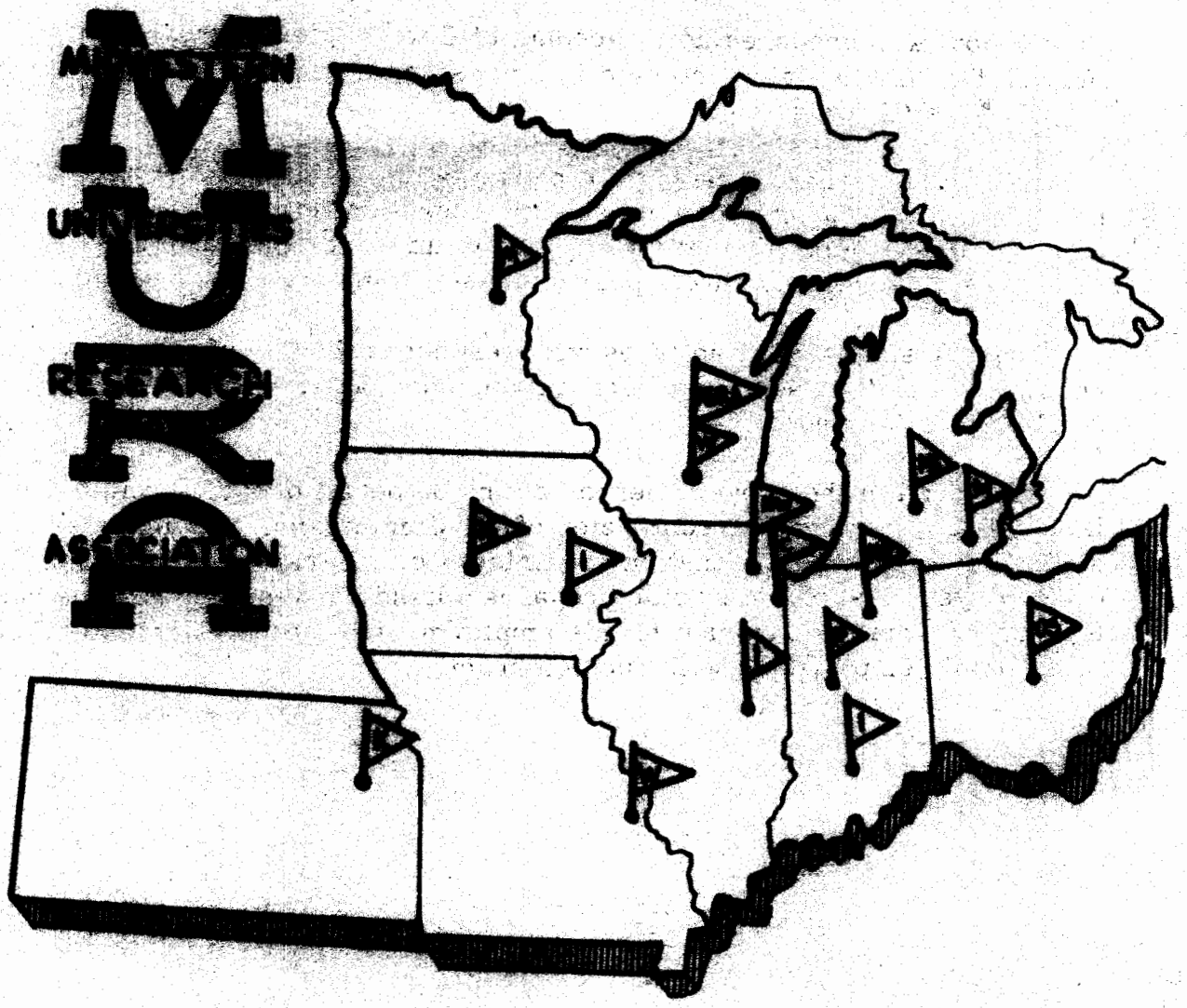

ON THE SUITABILITY OF THE 500 MEV FFAG SYNCHROTRON AS AN INJECTOR FOR THE AGS AND ZGS

E. M. Rowe

REPORT NUMBER 701

MURA-701
UC-28: Particle Accelerators and
High Voltage Machines
TID-4500 (32nd Edition)

