604

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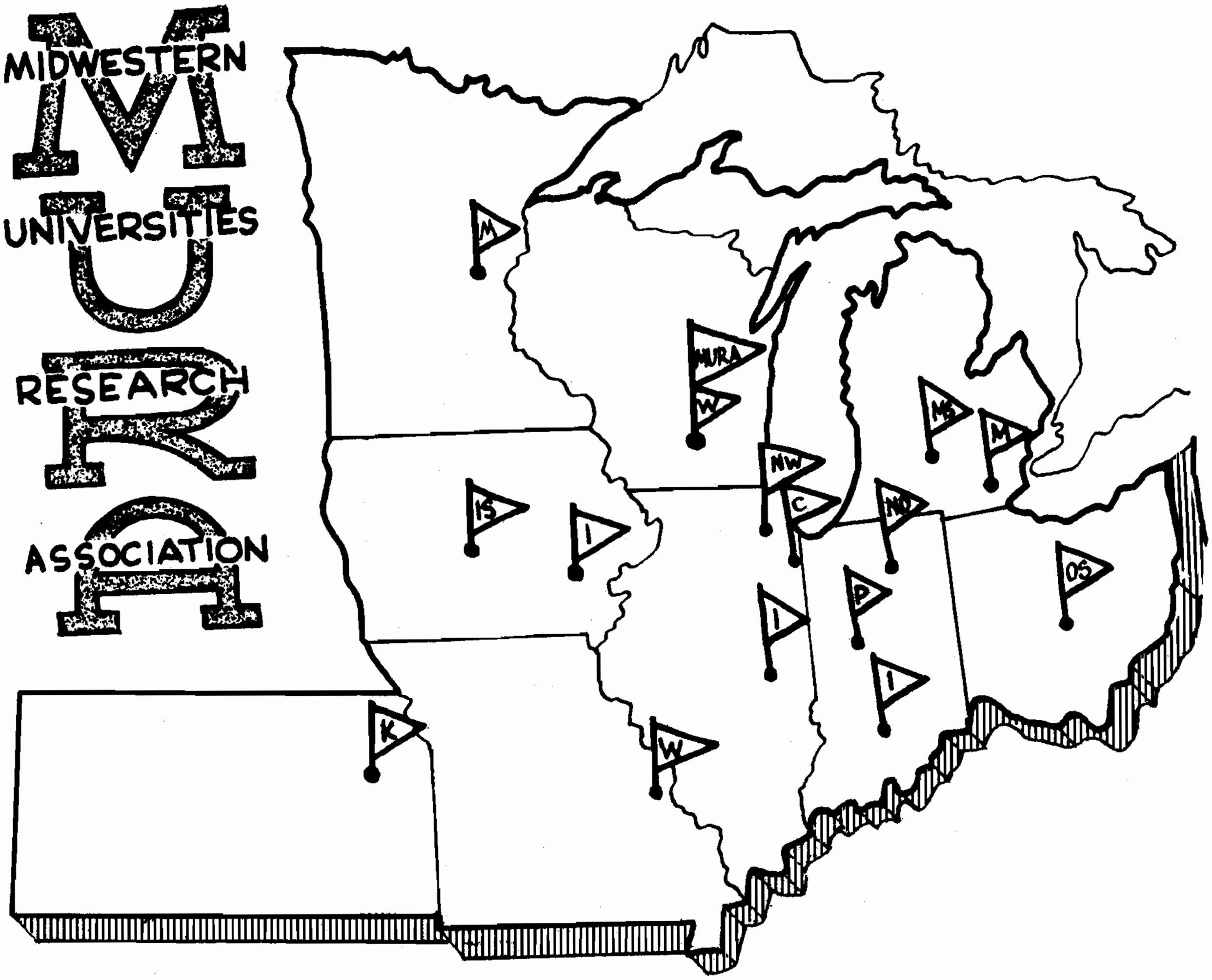

REPORT FLEXIBLE FIVER
(PROGRAM 280)
November 1960

NUMBER 604
COMPUTER
PROGRAM
INTERNAL

DISCLAIMER
Although this program has been carefully tested by its contributor, no guarantee is made of its correct functioning under all conditions, and no responsibility is taken by him in case of possible failure.

FLEXIBLE FIVER
(Program 280)

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FLEXIBLE FIVER
(Program 280)

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INTRODUCTION

FLEXIBLE FIVER is a program for the IBM 704[1] to solve the equations of motion describing the betatron oscillations of particles in a fixed field, alternating gradient accelerator. These equations are shown below.

$$\frac{dx}{d\theta} = \frac{(1+x)p_x}{\sqrt{P^2 - p_x^2 - p_y^2}}$$

$$\frac{dy}{d\theta} = \frac{(1+x)p_y}{\sqrt{P^2 - p_x^2 - p_y^2}}$$

$$\frac{dp_x}{d\theta} = \sqrt{P^2 - p_x^2 - p_y^2} + \lambda(1+x)\left[\frac{B_y}{B_o} - \frac{p_y}{\sqrt{P^2 - p_x^2 - p_y^2}}\,\frac{B_\theta}{B_o}\right] \qquad (1)$$

$$\frac{dp_y}{d\theta} = \lambda(1+x)\left[\frac{p_x}{\sqrt{P^2 - p_x^2 - p_y^2}}\,\frac{B_\theta}{B_o} - \frac{B_x}{B_o}\right]$$

If, for example, x and y are defined in the usual way as $x = \frac{r - r_o}{r_o}$, $y = \frac{z}{r_o}$, and P is put into units of m_oc, then λ becomes

$$\lambda = \frac{er_o B_o}{m_oc} \qquad (2)$$

In the following B_x, B_y, B_θ will be written for B_x/B_o, B_y/B_o, etc. (B_o is the value of the field at some arbitrary reference point.).

