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IDENTIFICATION

EXPLICIT SOLUTION OF THE GENERAL CUBIC EQUATION - MU CUBIC
704 FORTRAN SUBROUTINE
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PURPOSE

To rigourously solve the general cubic equation with real coefficients:

$$C_1 x^3 + C_2 x^2 + C_3 x + C_4 = 0.$$

METHOD

MU CUBIC computes the intermediate quantities

$$p = C_2/C_1 \qquad q = C_3/C_1 \qquad r = C_4/C_1$$

$$a = q - p^2/3 \qquad b = (2p^3 - 9pq + 27r)/27,$$

after which investigation is made of the discriminant:

$$\Delta = (b/2)^2 + (a/3)^3.$$

The (signed) magnitude of Δ is indicative of the nature of the roots of the original cubic equation. Accordingly, the value of the third argument, MTYPE, in the calling sequence or statement is set:

Δ	Nature of roots	MTYPE
positive	1 real, 2 complex (conjugate)	1
zero	3 real, 2 identical	2
negative	3 real, all distinct	3

For Δ positive or zero, the following additional quantities are required:

$$A = \left[-b/2 + \sqrt{\Delta}\right]^{1/3} \qquad B = \left[-b/2 - \sqrt{\Delta}\right]^{1/3},$$

whereupon the roots are given by:

$$x_1 = (A+B)/2 - p/3$$

$$x_2 = -(A+B)/2 + (A-B)\cdot\sqrt{-3}/2 - p/3$$

$$x_3 = -(A+B)/2 - (A-B)\cdot\sqrt{-3}/2 - p/3.$$

For Δ negative, a trigonometric solution is obtained:

$$x_i = 2\sqrt{-a/3}\ \cos\left[\phi/3+(i-1)\pi/3\right] - p/3,\ i=1,2,3$$

where $\cos\phi = (-b/2)\Big/(-a/3)^{3/2}$.

USAGE

MU CUBIC is entered via the FORTRAN statement

CALL CUBIC (CR, XR, MTYPE)

where CR and XR are one-dimensional arrays of 4 and 3 elements respectively, and where MTYPE is fixed point variable.

CR contains the original coefficients $C_1 \ldots C_4$,
XR contains the derived roots x_1, x_2, x_3 in the orders indicated above, except that for Δ positive, x_2 and x_3 contain, respectively, the real and imaginary parts of the complex conjugate pair of roots, MTYE is a signal indicating the nature of the roots, and is equal to the number of distinct real roots.

CREDITS

MU CUBIC is a plagiarism of VIETA (Share Distribution No. 1028) except for the definition of MTYPE.

AUXILIARY ROUTINES

MU CUBIC requires the following subroutines:

1. Floating Point COS (Floating point arguement).
2. Floating Point ASIN (Floating point arguement).
3. Floating Point ACOS (Floating point arguement).
4. Floating Point SQRT (Floating point arguement).
5. Floating Point CRT (Floating point arguement).

CODING INFORMATION

MU CUBIC requires 238 locations of core storage.

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C     EXPLICIT SOLUTION OF THE GENERAL CUBIC EQUATION - EDWARDS
      SUBROUTINE CUBIC (CR,XR,MTYPE)
      DIMENSION CR(4),XR(3)
      P=CR(2)/CR(1)
      Q=CR(3)/CR(1)
      R=CR(4)/CR(1)
      POV3=P/3.0
5     A=Q-P*POV3
      B=2.0*POV3**3-POV3*Q+R
      AOV3=A/3.0
      BOV2=B/2.0
      CUAOV3=AOV3**3
      SQBOV2=BOV2**2
      DEL=SQBOV2+CUAOV3
      IF (DEL) 30,20,10
10    MTYPE=1
      RTDEL=SQRTF(DEL)
      AA=CRTF(-BOV2+RTDEL)
      BB=CRTF(-BOV2-RTDEL)
      XR(1)=AA+BB-POV3
      XR(2)=-(AA+BB)/2.0-POV3
      XR(3)=(AA-BB)*0.86602540
      GO TO 60
20    MTYPE=2
      AB=CRTF(-BOV2)
      XR(1)=AB+AB-POV3
      XR(2)=-AB-POV3
      XR(3)=XR(2)
      GO TO 60
30    MTYPE=3
      ROOT=SQRTF(-SQBOV2/CUAOV3)
      IF (B) 35,40,40
35    PHI=ACOSF(ROOT)
      GO TO 45
40    PHI=1.5707963+ASINF(ROOT)
45    FACT=2.0*SQRTF(-AOV3)
      DO 50 I=1,3
      FM=I-1
50    XR(I)=FACT*COSF(PHI/3.0+FM*2.0943951)-POV3
60    RETURN
      END (0,1,1,1,0)
```