MIDWESTERN UNIVERSITIES RESEARCH ASSOCIATION*

P. O. Box 6, Stoughton, Wisconsin

ON THE SUITABILITY OF THE 500 MEV FFAG SYNCHROTRON AS AN INJECTOR FOR THE AGS AND ZGS

E. M. Rowe
September 25, 1964

ABSTRACT

A 500 MeV FFAG synchrotron has been designed at MURA as a possible injector for the AGS and ZGS accelerators. In order for the FFAG to function efficiently as an injector for these machines, certain requirements as to the emittance and energy spread of the FFAG beam must be met. In this report a number of schemes for injecting the beam from the FFAG into both the AGS and ZGS are described and estimates as to how well these requirements are met are made for each scheme.

*Supported by the U. S. Atomic Energy Commission Contract No. AT(11-1)-384. AEC Research and Development Report.

INTRODUCTION

The design of a 500 MeV FFAG synchrotron has been brought to a high degree of completion at MURA.[1] This design study was motivated by the current interest at BNL and ANL in raising the injection energies of the AGS and ZGS machines. The purpose of the study was to arrive at a workable design, and from this design make reasonable estimates of the construction cost and the performance of an FFAG accelerator as an injector for the AGS and ZGS. This report deals with the latter subject.

A number of modes of operation of the FFAG as an injector into both machines are considered and estimates are made in order to determine their practicability. We will consider injection into the AGS first.

I. INJECTION OF A SINGLE-TURN EXTRACTED STACKED BEAM INTO THE AGS

In this mode a stack of 16 groups of protons would be accumulated in the FFAG. This would amount to 1.3×10^{13} protons which is a factor of 4 below the calculated space-charge limit of the FFAG at 500 MeV. This is to be compared with the design goal of 2×10^{13} protons injected at 500 MeV in the AGS improvement program.[2] After the sixteenth group of protons has been deposited in the stack and allowed to debunch, the FFAG rf system would be turned on, rebunching the beam, so that the beam could be made to occupy π radians of azimuth in the FFAG, allowing some 95 ns for turning on the kicker magnet. The whole beam would then traverse the FFAG extraction channel and be injected into the AGS. Since the beam would occupy only a small portion of AGS circumference initially, the space-charge limit in the AGS would be exceeded locally.

The factor by which the space-charge limit would be exceeded in the AGS is given approximately by

$$\frac{N_{in}}{N_{sc}} \cdot \frac{2\pi r_2}{\Delta\varphi\, r_1} = 12 \qquad (1)$$

where N_{in} and N_{sc} are the number of protons injected and the number of protons corresponding to the space-charge limit in the AGS, r_1 and r_2 are the FFAG and AGS radii and $\Delta\varphi$ is the azimuthal dimension of the beam in the FFAG. The time required for this beam to unbunch in the AGS is given by

$$t \geqslant \frac{\nu_x \beta^2 E}{f \Delta E} = 5.77 \times 10^{-4} \text{ seconds} \qquad (2)$$

where ν_x is the radial tune of the AGS, f is the proton revolution frequency and ΔE, the energy spread of the injected beam, is taken as 15 MeV. Since this would give the beam time for some 160 revolutions, there is little reason to believe that the beam would "live" long enough to unbunch. Accordingly, this method does not appear to be practical in the AGS.

II. SYNCHRONOUS INJECTION INTO THE AGS

In this mode of operation a single group of protons is accelerated in the FFAG and extracted, still bunched, in a single turn as in mode I. This bunch of protons is then injected into an empty bucket in the AGS. Since there are 12 such buckets in the AGS, 12 bunches only may be accepted. Hence, the injected beam falls about 50% short of the design goal. The time of extraction has to be so chosen that the "center of gravity" of the group of protons arrives at the AGS inflector at the same time as the synchronous phase point of an empty bucket in the AGS.

To fill all 12 buckets in the AGS without disturbing the protons already captured we will require a fast pulsed inflector. This inflector would be turned on as an empty bucket enters it and off as the bucket, now filled, leaves it. This inflector would be essentially the same device as would be used in the single-turn extraction system from the FFAG.

