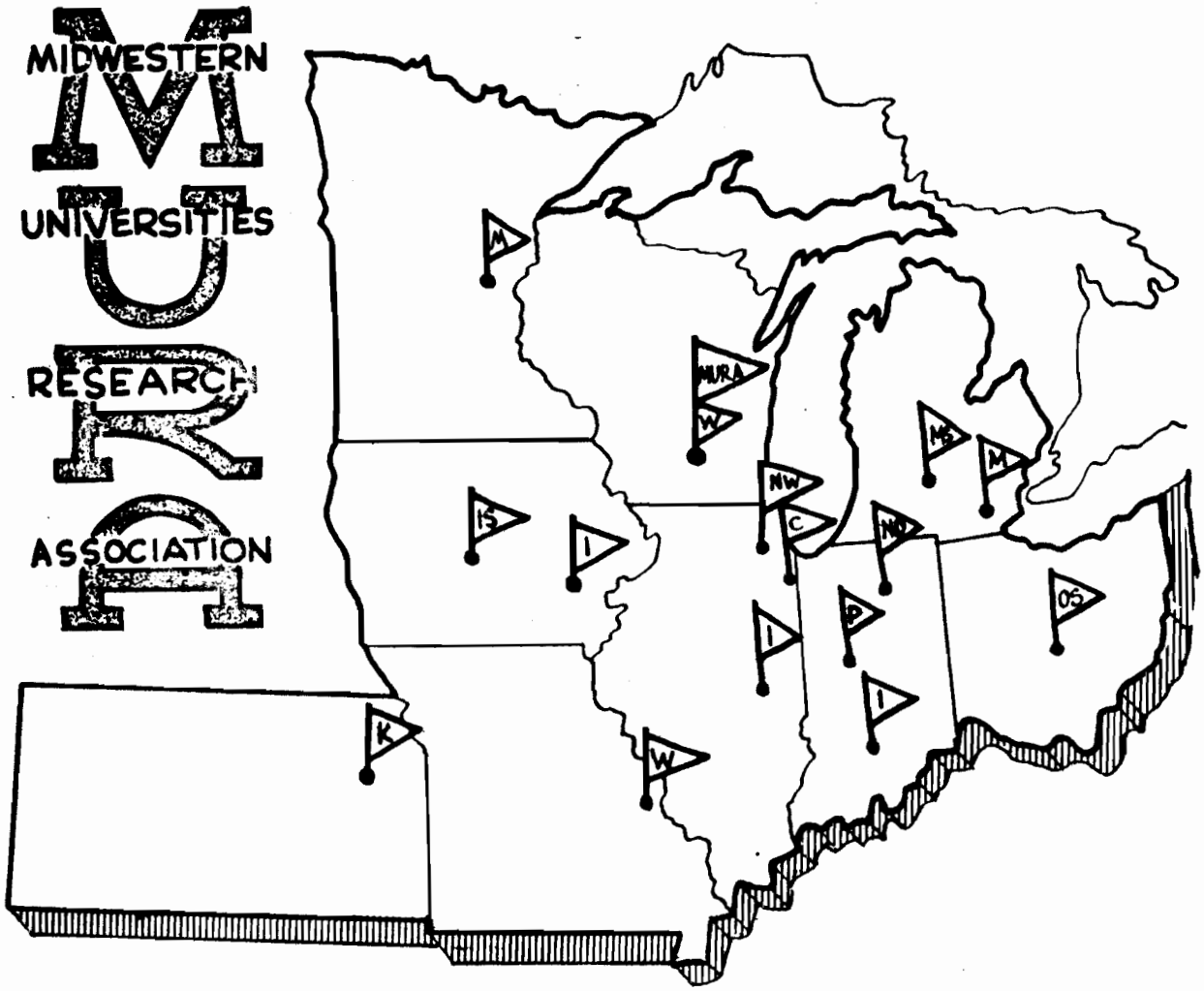

NEUTRINO FLUXES WITHOUT FOCUSING AND WITH "IDEAL" FOCUSING

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ABSTRACT

The problems to be solved in reducing the muon shield thickness are discussed, but no solution is offered. "Ideal" focusing, as seen by this writer, is defined. By direct evaluation it is found that an-order-of-magnitude increase in neutrino flux can be obtained with "ideal" focusing as compared with no-focusing. This increase is approximately independent of accelerator energy in the range 10 to 1000 Bev.

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I. POSSIBILITIES OF REDUCING THE SHIELD THICKNESS

In discussing accelerators of different energies as to their usefulness in high-energy physics, comparisons of neutrino fluxes have been made. The neutrino flux calculations have been made under the assumptions of:

1. No focusing on the decaying π and K mesons and
2. a μ-meson shield of thickness proportional to the accelerator energy.