EQUATIONS

The equations used in this program and described below are given by

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1. 8192 words of core storage, 1 tape unit <u>if</u> it is desired to use the resumption procedure.

F. T. Cole in MURA Report No. 377, <u>Field Calculations for Digital Computation with Non-Scaling Dynamics.</u>

The field components, B_x, B_y and B_θ are computed according to the following equations.

$$B_x = (1+x)^k \sum_{i=1}^{\infty} \sum_{m=0}^{\infty} (\alpha_{i,m} c_m + \beta_{i,m} s_m) \frac{1}{(2i-1)!} \left(\frac{y}{1+x}\right)^{2i-1}$$

$$B_y = -(1+x)^k \sum_{i=0}^{\infty} \sum_{m=0}^{\infty} (\mu_{i,m} c_m + \nu_{i,m} s_m) \frac{1}{(2i)!} \left(\frac{y}{1+x}\right)^{2i} \qquad (3)$$

$$B_\theta = (1+x)^k \sum_{i=1}^{\infty} \sum_{m=0}^{\infty} (\gamma_{i,m} c_m + \delta_{i,m} s_m) \frac{1}{(2i-1)!} \left(\frac{y}{1+x}\right)^{2i-1}$$

where

$$c_m = \cos m \Phi$$

$$s_m = \sin m \Phi$$

$$\Phi = Kq - N\theta, \qquad K = \frac{1}{w} \qquad (4)$$

$$q = \ln(1+x)$$

These expressions for B_x, B_y and B_θ are truncated by specifying i_{max}. All terms in the series with $i > i_{max}$ are neglected.

The parameters appearing in (3) are obtained from the relations:

$$\mu_{i,m} = \sum_{j=0}^{j_{max}} \left(\mu_{i,m,j}\right) x^j \cdots$$

$$\nu_{i,m} = \sum_{j=0}^{j_{max}} \left(\nu_{i,m,j}\right) x^j$$

$$\alpha_{i,m} = \sum_{j=0}^{j_{max}} \left(\alpha_{i,m,j}\right) x^j$$

$$\beta_{i,m} = \sum_{j=0}^{j_{max}} \left(\beta_{i,m,j}\right) x^j$$

$$\gamma_{i,m} = mN \nu_{i-1,m} \tag{5}$$

$$\delta_{i,m} = -mN \mu_{i-1,m}$$

$$k = \sum_{j=0}^{j_{max}} k_j x^j$$

$$K = \sum_{j=0}^{j_{max}} K_j x^j$$

The user of this program characterizes the magnetic field for the problem to be solved by specifying its value on the median plane (y = 0). In particular he gives values for the k_j, K_j, $\mu_{o,m,j}$ and the $\nu_{o,m,j}$ ($m = m_1, m_1, \ldots, m_{32}$, where the m's may have any rational value; $j = 0, 1, \ldots, j_{max} \leq 64$). The FLEXIBLE FIVER program then computes the $\alpha_{i,m,j}$, $\beta_{i,m,j}$, $\mu_{i,m,j}$ and $\nu_{i,m,j}$ from the following recursion relations.

$$\alpha_{i+1,m,j} = -\sum_{\ell=0}^{j} \left(\varphi_{1\ell}\, \mu_{i,m,j-\ell} + m\varphi_{2\ell}\, \nu_{i,m,j-\ell}\right) - j\mu_{i,m,j} - (j+1)\mu_{i,m,j+1}$$

$$\beta_{i+1,m,j} = -\sum_{\ell=0}^{j}\left(\varphi_{1\ell}\, \nu_{i,m,j-\ell} - m\varphi_{2\ell}\, \mu_{i,m,j-\ell}\right) - j\nu_{i,m,j} - (j+1)\,\nu_{i,m,j+1}$$

$$\alpha_{0,m,j} = \beta_{0,m,j} = 0$$

$$\mu_{i+1,m,j} = m^2N^2\mu_{i,m,j} + \sum_{\ell=0}^{j}\left(\varphi_{1\ell}\,\alpha_{i+1,m,j-\ell} + m\varphi_{2\ell}\,\beta_{i+1,m,j-\ell}\right) + j\,\alpha_{i+1,m,j} + (j+1)\,\alpha_{i+1,m,j+1}$$

$$\nu_{i+1,m,j} = m^2N^2\nu_{i,m,j} + \sum_{\ell=0}^{j}\left(\varphi_{1\ell}\,\beta_{i+1,m,j-\ell} - m\,\varphi_{2\ell}\,\alpha_{i+1,m,j-\ell}\right) + j\beta_{i+1,m,j} + (j+1)\beta_{i+1,m,j+1}$$

where

$$\varphi_{10} = k_0 - 2i$$

$$\varphi_{1j} = k_j + \sum_{n=1}^{j}(-1)^{n-1}\binom{j-n+1}{n}k_{j-n+1} + \sum_{n=1}^{j-1}(-1)^{n-1}\binom{j-n}{n}k_{j-n}$$

$$\text{for } j = 1, 2, \cdots, j_{max}$$

and

$$\varphi_{20} = K_0$$

$$\varphi_{2j} = K_j + \sum_{n=1}^{j}(-1)^{n-1}\binom{j-n+1}{n}K_{j-n+1} + \sum_{n=1}^{j-1}(-1)^{n-1}\binom{j-n}{n}K_{j-n}$$

$$\text{for } j = 1, 2, \cdots, j_{max}$$

At each integration step the program calculates k, K, $\mu_{i,m}$, $\nu_{i,m}$, $\alpha_{i,m}$, $\beta_{i,m}$ from (5) using the current values of x and y. If $j_{max} = 0$, these quantities are computed only once, at the start of the problem or series of problems. These quantities are used in (3) to compute the fields which occur in the equations of motion.