A bunched single pulse beam in the FFAG would have a radial width due to betatron motion and energy spread of 2.58 cm. This would become in the AGS greater by a factor of

$$\left[\frac{\nu_{x_1}}{\nu_{x_2}} \frac{r_2}{r_1} \right]^{1/2} = 2.62 , \tag{3}$$

where the subscripts 1 and 2 refer to the FFAG and the AGS. Thus, this beam in the AGS would have width of 6.75 cm. However, to this must be added an increment in radius due to the energy spread of the beam. The energy spread of the bunched beam in the FFAG is expected to be 4.7×10^5 eV. Since the radius, harmonic number, accelerating voltage, and tune of the two machines differ markedly, we can expect the energy spread of the bunched beam when captured in a bucket in the AGS to differ from what it was in the FFAG. It will, in fact, turn out to be greater. To show this we make the following calculation.

It may be shown[3] that the maximum excursion in energy from the synchronous energy of a particle traveling on a stable phase oscillation of small amplitude in a stationary bucket is given by

$$\Delta E = \frac{\Delta \varphi_m}{\pi} \left[\frac{2 \text{ eV } E_s \cos \varphi_o}{\pi \cdot h \ |K|} \right]^{1/2} \tag{4}$$

where $\Delta\varphi_m$ is the maximum excursion in phase from the stable phase, φ_o is the stable phase, in this case 0, h is the harmonic number, V is the total accelerating voltage per turn, E_s is the synchronous energy and K is defined as

$$K = \frac{E}{f}\frac{df}{dE} = \frac{1}{\beta^2}\left(\frac{1}{\gamma^2} - \frac{1}{\nu_x^2}\right) = 0.745 .$$

Assuming that the half azimuthal dimension of the beam, and therefore $\Delta\varphi_{m_1}$, in the FFAG is $\pi/2$ radians as before, the value of $\Delta\varphi_{m_2}$ for this group of protons properly centered in a stationary bucket in the AGS becomes

$$\Delta\varphi_{m_2} = \Delta\varphi_{m_1}\, h_2 \frac{r_1}{r_2} = 0.322\,\pi . \tag{5}$$

Then taking V as 2×10^5 volts and $\cos\varphi_o = 1$, the maximum energy spread is 2.84 MeV. Actually this value for the energy spread is somewhat small since the model that the calculation is based on is one in which the beam bunch entering the AGS has zero energy spread. Since the energy spread of the beam from the FFAG is small compared to 2.84 MeV, the resultant error is not large. This latter effect also permits rather large errors in matching energy before the energy spread of the beam in the AGS increases appreciably. The increase in radial size of the beam due to this energy spread is

$$\Delta r = \frac{r}{\nu_x\,\beta^2}\frac{\Delta E}{E} = 0.52 \text{ cm}, \tag{6}$$

which results in a total width of the beam of 7.3 cm.

The accuracy with which the bunch of protons is centered in phase on the AGS bucket will also affect the final energy spread. For small errors

in phase, the energy spread will increase at least as fast as

$$\Delta E \left[1 + \frac{\delta \varphi}{\Delta \varphi_{m1}}\right] \tag{7}$$

where $\delta \varphi$ is the phase error. This effect may be reduced by reducing the voltage during the injection period. However, the requirement for accuracy still remains because as the bucket is made smaller errors in matching phase and energy can result in loss of particles as well as increased energy spread. If the machine parameters were such that their revolution frequencies were harmonically related, the problem of phase matching would be considerably easier.

It would also be necessary to add a "front porch" lasting about 1/8 second to the AGS guide field during injection. The accuracy with which this front porch would have to be controlled may be estimated from

$$\frac{\Delta E}{E} = \frac{1}{\beta^2} \frac{\Delta B}{B} . \tag{8}$$

From this we have that a change $\Delta B/B$, of 0.15% during injection would just double the energy spread of the beam.

III. INJECTION OF A STACKED BEAM BY MEANS OF RESONANT EXTRACTION FROM THE FFAG INTO THE AGS

Resonant extraction has been performed on an electron synchrotron with good efficiency ($\sim$70%) for a reasonable number of turns at the Cambridge Electron Accelerator.[4] Theoretical and computer studies of this process have been done by Hammer, Laslett, and Symon.[5,6] Briefly, the scheme involves driving the tune of the beam to a half-integral resonance by

the use of guide field perturbations. As the half-integral resonance is reached, the radial motion of the beam grows exponentially, and if the harmonics of the field perturbations are suitably chosen, there will be one region of peak departure of the beam from the unperturbed motion which will occur at precisely the same azimuth on every turn. Since the beam is on a half-integral resonance, a particular proton in the beam will on alternate revolutions be displaced outward and inward from its unperturbed orbit. Thus, if the rate of growth per turn is large enough, the proton may "step across" a reasonably thick septum and into an extraction channel. The efficiency of the scheme depends on the ratio of growth rate to septum thickness.