Before proceeding with the main subject of this report, namely the effect of focusing on neutrino flux spectra, a few remarks can be made on the second assumption. If the μ mesons could be prevented from disturbing the experiment by means other than the assumed brute-force method of providing a great enough thickness so that dE/dx losses finally stop the muons, it is likely that the ν detector could be moved closer to the decay path, resulting in a larger solid angle and, therefore, a higher ν flux. The shield cannot be completely eliminated because of the presence of the strongly interacting particles. At an accelerator with 10^{14} protons/sec and a "low" energy, say 10 Bev, the range of the highest energy muon (about 7.5 meters in steel) is about the same as the thickness required to make the neutron flux negligible. In this case the dE/dx method is probably the best one to stop the muons. But, with increasing energy the muon range goes up linearly, and the neutron shield thickness approximately logarithmically. At 1000 Bev the muon range is 750 meters in steel and the neutron shield thickness is less than 10 meters. There is, obviously, some room for improvement at very high energies.

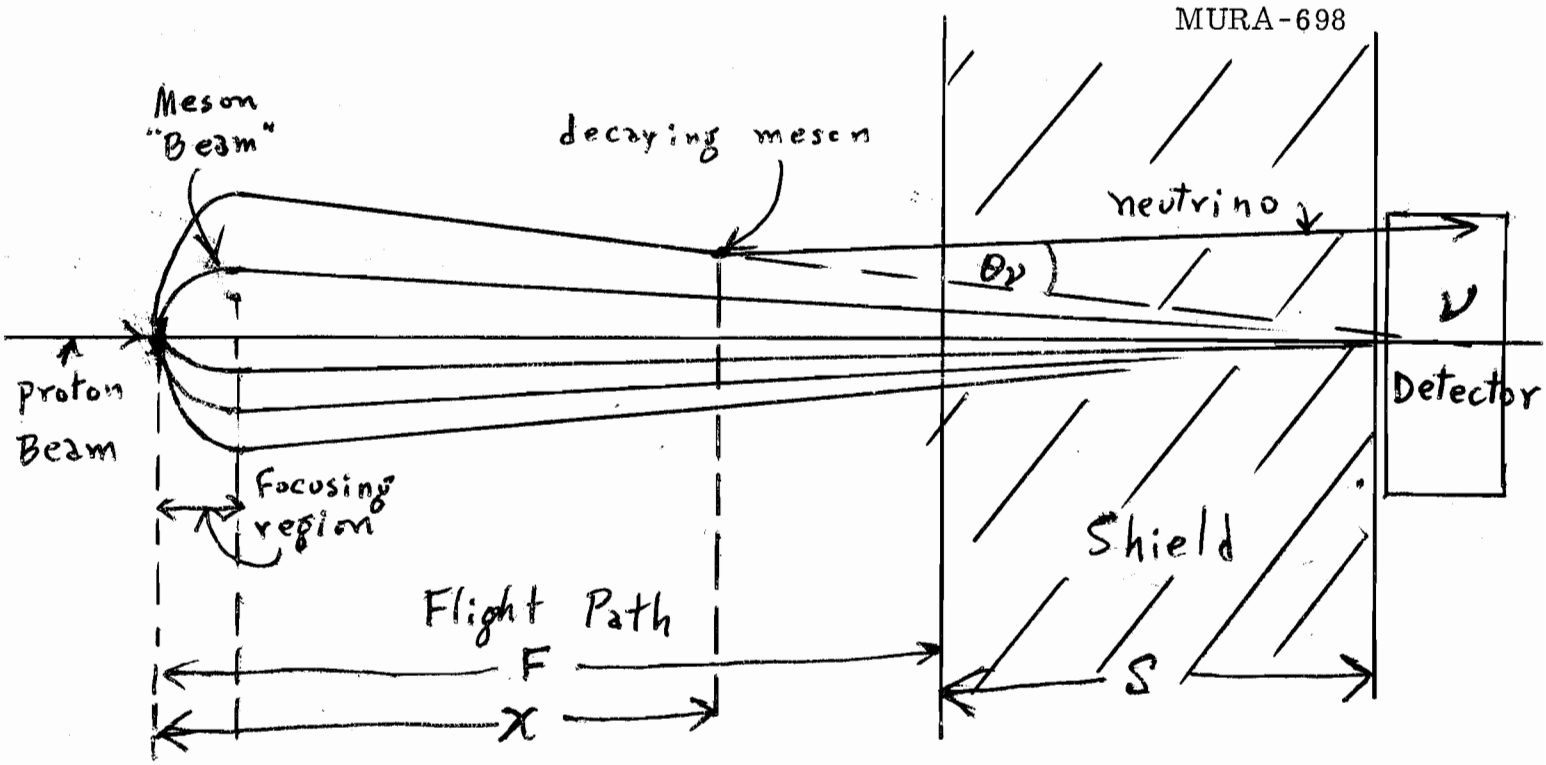
By using magnetic deflection of the muons from π and K decay it should be possible to reduce the total distance from the end of the decay path to the detector to something below the muon range. A solution to this problem has not been worked out. Some of the problems involved in using magnetic deflection are:

1. Divergence of the decay muons, especially from K decay, making it difficult to cleanly deflect them.
2. Divergence in the production process of the π's and K's which produce the muons.
3. Coulomb scattering of muons in the neutron shield after deflection (or during deflection if the shield is magnetized iron).
4. If the π's and K's are focused, the π's, K's, and μ's of the opposite charge are defocused.
5. Decay of π's and K's produced in the shielding by neutrons coming from the main target. (Other strongly interacting particles are assumed to be deflected with the muons.) High-energy, charge-exchange neutrons will be the most troublesome. They should produce about 10^{-7} high-energy muons per accelerator proton.
6. A rough vacuum is required in the region forward from the target to reduce production of nonaxial π's and K's from neutron interactions in the air.

A design which is a substantial improvement over assumption 2. requires a careful study. We have considered only the effect of introducing focusing. Shield thickness is a parameter in the result, which we have evaluated with a thickness in meters of steel equal to 0.76 E_p, where E_p is the accelerator energy in Bev.

II. "IDEAL" FOCUSING

In order to adequately allow for future ingenuity in the design of neutrino focusing devices, and to avoid making a specific design, we have studied the case of "ideal" focusing only. By ideal focusing we mean focusing in which all of the π and K mesons (of one charge state) produced by the proton beam are aimed so as to converge at a spot in the neutrino detector or, perhaps, at infinity. We will assume that the size of the spot is small enough compared to the size of the detector that the spot can be approximated by a point. This is equivalent to assuming zero emittance for the original proton beam. A sketch of a possible geometry is given below. In ideal focusing the length of the focusing region is zero. This also is assumed in the calculations to follow. Then it is entirely equivalent to replace the converging meson beam by a paraxial pencil beam.



III. THE MESON SPECTRA

We start with the Cocconi, Koester, and Perkins-type meson spectrum discussed on pp. 161 ff. of the revised edition, September 30, 1963, of the MURA proposal. The spectrum per interacting proton for production of a meson of momentum P at angle θ is given by

$$\frac{d^2N}{dP\,d\Omega} = BP^2 \left[1 - \frac{P}{P_m(\theta)}\right] e^{-P/T} e^{-P\theta/P_0} , \qquad (1)$$

where B, T, and P_0 are parameters with a predictable dependence on the energy of the incident proton, and $P_m(\theta)$ is the maximum possible value of P at θ. For π mesons the values of B and T used in this report are given

in Fig. 1 for proton energies between 10 and 1000 Bev. The value of P_0 was chosen as 0.24 Bev/c for all proton energies. On the basis of observed K - π ratios from protons between 10 and 30 Bev, we reduced B by a factor of 10 and increased P_0 to 0.36 for K^+ mesons. The same values of T were used for both pions and kaons.

The angles of meson production are irrelevant with ideal focusing since all mesons are directed toward the center of the detector. Hence, we integrate over θ and get the meson production spectrum

$$\frac{dN}{dP} \cong Ae^{-P/T}\left\{\left[1 - \frac{P}{P_m(0)} - \frac{C}{P}\right]\left[1 - e^{-P/P_0}\left(P/P_0 + 1\right)\right] + \frac{CP^2}{6P_0^3} e^{-P/P_0}\left(1 + 3\,P_0/P\right)\right\} , \tag{2}$$

where

$$A = 2\pi B P_0^2$$

$$C = 3 \frac{E_p}{M_p} \frac{P_0^2}{P_m(0)};$$ E_p and M_p are proton kinetic energy and mass, respectively. Since $E_p/P_m(0) \cong 1$, C depends only on P_0, and has the values 0.184 for pions and 0.415 for kaons. For very high momenta and very low

momenta, we get the approximate meson spectra

$$\frac{dN}{dP} = A\, e^{-P/T} \left(1 - P/E_p \right) , \quad P >> P_0 \tag{3}$$

$$\frac{dN}{dP} = \frac{A}{2} \left(\frac{P}{P_0} \right)^2 \left(1 - \frac{2\,C}{3\,P_0} \right) , \quad P << P_0 \; . \tag{4}$$