INPUT

A standard agendum form is included on pages 8 and 9 . Two types of numbers appear on the agendum: fixed point integers and floating point numbers. Integers are written in the usual fashion; leading zeroes may be omitted, and $2^{35} - 1$ is the largest integer allowed. The value to be entered as a floating point number must be expressed in the form, value $= n \times 10^{exp}$, where $1 \leqslant |n| < 10$, or zero and n may contain from 1 to 9 (inclusive) decimal digits and $|exp| \leqslant 37$. All quantities are assumed positive unless designated negative by - . All quantities not specified are set equal to zero except for $p_{x_{max}} = p_{y_{max}} = 1$, $x_{max} = 1/2$, $y_{max} = 1/2$, $P = 1, \lambda = 1$, $N_p = 1$, and $i_{max} = 1$.

The user must compute the addresses for the Fourier coefficients of the median plane field and enter them on the agendum. This is done in the following way.

PARAMETER	ADDRESS
$\mu_{0, m_1, 0}$	8191
$\mu_{0, m_1, 1}$	8190
$\vdots$	$\vdots$
$\mu_{0, m_1, j_{max}}$	$8191 - j_{max}$
$\mu_{0, m_2, 0}$	$8191 - (j_{max} + 1)$
$\mu_{0, m_2, 1}$	$8191 - (j_{max} + 1) - 1$
$\vdots$	$\vdots$
$\mu_{0, m_2, j_{max}}$	$8191 - (j_{max} + 1) - j_{max}$
$\mu_{0, m_3, 0}$	$8191 - 2(j_{max} + 1)$
$\nu_{0, m_1, 0}$	8191 - (No. of harmonics) $(j_{max} + 1)$
$\nu_{0, m_1, 1}$	8191 - (No. of harmonics) $(j_{max} + 1) - 1$
$\vdots$	$\vdots$
$\nu_{0, m_2, 0}$	8191 - (No. of harmonics + 1) $(j_{max} + 1)$

The limits on i_{max}, j_{max}, and the number of harmonics must be such that

$$(i_{max} + 1)(j_{max} + 1)\,(\text{No. of harmonics}) \leqslant 1120,$$

$$i_{max} \leqslant 6,$$

$$j_{max} \leqslant 64,$$

and

$$\text{No. of harmonics} \leqslant 32.$$

SERIES

As in other MURA programs a series of problems may be submitted by stapling a "General Series Agendum" sheet to the standard agendum form. In a series the parameter values are not reset to zero as each problem is completed and computations on the next run are started. The parameters retain the values they had on the previous run if no subsequent entry is made on the agendum. There are four exceptions, namely the initial values of four counters, $(C_p)_o$, $(C_e)_o$, $(C_{1,p})_o$, $(C_{2,p})_o$, which are always reset to zero; if other values are desired they must be specified for each run of the series.

The sense switch settings for every run of a series must be the same. The quantities which it is permissible to change in a series are: Run ID, N_p, N_e, p_{x_o}, p_{y_o}, x_o, y_o, $\left(\frac{N\theta}{2\pi}\right)_o$, $p_{x_{max}} = p_{y_{max}}$, x_{max}, y_{max}, P, λ [2].

The "search" feature is included in this program. Under the search feature successive problems of a series will be read in by the computer and computations will be carried out on each one until p_x, p_y, x, or y exceeds its upper bound. Then without interruption the program will proceed to the next problem of the series. This will continue until a problem is completed, that is $C_e = N_e$, at which time the computer will stop and succeeding problems of the series can be ignored. This feature is useful when exploring the phase plane for regions of stability. The search feature is put into operation by putting sense switch 6 up.

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2. It is impossible to submit series agenda in which the $\mu_{o,m,j}$, $\nu_{o,m,j}$ are changed.

FLEXIBLE FIVER FIELD AGENDUM
Program 280

FIELD ID

Parameter	Address	Value n	exp	Remarks
		n	exp	
N	3204			
	3206			The number of non-zero field harmonics $\leq$ 32.
j_{max}	3207			Last term in power series for $\mu_{i,m}$, $\nu_{i,m}$, $\alpha_{i,m}$, $\beta_{i,m}$ etc. $\leq$ 64.

List of Harmonic Numbers

Parameter	Address	Value		Parameter	Address	Value	
		n	exp			n	exp
m_1	3303			m_9	3295		
m_2	3302			m_{10}	3294		
m_3	3301			m_{11}	3293		
m_4	3300			m_{12}	3292		
m_5	3299			m_{13}	3291		
m_6	3298			m_{14}	3290		
m_7	3297			m_{15}	3289		
m_8	3296			m_{16}	3288		

$k = \sum_j k_j x^j$				$K = \sum_j K_j x^j$ $\left(K = \frac{1}{w}\right)$			
k_0	3368			K_0	3433		
k_1	3367			K_1	3432		
k_2	3366			K_2	3431		
k_3	3365			K_3	3430		
k_4	3364			K_4	3429		
k_5	3363			K_5	3428		
k_6	3362			K_6	3427		
k_7	3361			K_7	3426		
k_8	3360			K_8	3425		
k_9	3359			K_9	3424		
k_{10}	3358			K_{10}	3423		
k_{11}	3357			K_{11}	3422		
k_{12}	3356			K_{12}	3421		

Fourier Coefficients of the Median Plane Field

	8191						

FLOATING POINT NUMBERS

NOTE: If it is desired to enter values higher than those printed here, firmly staple another page.