A particular advantage of this scheme for extracting the beam is that the emittance of the FFAG improves. This results for the following two reasons. First, all the particles become locked together in phase and second, only particles of nearly the same amplitude traverse the extraction channel. As a result the emittance of the FFAG decreases by a factor at least as great as the ratio of the beam width to the amplitude increase per turn at the time that the beam "steps across" the septum.

As in the first mode, 16 groups of protons would be accumulated at the 500 MeV radius in the FFAG. The field perturbations would then be turned on and the radial growth would then set in.

The radial extent of the stacked beam in the FFAG due to betatron motion alone would be 2.54 cm. To this we must add an increment due to the energy spread of the stacked beam. The energy spread of the beam injected into the FFAG is expected to be 50 kV. Upon arriving at the stacking radius, this

energy spread will be 150 kV since the revolution frequency increases by a factor of about 3. The stacked beam must have an energy spread of at least 16 times this or 2.4 MeV. To be realistic we cannot expect the capture, acceleration and stacking process to proceed with no loss of phase space density, hence we will take an energy spread of twice this, or 4.8 MeV, for the unbunched stacked beam. Then the total width of the stacked beam in the FFAG would be 3 cm.

If we take 0.15 cm as a growth rate, a not unreasonable assumption in view of recent studies of this effect at MURA, and use an electrostatic septum as the first element in the extraction channel, an efficiency of 80% or better over a period of 20 turns should be obtainable. The width of the beam to be transferred to the AGS would then be 0.15 cm instead of 3 cm. Upon injection into the AGS the width would increase to 0.4 cm.

Since the increment in beam width in the FFAG caused by energy spread is small compared to the betatron motion, we can expect that the energy spread of the extracted beam will be constant during extraction and nearly equal to the full energy spread of the stacked beam. Then the increment in radius in the AGS contributed by energy spread would be 1.3 cm. Thus, the entire stacked beam may be transferred to the AGS and lie on an increment of radius about 2 cm wide. If this beam is captured adiabatically, the width will increase to 4.8 cm. If the capture is completely nonadiabatic in a moving bucket with $\Gamma = 0.5$, the width will be about 6 cm, assuming again 200 kV as the peak accelerating voltage. However, half the beam would be lost.

It should be pointed out here that it may actually be necessary to degrade the emittance of the injection system in this mode. The beam would only be 2 cm wide and might exceed the space-charge limit because of its small radial size.

IV. INJECTION INTO THE ZGS

Because the ZGS is a weak-focusing machine, injection of a beam of appreciable energy spread from a source with fairly large emittance, such as the FFAG, poses difficult problems. Evaluating Eqs. (3) and (4) for the FFAG-ZGS system gives $\Delta r_2 = 3.68\ \Delta r_1$ (Eq. (3)) and $\Delta r_2 = 3.5 \times 10^{-6}$ cm/eV (Eq. (4)). Thus a single pulse from the FFAG having an energy spread of 1.5×10^5 eV and a width due to betatrom motion of 2.54 cm in the FFAG would be 9.88 cm wide after injection into the ZGS. If the beam was bunched in the FFAG as would be required to make single-turn extraction efficient, the energy spread would be 4.7×10^5 eV and the width in the ZGS would be 11 cm.

Furthermore, if a single pulse was extracted from the FFAG while bunched and injected and allowed to unbunch in the ZGS, it would occupy an area of

$$\frac{2\pi \Delta E}{f} = 2.2 \text{ volt seconds} \tag{9}$$

of ZGS synchrotron phase space. The total ZGS synchrotron phase space available at 500 MeV with 2×10^4 volts per turn and $\Gamma = 0.5$ is

$$\frac{8}{f}\left[\frac{2\,V\,E}{h\,\pi\,|K|}\right]^{1/2} \alpha\ (\Gamma) = 2.0 \text{ volt seconds.} \tag{10}$$

Thus, increasing the intensity of the pulses injected into the ZGS by stacking in the FFAG would quickly result in exceeding the available synchrotron

phase space. On the other hand, injecting consecutive pulses into the ZGS would soon exhaust the available radial aperture. However, by utilizing both synchrotron and betatron phase spaces a considerable improvement may be realized.