IV. THE NEUTRINO SPECTRUM

At a distance x from the target the number of mesons with momentum between P and P + dP is

$$\frac{dN}{dP} \; e^{-x/L\,(P)} \; dP \; ,$$

where the decay length

$$L\,(P) = \frac{P\,c\,\tau}{M} \; ,$$

τ being the mean life of the meson and M its mass. The number of these mesons decaying between x and x + dx is

$$\frac{dN}{dP} \; e^{-x/L} \; dP \; \frac{dx}{L} \; . \tag{5}$$

The energy spectrum of the neutrinos is uniform, the number with energy between E_ν and $E_\nu + dE_\nu$ being

$$dE_\nu / aP, \tag{6}$$

where the constant $a = 2\,\overline{E}_\nu / M = 1 - (M_\mu / M)^2$. $\overline{E}_\nu$ is the neutrino energy in the rest system of the decaying meson and M_μ is the muon mass. Only two-body decay is considered here, that is meson $\rightarrow \mu + \nu$.

Combining (5) with (6) and integrating over meson momentum and over a flight path of length F, we get the neutrino spectrum per proton

$$N(E_\nu) = \int_0^F dx \int_{P_{min}}^{P_m(0)} \frac{dN}{dP} \frac{e^{-x/L(P)}}{L(P)} \frac{dP}{aP}, \tag{7}$$

where $P_{min} = E_\nu / a - \frac{M}{2} \overline{E}_\nu / E_\nu = \frac{M}{2} \left(\frac{E_\nu}{\overline{E}_\nu} - \frac{\overline{E}_\nu}{E_\nu} \right)$.

This is not the spectrum of neutrinos at the detector. If the detector has radius R, it subtends an angle $\theta(x)$ with the meson beam at the decay point x given by

$$\tan \theta(x) = \frac{R}{F + S - x}, \tag{8}$$

where S is the shield thickness. Of the neutrinos originating at x only those with decay angle $\theta_\nu(x)$ satisfying

$$\theta_\nu(x) \leqq \theta(x) \tag{9}$$

can reach the detector. For a given neutrino energy, E_ν, θ_ν is a function of P. The relation between these quantities is obtained from the Lorentz transformation

$$\bar{E}_\nu = \gamma E_\nu \quad (1 - \beta \cos \theta_\nu)$$

or (10)

$$\cos \theta_\nu = \frac{1}{\beta} (1 - \bar{E}_\nu / \gamma E_\nu) ;$$

where the meson relativistic factors, β and γ, are given in terms of P by

$$\beta = \frac{P}{P^2 + M^2} , \qquad \gamma = \frac{1}{M} \sqrt{P^2 + M^2} . \tag{11}$$

Curves of θ_ν vs P_π and θ_ν vs P_K are given in Figs. 2 and 3. For either type of meson, there are two meson momenta that can produce a neutrino of energy E_ν at angle θ_ν. (For $E_\nu \leqq \bar{E}_\nu$, P is a single-valued function of θ_ν.) If a horizontal line is drawn in either figure at the value of θ (x), the curves, or branches of curves, below that line give the ranges of P and E_ν reaching the detector. Any branch of a curve above that line represents neutrinos with too large a decay angle to enter the detector. If the entire curve for a given value of E_ν is below θ (x), the upper limit of integration over P is P_m (0), (for that value of x) exactly as in Eq. (7).

Otherwise there are two points of intersection, given by solving (10) and (11) for P with $\theta_\nu = \theta(x)$. The result is

$$P(x) = \frac{M}{\sin^2\theta(x)}\left[\frac{\bar{E}_\nu}{E_\nu}\cos\theta(x) \pm \sqrt{\left(\frac{\bar{E}_\nu}{E_\nu}\right)^2 - \sin^2\theta(x)}\right] \tag{12}$$

The integral over momentum now extends from P_{min} to $P_-(x)$ and from $P_+(x)$ to $P_m(0)$, where $P_-(x)$ and $P_+(x)$ are the smaller and larger values, respectively, of $P(x)$ in (12). Through Eq. (8) for $\theta(x)$ these limits of integration are, obviously, functions of x.

