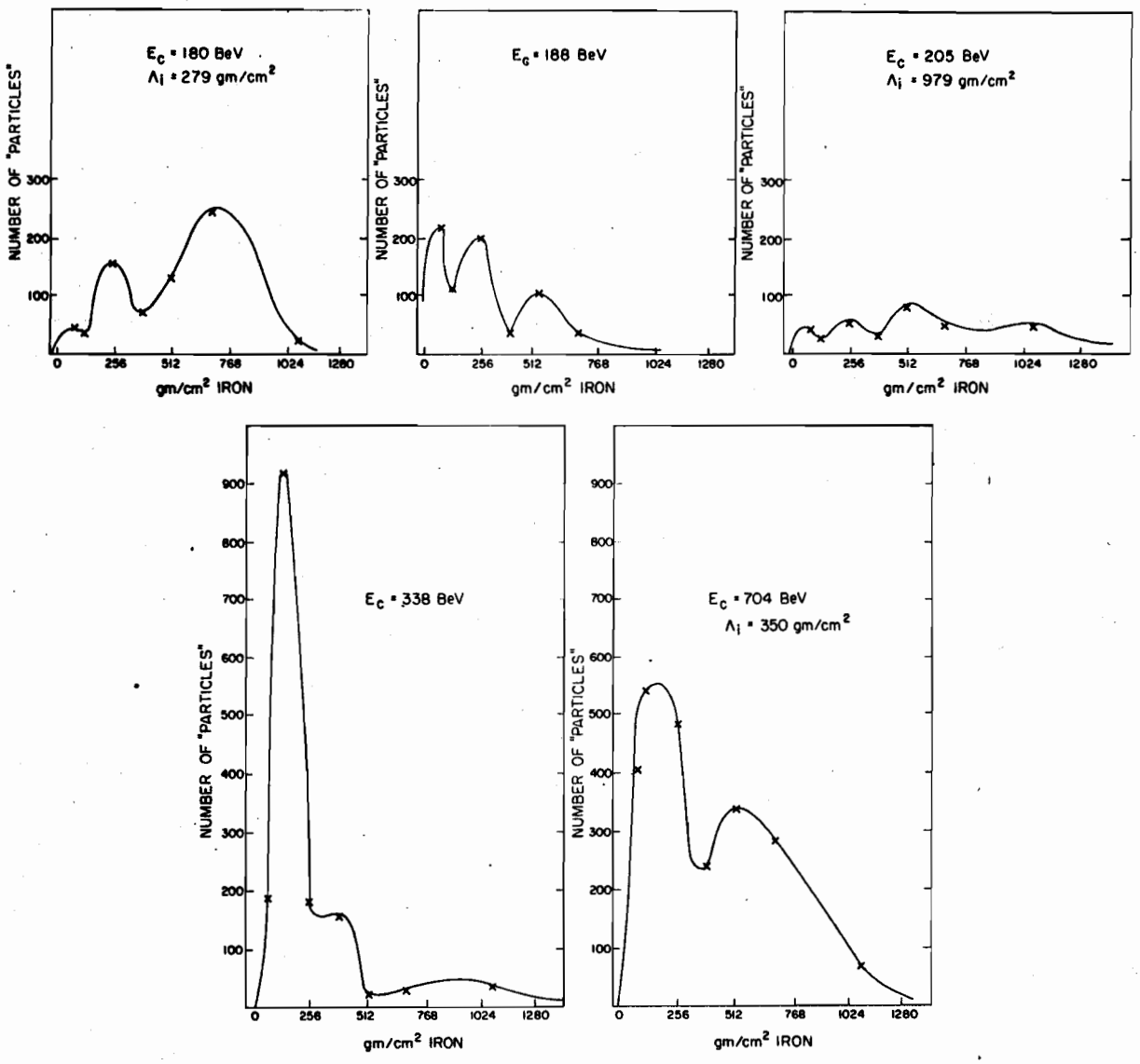
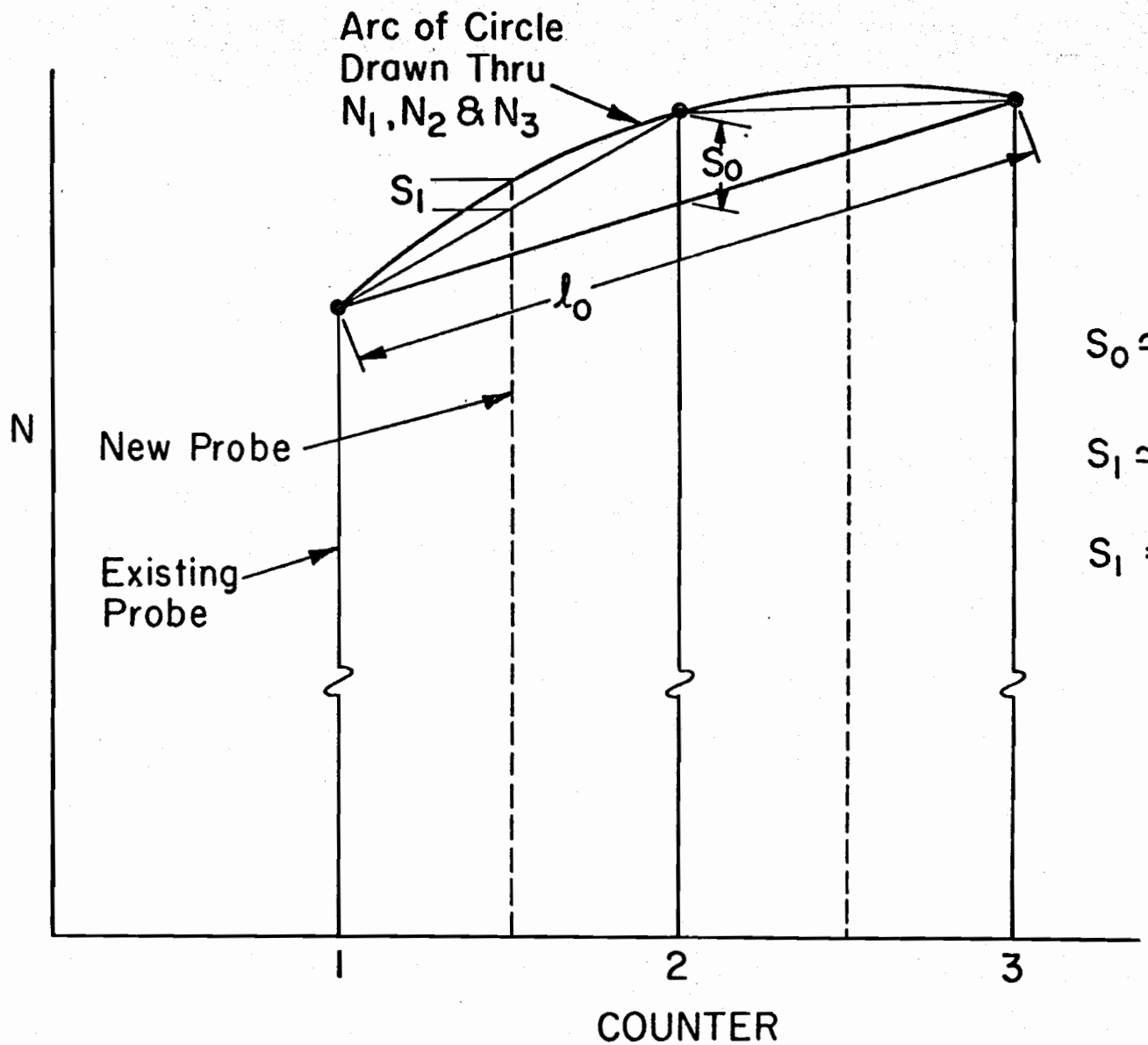

E_c = 180 BeV
Λ_i = 279 gm/cm^2
E_c = 188 BeV
E_c = 205 BeV
Λ_i = 979 gm/cm^2
E_c = 338 BeV
E_c = 704 BeV
Λ_i = 350 gm/cm^2
NUMBER OF "PARTICLES"
gm/cm^2 IRON
0
256
512
768
1024
1280
100
200
300
400
500
600
700
800
900

FIG 4.

Fig 5.



$$S_0 \simeq \frac{l_0^2}{8r} = \langle \Delta N_2 \rangle_{rms}$$

$$S_1 \simeq \frac{(\frac{1}{2} l_0)^2}{8r}$$

$$S_1 = \frac{1}{4} S_0$$