INVARIANTS

The program will compute zero, one or two invariants as requested by the user on the invariants agendum (page 11). If one invariant is desired the program will compute

$$K_{1,x} = \sqrt{\xi_{1,x}(x - x_{1,f})^2 + \eta_{1,x}(x - x_{1,f})(x' - x'_{1,f}) + \zeta_{1,x}(x' - x'_{1,f})^2},$$

and

$$K_{1,y} = \sqrt{\xi_{1,y}(y - y_{1,f})^2 + \eta_{1,y}(y - y_{1,f})(y' - y'_{1,f}) + \zeta_{1,y}(y' - y'_{1,f})^2} \tag{10}$$

If two invariants are requested the program will compute, in addition to the above, the quantities

$$K_{2,x} = \sqrt{\xi_{2,x}(x - x_{2,f})^2 + \eta_{2,x}(x - x_{2,f})(x' - x'_{2,f}) + \zeta_{2,x}(x' - x'_{2,f})^2}$$

$$K_{2,y} = \sqrt{\xi_{2,y}(y - y_{2,f})^2 + \eta_{2,y}(y - y_{2,f})(y' - y'_{2,f}) + \zeta_{2,y}(y' - y'_{2,f})^2} \tag{11}$$

The quantities $\xi_{1,x}$, $\eta_{1,x}$, $\zeta_{1,x}$, $x_{1,f}$, $x'_{1,f}$, $\xi_{1,y}$, $\eta_{1,y}$, $\zeta_{1,y}$, $y_{1,f}$, $y'_{1,f}$, etc. are specified by the user on the invariants agendum. The values of the independent variable at which computation of the invariants is desired is specified by giving the number, $N_{1,p}$, of integration steps between successive computations of the invariants, and a number, $(C_{1,p})_o$, which gives the initial reading for the counter counting the number of integration steps between successive computations of the invariants. A corresponding pair of numbers, $N_{2,p}$ and $(C_{2,p})_o$ must also be specified when a second invariant is called for by the user.

For more details on these invariants, see MURA Report No. 206, Remarks on the Invariant Quadratic Forms Pertaining to Motion Characterized by a Linear Differential Equation with Periodic Coefficient by L. Jackson Laslett.

FLEXIBLE FIVER

Invariants Agendum

Program 280A

Parameter	Address	Value n	Value exp	Remarks
$\xi_{1,x}$	3232			
$\eta_{1,x}$	3233			PARAMETERS FOR K_{1x}
$\zeta_{1,x}$	3234			$K_{1,x} = \sqrt{\xi_{1,x}(x-x_{1,f})^2 + \eta_{1,x}(x-x_{1,f})(x'-x'_{1,f}) + \zeta_{1,x}(x'-x'_{1,f})^2}$
$x_{1,f}$	3235			
$x'_{1,f}$	3236			
$\xi_{1,y}$	3237			PARAMETERS FOR K_{1y}
$\eta_{1,y}$	3238			$K_{1,y} = \sqrt{\xi_{1,y}(y-y_{1,f})^2 + \eta_{1,y}(y-y_{1,f})(y'-y'_{1,f}) + \zeta_{1,y}(y'-y'_{1,f})^2}$
$\zeta_{1,y}$	3239			
$y_{1,f}$	3240			
$y'_{1,f}$	3241			
$\xi_{2,x}$	3242			PARAMETERS FOR K_{2x}
$\eta_{2,x}$	3243			$K_{2,x} = \sqrt{\xi_{2,x}(x-x_{2,f})^2 + \eta_{2,x}(x-x_{2,f})(x'-x'_{2,f}) + \zeta_{2,x}(x'-x'_{2,f})^2}$
$\zeta_{2,x}$	3244			
$x_{2,f}$	3245			
$x'_{2,f}$	3246			
$\xi_{2,y}$	3247			PARAMETERS FOR K_{2y}
$\eta_{2,y}$	3248			$K_{2y} = \sqrt{\xi_{2,y}(y-y_{2,f})^2 + \eta_{2,y}(y-y_{2,f})(y'-y'_{2,f}) + \zeta_{2,y}(y'-y'_{2,f})^2}$
$\zeta_{2,y}$	3249			
$y_{2,f}$	3250			
$y'_{2,f}$	3251			

FLOATING POINT NUMBERS

OUTPUT

At the start of every run, the input parameters are printed. The initial print format is shown on page 14. The FLEXIBLE FIVER program operates in the floating point mode; therefore all numbers other than fixed point integers will be printed in floating point form. A floating point number is printed as an 8 digit decimal fraction and a signed two digit exponent. The decimal point is understood to be between the first and second decimal digits, counting from left to right. If the decimal fraction is positive the sign is omitted and if the fraction is negative the sign is printed. If the floating point number is identically zero, then an eleven digit zero is printed. For example, .03124 is printed as 31240000- 2 and 0 as 00000000000.

The printed output will have two distinct forms depending on the presence or absence of invariants. When no invariants are called for ($N_1 = 0$), then every N_p integration steps, one line is printed. The quantities p_x, p_y, x, y, and $\frac{N\theta}{2\pi}$ appear in columns one through five, respectively, of each line, with an index at the left of each line which counts the lines, starting with the first line numbered zero. A sample output of this type appears on page 15.