For a given accelerator energy (which fixes $P_m(0)$) the contribution from the high-momentum integral, that is, in the $P_+(x)$ to $P_m(0)$ interval, vanishes when θ, or R, is small enough. In this case the entire contribution comes from P_{min} to $P_-(x)$, and for θ (or R) $\longrightarrow 0$,

$$P_-(x) - P_{min} \xrightarrow[\theta \to 0]{} \frac{M}{8}\,\frac{\bar{E}_\nu}{E_\nu}\,\sin^2\theta\left[\left(\frac{E_\nu}{\bar{E}}\right)^4 + \frac{1}{3}\right]. \tag{13}$$

Then the neutrino flux spectrum becomes

$$F(E_\nu) \equiv \frac{N(E_\nu)}{\pi R^2} \xrightarrow[R \to 0]{} \frac{M^2\, dN/dP(P_{min})}{16\pi E_\nu\, P_{min}\, L(P_{min})}\left[\left(\frac{E_\nu}{\bar{E}_\nu}\right)^4 + 1/3\right]\int_0^F \frac{dx\, e^{-x/L(P_{min})}}{(F+S-x)^2}. \tag{14}$$

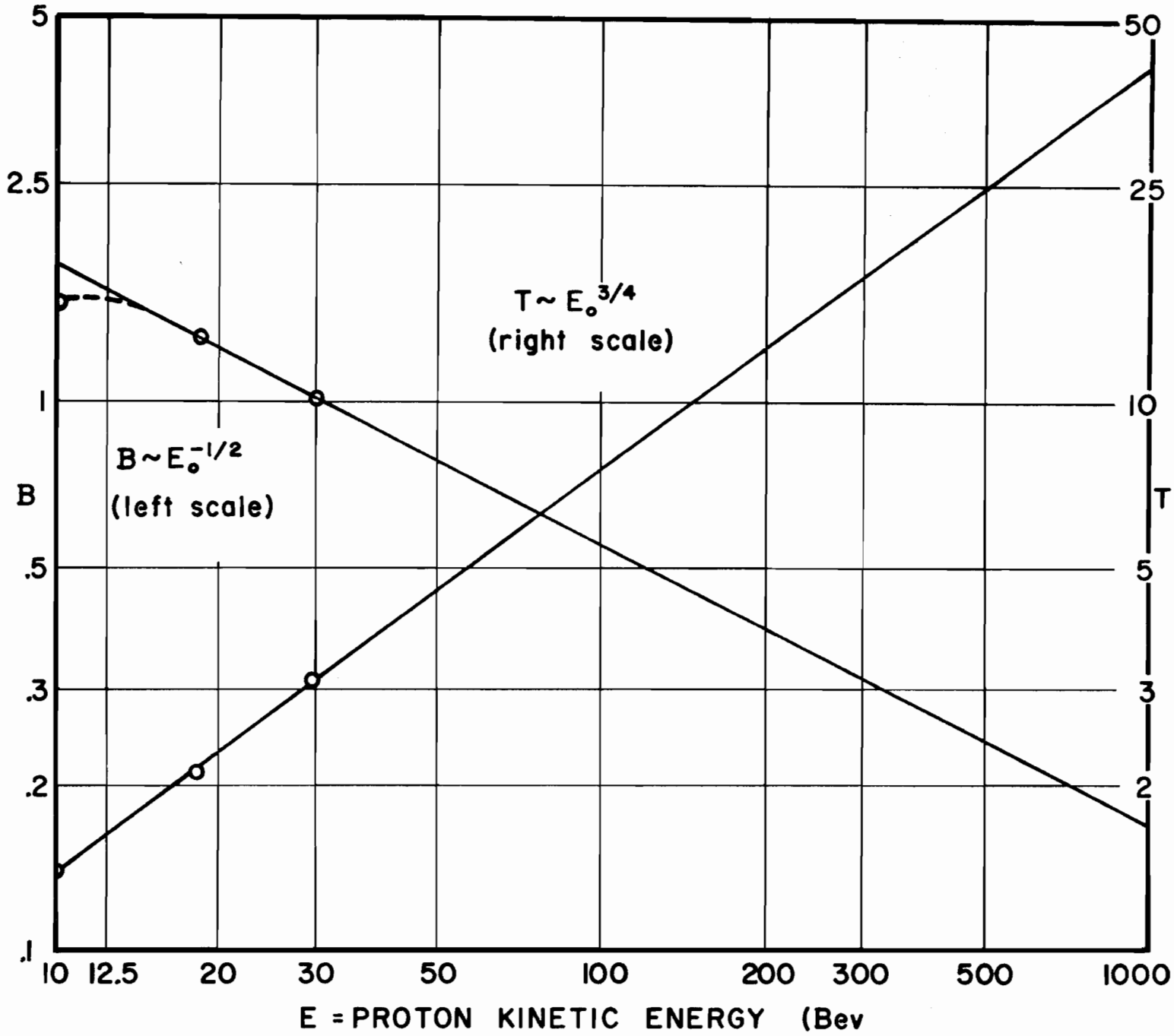
V. RESULTS

A Fortran program (F109) was written for evaluation of the neutrino flux spectrum in the general case where E_p, R, F and S could be assigned any values. The choice of S values has already been discussed. Solutions for a variety of values of F and R were obtained for E_p = 10, 100, and 1000 Bev. At each of these three accelerator energies the results were insensitive to the flight path F in the range S to 4S, where maximum fluxes were obtained. The curves in Fig. 4 are for E_p = 10, 100, and 1000 Bev with F = S. The dashed curves are for the case of no focusing. They were computed with a Fortran program (F73) written by K. R. Symon. The solid curves, two for each accelerator energy, have "ideal" focusing with R = 0.1 meter and R = 1 meter. The cases of R = 0.1 are very similar to $R \rightarrow 0$ (not shown). At the lower neutrino energies, where the neutrino decay angles are large, there is some loss of flux as $R \rightarrow 0$, but in general a flux loss occurs for large R. The no-focusing curves are dependent on R only if the angle subtended by the detector is much greater than the meson production angles. This is not the case here.

The main conclusions to be drawn from Fig. 4 are:

(1) The flux can be enhanced by an-order-of magnitude over most of the neutrino spectrum with "ideal" focusing as compared to no focusing.

(2) The relative enhancement is approximately independent of accelerator energy in the range 10 to 1000 Bev.



ENERGY DEPENDENCE OF THE PARTICLE PRODUCTION PARAMETERS

Points obtained by fitting eq. (4.2.2) to experiment. Straight lines have slopes in accord with Cocconi, Koester, and Perkins theory.