To do this we must make use of the techniques of both synchronous injection and pulsed bump, multiturn injection. A single group of protons would be accelerated in the FFAG and extracted while bunched tightly enough to fit into an rf bucket in the ZGS. Since there are eight such buckets, eight pulses may be accommodated. After eight pulses have been injected, the pulse bump which has distorted the equilibrium orbit in the ZGS so that it passes through the inflector is reduced sufficiently to move the beam over at the inflector by one beam width. The injected beam is now outside of the inflector and eight more pulses may be injected synchronously. If the beam energies of the ZGS and FFAG are kept equal, then the next eight pulses will go into the eight buckets with no increase in phase area. The whole cycle of synchronous injection and beam movement may be repeated four times before the radial aperture of the ZGS is exceeded. The total beam accumulated in the ZGS would be about 2.5×10^{13} protons, a factor of four below the goal of the improvement program.

This mode is probably practical but hardly represents a great improvement over the design intensity of the ZGS with 50 MeV injection. It should be noted, however, that the number of protons accelerated per pulse in the FFAG is taken to be a factor of <u>four</u> under the calculated space-charge limit of the FFAG. A reasonable relaxation of this safety factor and minor changes in the

design of the FFAG, for example increased vertical aperture at the injection radius, could make up a large part of the discrepancy.

The guide field of the ZGS would have to be controlled very rigidly during injection. From Eq. (8) we have that a change of 0.06% in the ZGS guide field during injection would add 10% to the radial width of the beam.

CONCLUSIONS

There are two methods of matching the FFAG to the AGS as an injector. There are certain technical problems to be solved in both methods. Of these, the most important appears to be the need for accurate phase matching for one method and the need for fairly high efficiency multiturn extraction from an FFAG for the other. Since both the CERN and Berkeley groups feel that the former is solvable, and since high efficiency resonant extraction has been demonstrated on at least one machine,[4] it is felt that these problems present no great obstacle.

The point to be made here is that there appears to be no fundamental difficulty in matching the FFAG to the AGS by either of these schemes. It seems reasonable to expect that the expenditure of a small part of the price differential between the FFAG and the proposed BNL 500 MeV linac (about a factor of two) would go a long way towards solving the technical problems.

With regard to the ZGS, the case is not so clear. Assuming that the various complicated gymnastics required during injection can be performed, within the safety factor included in the calculations of the FFAG intensity (a factor of 4), the FFAG is still not a particularly exciting injector from either the standpoint of cost or performance. However, with a reasonable relaxation

of safety factors and some redesign, the FFAG can probably come close to the performance of a 200 MeV linac as an injector for the ZGS at about the same price. Furthermore, it would do this at an energy sufficiently high (500 MeV) so that the improvement program goal for the ZGS intensity, which is the space-charge limit at 200 MeV,[7] can probably be reached.

REFERENCES

1. S. C. Snowdon, "Study of a 500 MeV High Intensity Injector", MURA Report No. 700 (unpublished, September 8, 1964).
2. A Proposal for Increasing the Intensity of the Alternating-Gradient Synchrotron at the Brookhaven National Laboratory. BNL 7956 (1964).
3. K. R. Symon and A. M. Sessler. Proceedings of CERN Symposium on High Energy Accelerators and Pion Physics (CERN, Geneva, 1956), Vol. 1, p. 44.
4. Brasse, Fischer, Fotino and Robinson. Cambridge Electron Accelerator Report, CEAL-1006 (1963).
5. C. L. Hammer and L. J. Laslett. Rev. Sci. Instr. 32, 144 (1961).
6. L. J. Laslett and K. R. Symon. Proceedings of International Conference on High Energy Accelerators and Instrumentation (CERN, Geneva, 1959), p. 38.
7. Expected Beam Intensities of the FFAG and ZGS Accelerators. Report to the AEC of the Ad Hoc Committee, July 31, 1963.