If one invariant is called for, the lines of print containing p_x, p_y, x, y, $N\theta/2\pi$ will be mixed with lines of print of $K_{1,x}$ and $K_{1,y}$ which will appear in columns one and two, respectively. The order of printing on successive lines is in the order of increasing $N\theta/2\pi$. When p_x, p_y, x, y, $N\theta/2\pi$ and K_x, K_y are to be printed for the same $N\theta/2\pi$, then p_x, p_y, etc. is printed first. Lines of printing of the first invariant are separately numbered 10000, 10001, 10002, etc.; thus the nth line of print of the first invariant is numbered 10000 + n.

Similarly, if two invariants are requested, additional lines of print containing $K_{2,x}$ and $K_{2,y}$ in columns one and two, respectively, are interspersed among the lines of print of p_x, p_y, etc. and the lines containing $K_{1,x}$ and $K_{1,y}$. As always, the order of printing is in the order of increasing $N\theta/2\pi$. If p_x, p_y, etc. and $K_{1,x}$, $K_{1,y}$ and $K_{2,x}$, $K_{2,y}$ are all to be printed at the same value of $N\theta/2\pi$, then p_x, p_y, x, y, $\frac{N\theta}{2\pi}$ are printed first, $K_{1,x}$, $K_{1,y}$ are printed second and $K_{2,x}$, $K_{2,y}$ are printed third. Lines of printing of the second invariant are separately numbered 20000, 20001, 20002, etc.; thus the nth line of print of the second invariant is numbered 20000 + n.

Samples of outputs with one invariant and with two invariants appear on pages 16 and 17.

When a problem comes to the end ($C_e = N_e$), the word END is printed below the last line of the output at the right side of the page.

When p_x, p_y, x or y exceed their bounds, as set by $p_{x_{max}} = p_{y_{max}}$, x_{max} and y_{max}, computations on a run (problem) are stopped and an identifying symbol is printed in the line numbering column before proceeding to the next run. These symbols are:

00001	p_x overflow,
00002	p_y overflow,
00003	x overflow,
00004	y overflow,

If at some time during the computations $\sqrt{P^2 - p^2_x - p^2_y}$ should become negative or zero, the program prints the line

SQUARE ROOT P2 - PX2 - PY2 NEGATIVE

INITIAL PRINT FORMAT

	Line Label	Col. #1	Col. #2	Col. #3	Col. #4	Col. #5
I*	00000	00000000280	Human ID	Field ID	Run ID	
	00001	N	n_{RK}	No. of harmonics	j_{max}	
I	00002	N_p	N_e	i_{max}	$(C_p)_o$	$(C_e)_o$
	00003	P	λ	$p_{x_{max}} = p_{y_{max}}$	x_{max}	y_{max}
	00004**	N_I	N_{I_p}	N'_{I_p}	$(C_{I_p})_o$	$(C'_{I_p})_o$
	00005	$\xi_{1,x}$	$\eta_{1,x}$	$\zeta_{1,x}$	$x_{1,f}$	$x'_{1,f}$
	00006	$\xi_{1,y}$	$\eta_{1,y}$	$\zeta_{1,y}$	$y_{1,f}$	$y'_{1,f}$
	00007	$\xi_{2,x}$	$\eta_{2,x}$	$\zeta_{2,x}$	$x_{2,f}$	$x'_{2,f}$
	00008	$\xi_{2,y}$	$\eta_{2,y}$	$\zeta_{2,y}$	$y_{2,f}$	$y'_{2,f}$
	Space					
	00009	$k_{j_{max}}$	$k_{j_{max-1}}$	$k_{j_{max-2}}$	$k_{j_{max-3}}$	$k_{j_{max-4}}$
	⋮	⋮				
	00009	k_4	k_3	k_2	k_1	k_o
	Space					
	00010	$K_{j_{max}}$	$K_{j_{max-1}}$	$K_{j_{max-2}}$	$K_{j_{max-3}}$	$K_{j_{max-4}}$
	⋮	⋮				
	00010	K_4	K_3	K_2	K_1	K_o
	Space					
	0000m#	$\mu_{o,m,j_{max}}$	$\mu_{o,m,j_{max-1}}$	$\mu_{o,m,j_{max-2}}$	$\mu_{o,m,j_{max-3}}$	$\mu_{o,m,j_{max-4}}$
	⋮	⋮				
	0000m	$\mu_{o,m,4}$	$\mu_{o,m,3}$	$\mu_{o,m,2}$	$\mu_{o,m,1}$	$\mu_{o,m,o}$
	Space					
	1000m#	$\nu_{o,m,j_{max}}$	$\nu_{o,m,j_{max-1}}$	$\nu_{o,m,j_{max-2}}$	$\nu_{o,m,j_{max-3}}$	$\nu_{o,m,j_{max-4}}$
	⋮	⋮				
	1000m	$\nu_{o,m,4}$	$\nu_{o,m,3}$	$\nu_{o,m,2}$	$\nu_{o,m,1}$	$\nu_{o,m,o}$

* I indicates a line of fixed point integers. All other lines contain floating point numbers.

** Lines 4-8 will be omitted and replaced by a single line space if $N_I = 0$ (no invariants).

$m = m_1, m_2, \ldots, m_{32}$.