Figure 1

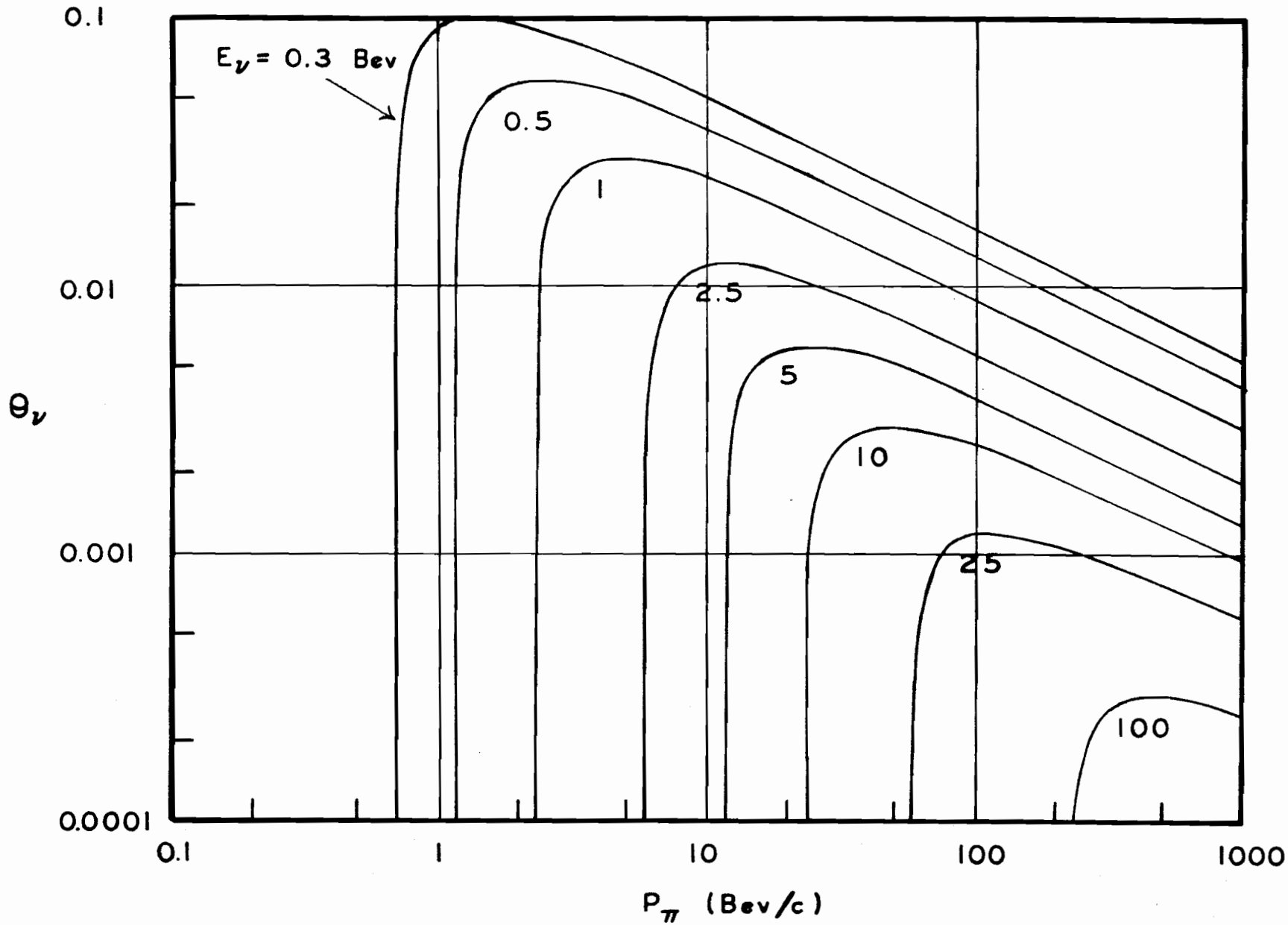


FIG. 2 NEUTRINO ANGLE VS. π-MESON MOMENTUM

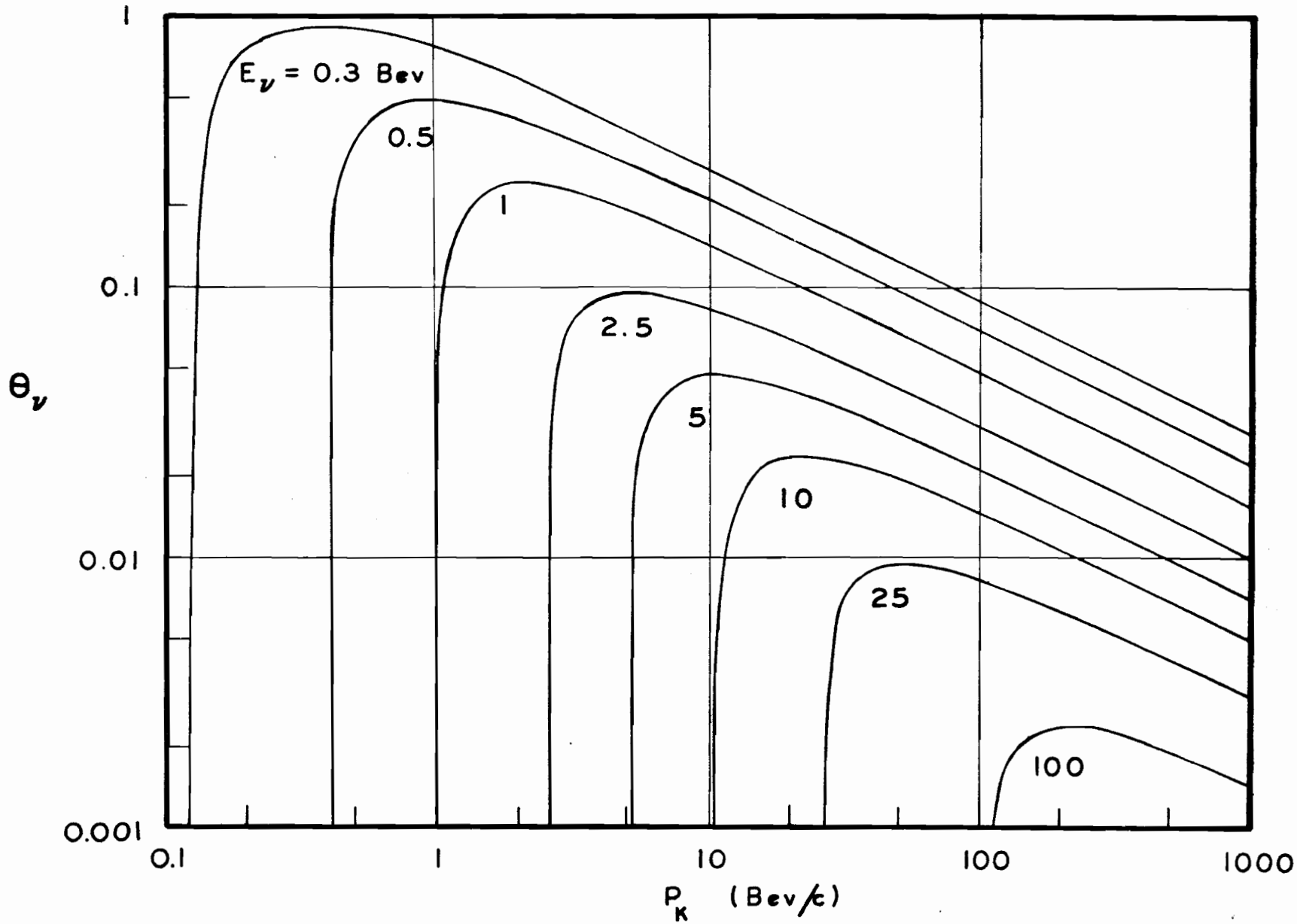


FIG. 3 NEUTRINO ANGLE VS. K-MESON