00000	-19547170- 3	00000000000	63500400- 4	10000000- 4	00000000000
00001	-32162218- 3	-14149084- 3	45561659- 4	16250009- 4	99999999- 1
00002	-36012379- 3	-11488539- 3	32928769- 4	30602700- 5	20000000+ 0
00003	-28091627- 3	50430009- 4	35197711- 4	-13655488- 4	30000000+ 0
00004	-13882325- 3	15630461- 3	50687012- 4	-13798211- 4	40000000+ 0
00005	-39959332- 4	66353326- 4	67827437- 4	32787387- 5	49999999+ 0
00006	-52066310- 4	-10607697- 3	74821972- 4	16492737- 4	59999999+ 0
00007	-16877157- 3	-14568413- 3	66355008- 4	92793218- 5	69999999+ 0
00008	-30531752- 3	-10305895- 4	48531443- 4	-91045064- 5	79999999+ 0
00009	-36140167- 3	13703997- 3	34346571- 4	-16343653- 4	89999998+ 0
00010	-30151584- 3	12138757- 3	33633987- 4	-40108186- 5	99999998+ 0

Sample Computer Output with $N_I = 0$

00000	-19547170- 3	00000000000	63500400- 4	10000000- 4	00000000000
10000	21691703- 4	16723038- 4			
00001	-32162218- 3	-14149084- 3	45561659- 4	16250009- 4	99999999- 1
10001	21517257- 4	16628601- 4			
00002	-36012379- 3	-11488539- 3	32928769- 4	30602700- 5	20000000+ 0
10002	21488175- 4	16729498- 4			
00003	-28091627- 3	50430009- 4	35197711- 4	-13655488- 4	30000000+ 0
10003	21481664- 4	16512228- 4			
00004	-13882325- 3	15630461- 3	50687012- 4	-13798211- 4	40000000+ 0
10004	21459094- 4	16731527- 4			
00005	-39959332- 4	66353326- 4	67827437- 4	32787387- 5	49999999+ 0

Sample Computer Output with $N_I = 1$

00000	-19547300- 3	00000000000	63500400- 4	10000000- 4	00000000000
10000	21691703- 4	16723038- 4			
20000	19354163- 4	18231989- 4			
00001	-32162536- 3	-14149083- 3	45561318- 4	16250006- 4	99999999- 1
10001	21517257- 4	16628598- 4			
20001	19839453- 4	18428789- 4			
00002	-36012997- 3	-11488536- 3	32928033- 4	30602708- 5	20000000+ 0
10002	21488860- 4	16729493- 4			
20002	19619003- 4	17826341- 4			
00003	-28091826- 3	50429951- 4	35197140- 4	-13655467- 4	30000000+ 0
10003	21482627- 4	16512200- 4			
20003	19808596- 4	18328304- 4			
00004	-13882318- 3	15630440- 3	50686787- 4	-13798183- 4	40000000+ 0
10004	21459934- 4	16731504- 4			
20004	19350005- 4	18248206- 4			
00005	-39958937- 4	66353131- 4	67827376- 4	32787442- 5	49999999+ 0
10005	21383501- 4	16590242- 4			
20005	19503081- 4	18462508- 4			
00006	-52061625- 4	-10607690- 3	74822326- 4	16492715- 4	59999999+ 0
10006	21267331- 4	16787836- 4			

Sample Computer Output with $N_I = 2$.

PREPARING THE DATA DECK

Two kinds of input data cards, fixed point integers and floating point numbers are used with FLEXIBLE FIVER. With both of these types the following holds: commas and decimal points are not punched; each card has an address followed by the value to be assigned to a parameter.

INTEGERS

Starting in column 1 of the data card the address (from 1-4 decimal digits) is punched followed by a space followed by a sign followed by an integer of up to 11 decimal digits. The sign may be omitted if the integer is positive. (Note: omission means "leaving out and closing up") For example N_p = 32 would be punched as

```
 1 2 3 4 5 6 7 8 9        (Column Number)
 3 2 0 8   3 2
```

To signal the end of the integers a card is punched with a 1 in column 1 and in columns 37-48, END INTEGERS.

```
 1 2 3 4 ...      36 37 38 39 40 41 42 43 44 45 46 47 48
 1                    E  N  D     I  N  T  E  G  E  R  S
```

FLOATING POINT NUMBERS

Starting in column 1 the address (from 1-4 decimal digits) is punched followed by a space followed by a sign followed by a fraction of up to 9 decimal digits. Immediately after this, the sign of the exponent is punched, followed by an exponent of up to two decimal digits. The sign of the fraction may be omitted if the fraction is positive. A positive exponent sign, however, must be either a blank space or a plus sign.

Examples:

	1	2	3	4	5	6	7	8	9	10	11
$x_o = 5.147 \times 10^{-4}$	3	2	1	5		5	1	4	7	-	4
$n_{RK} = 3.2 \times 10^{1}$	3	2	0	5		3	2		1		
$p_{x_o} = -3.51 \times 10^{-3}$	3	2	1	3		-	3	5	1	-	3
$N = 3$	3	2	0	4		3					
$E = 1.0034$	3	2	2	6		1	0	0	3	4	

To signal the end of the floating point numbers, a card is punched with a 1 in column 1 and in columns 37-45, END FLOAT.

1	2	3	...	36	37	38	39	40	41	42	43	44	45
1					E	N	D		F	L	O	A	T

To signal the end of a series a card is punched in columns 37-46, STOPSERIES.