MOMENTUM

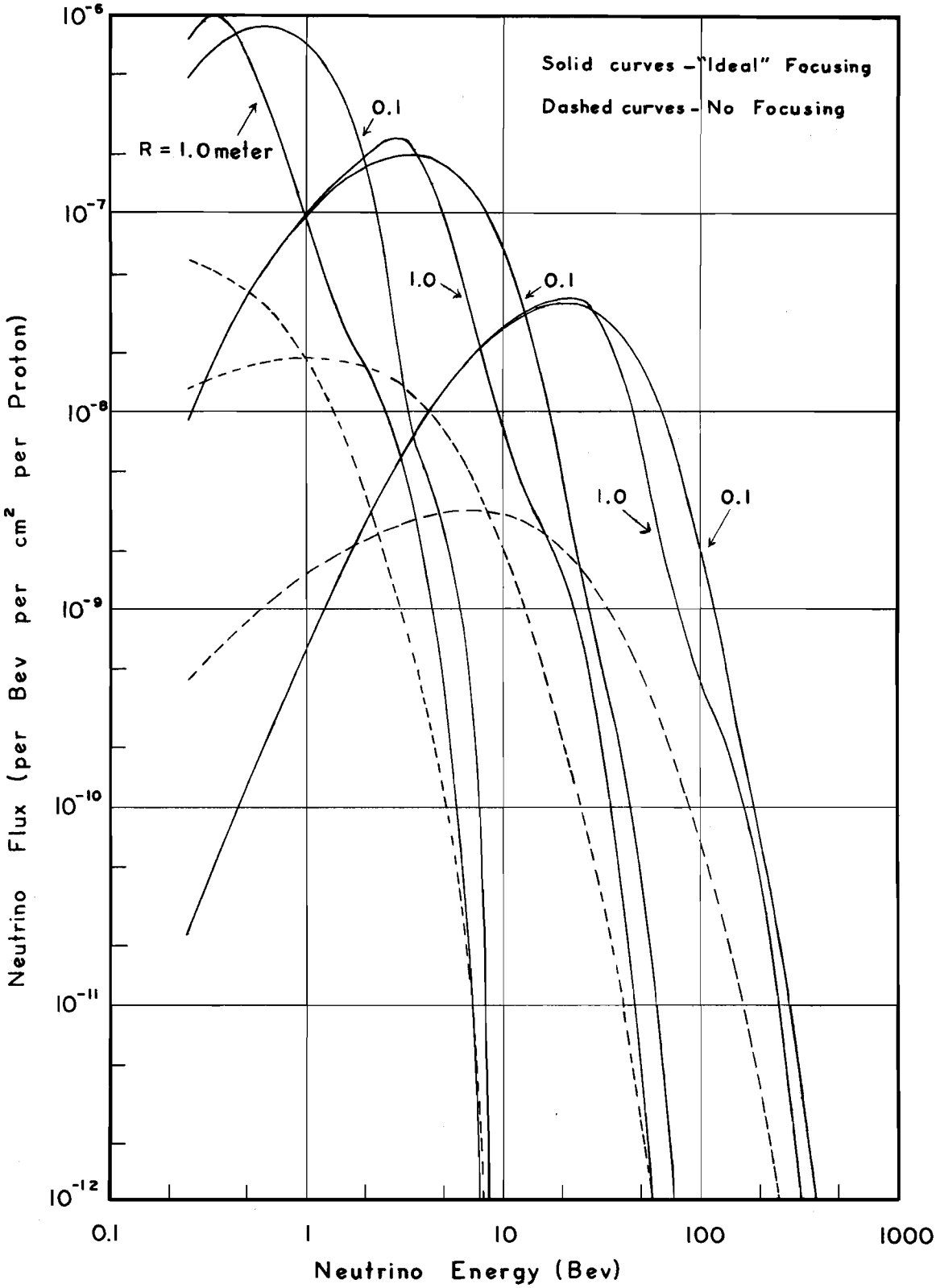


FIG. 4 NEUTRINO FLUX SPECTRA