1	2	3	...	36	37	38	39	40	41	42	43	44	45	46
1					S	T	O	P	S	E	R	I	E	S

Cards are arranged in the data deck in the following sequence:

$$\left.\begin{array}{l}\text{integers}\\ \text{END INTEGERS card}\\ \text{Floating point numbers}\\ \text{END FLOAT card}\end{array}\right\}\ \text{first run}$$

$$\left.\begin{array}{l}\text{integers}\\ \text{END INTEGERS card}\\ \text{floating point numbers}\\ \text{END FLOAT card}\end{array}\right\}\ \text{second run}$$

⋮

STOPSERIES card

2 blank cards.

On all the data cards columns 73-80 are not used by the program and therefore may be used for identification purposes by the user.

OPERATING INSTRUCTIONS

Physically there exist two binary program decks for FLEXIBLE FIVER. The 280A deck will run standard FLEXIBLE FIVER problems, Overwrite 1, Overwrite 3, and Overwrite 4. The 280B deck will run only Overwrite 2 problems.

STARTING PROCEDURE

a. Place the data deck at the back of the binary program deck and ready the cards in the card reader.

b. Place the MURA 1 printer board in the printer and ready the printer.

c. Check the sense switch settings.

d. Press CLEAR, then LOAD CARDS.

SENSE SWITCH SETTINGS

The normal running condition is with all sense switches DOWN.

SS1 (tested once per integration step)

UP Stop with location counter = 05003_8
storage register = 042000000001_8

SS2 DOWN: For each time START is pressed one integration cycle will be completed. To resume normal operations, put SS1 down.

SS2 UP: A dump of the entire core memory onto logical tape 1 will be produced after which the normal stop in location 03625_8 will occur. To resume see TAPE RESUMPTION procedure (page).

DOWN Continuous integration.

SS2 (tested once per integration step)

UP Stop with location counter = 05007_8
storage register = 042000000002_8

Press START and if:

SS1 DOWN: Computations on the current run are stopped and END is printed.

SS6 DOWN: Computations will continue with the next run of the series.

SS6 UP: Stop with location counter = 04307_8, storage register = 042000077777_8, signaling the end of a search. If START is pressed now, computations will continue with the next run of the series.

SS1 UP: A dump of the entire core memory onto logical tape 1 will be produced after which the normal stop in location 03625_8 will occur. To resume see TAPE RESUMPTION procedure (page).

DOWN Continuous integration.

<u>SS3</u> (tested once per integration step)

<u>UP</u>

SS5 UP: See SS5 up, SS3 up below.

SS5 DOWN: B_x, B_o, B_y are printed at every integration step with the line label 00000.

DOWN: Normal operation.

<u>SS4</u>

<u>UP</u>

SS5 UP: See SS5 up, SS4 up below.

SS5 DOWN: Overwrite 1 is put into operation.

DOWN: Normal operation.

<u>SS5</u>

<u>UP:</u> Overwrite 2 is in operation (280B deck only)

SS4 UP: The equilibrium orbit print is omitted with the exception of the first and last lines.

SS4 DOWN:

SS3 UP: Following every print step of the equilibrium orbit, the quantities

$$\frac{px}{\sqrt{P^2-p_x^2}}, \frac{(1+x)}{\sqrt{P^2-p_x^2}}, \frac{(1+x)P^2}{(P^2-p_x^2)^{3/2}}, \lambda X, \lambda(\mathcal{X}-\Upsilon)$$

are printed with the line label 00000.

SS3 DOWN: Normal Overwrite 2 operation.

SS6 (tested whenever a run is completed in any of the following ways.)

1. $C_e = N_e$
2. SS2 has been put up.
3. An overflow (p_x, p_y, x, or y have exceeded their bounds) has occurred.

UP: Case 1 and 2, normal search stop in location 04307_8 with storage register = 042000077777_8 will occur signaling the end of the problem. If START is pressed now, computations will continue with the next run of the series. Case 3, computation is continued with the next run of the series without a stop.

DOWN: Normal operation.

PROGRAM STOPS

Twenty-four program stops are listed on pages 24 and 25. Stops 1, 2, 3, 4 are normal stops. The remaining stops are abnormal and indicate a data error, prohibited intermediate values, or a computer error.

There are a number of stops which are not listed. These are chiefly divide check stops which could occur because of prohibited values for certain quantities or could be the result of a computer error. If the computer should stop at one of these abnormal stops (listed or unlisted), it is important that it be noted by the operator -- the complete console configuration should be recorded and the problem removed from the computer.

FLEXIBLE FIVER
(Program 280)

Program Stops

Stop No.	Location Counter (Octal)	Storage Register (Octal)	Reason for Stop
1.	03625 (03614)B	000000077777	NORMAL stop, end of a series of runs (or end of a tape dump). #
2.	04307	042000077777	NORMAL stop, end of search (SS6 up). #
3.	05003 (04423)B	042000000001	SS1 up, end of one integration cycle.
4.	05007 (04427)B	042000000002	SS2 up.
5.	00027	000000000005	Check sum error on the card just read by MULBL3. Possibly machine error. Press START to ignore check sum error and continue loading cards.
6.	02413	000000000007	$\frac{\lvert \mu_{i,m,j} \rvert}{2i!} > 10^{35}$. *
7.	02414	000000000077	$\frac{\lvert \nu_{i,m,j} \rvert}{2i!} > 10^{35}$. *
8.	03265 (03222)B	000000000777	$(i_{max}+1)(j_{max}+1)$ (No. of harmonics) $>$ 1120. *
9.	03301	000000003556	Field data cards missing, ready corrected data deck in reader and press START.
10.	03351	000000007223	Square root error return, $\sqrt{E_i^2 + P_o^2 - E_o^2}$ 0, (Overwrite 1). *
11.	03434	000000003434	Logarithm error return, $(1 + x) \leq 0$. *

Stop No.	Location Counter (Octal)	Storage Register (Octal)	Reason for Stop
12.	03476	000000003476	Exponential error return, $\left\| (k+1) \ln (1+x) \right\| > 87.3$. *
13.	04173	000000000012	Square root error return, $(K_{1,x})^2$ or $(K_{2,x})^2 < 0$. *
14.	04221	000000004221	Square root error return, $(K_{1,y})^2$ or $(K_{2,y})^2 < 0$. *
15.	04354 (04066)B	000000004336	Tape check on, press START to try writing the tape again.
16.	04362 (04074)B	000000004373	Tape check on, press START to try loading the tape again.
17.	04535 (04247)B	000000004535	Cosine error return, $\left\| m\Phi \right\| > 2^{36}$. *
18.	04542 (04254)B	000000004542	Sine error return, $\left\| m\Phi \right\| > 2^{36}$. Note: This stop may indicate a computer error since the argument has been used earlier and presumably did not cause a stop.
19.	04743 (04377)B	000000000177	Overflow in forming one of the μ i, m, ν i, m, etc. sums. *
20.	04744	000000000377	$\frac{y}{1+x} > 2$. *
21.	05011	000000000006	Square root error return, $\sqrt{P^2 - P_x^2} < 0$, (Overwrite 2). *
22.	05224	000000000012	Arc cosine error return, $\left\| \cos\sigma \right\| > 1$. (Overwrite 2). Note: This stop may indicate a computer error since the program would not transfer to this subroutine if $\left\| \cos\sigma \right\| > 1$.
23.	05275	000000005275	Cosine error return, $\left\| m\Phi \right\| > 2^{36}$. (Overwrite 4). *

Stop No.	Location Counter (Octal)	Storage Register (Octal)	Reason for Stop
24.	05303	000000005303	Sine error return, $\lvert m'\Phi \rvert > 2^{36}$. (Overwrite 4).

B. Operator: These are the addresses for the location counter if the 280B deck (Overwrite 2) is used.

\# Operator: The contents of the AC and MQ registers will be the same as the storage register.

* Operator: Note the stop number on the output (return to the user) and remove the problem.

TAPE RESUMPTION

If it is desired to interrupt the computation at any time with the possibility of resuming later, set a tape unit to logical tape 1 and put sense switches 1 and 2 up. Almost immediately the computer comes to one of the following two stops depending on which sense switch is tested first.

a. SS1 tested first.
   Storage register = 042000000001_8
   Location counter = 05003_8

b. SS2 tested first.
   Storage register = 042000000002_8
   Location counter = 05007_8

Pressing START causes the other sense switch to be tested and if it is also up the entire core memory is written on logical tape 1. The normal program stop (location counter = 03625_8, storage register = 000000077777_8) then occurs.

At the time of resumption, the data cards for the remaining runs of the series (not including the current run) must be readied in the card reader. Also the resumption tape must be loaded on logical tape 1. Then press LOAD TAPE. The first line of print will contain the following information: program number, human ID, field ID, and run ID. Computation is then continued in the normal manner.

TIMING

Some typical times per 32 integration steps and one line of print, as a function of i_{max}, number of harmonics, j_{max} and y motion, are given below:

i_{max}	No. of harmonics	j_{max}	y-motion	Seconds per 32 steps and one line of present
1	6	0	No	10
1	6	0	Yes	18
1	6	6	No	16
1	6	6	Yes	29
3	6	12	No	35
3	6	12	Yes	68
4	3	0	No	9
4	3	0	Yes	

APPENDIX I

OVERWRITE 1

The purpose of this overwrite is to simulate the energy gain by a particle in a cyclotron. The quantity P, the total mechanical momentum of a particle, appears directly in the FLEXIBLE FIVER equations of motion. The energy gain is simulated by adding amounts ΔP_i to P at intervals specified by the user. The user specifies an energy increment ΔE_i and the program computes P_i from

$$E_i = E_{i-1} + \Delta E_i$$

$$P_i = \sqrt{E_i^2 - E_o^2 + P_o^2}$$

where units are used where c = 1, so that $E_o^2 - P_o^2$ is the square of the rest energy.

By putting sense switch 4 up at the beginning of a series of runs, this overwrite is placed in operation.

The user then specifies up to 50 sets of quantities N_{Ri} and ΔE_i, E_o, P_o and a larger periodicity N_B. A sample agendum is included on page 31. This data is printed after the standard initial print in the following format:

	Line Label	Col. 1	Col. 2	Col. 3	Col. 4	Col. 5
	01111					
	00000	E_o				
I	00001	N_B				
I	00002	N_{R_1}	N_{R_2}	N_{R_3}	N_{R_4}	N_{R_5}
	00003	ΔE_1	ΔE_2	ΔE_3	ΔE_4	ΔE_5

.
.
.

After line 00003, lines which contain only zeros are not printed.

At the beginning of the N_{R_i}th integration step, the program computes P_i and replaces P_{i-1} with this value. P_i and E_i are printed at this time with the line label of 30000 + i. The program continues through the sequence of sets until it reaches step N_B. Then the N_R counter is reset to zero and the whole incrementing process repeats.

It should be noted that as P increases the orbit radius will increase and the orbit will move to larger x values. In this process, x may eventually grow beyond the range of validity of the original expansion of the fields about the radius r_o.

FLEXIBLE FIVER

Overwrite 1 Agendum

Program 280 A

To be attached by staples to the <u>front</u> of a series of FLEXIBLE FIVER runs. (SENSE SWITCH 4 MUST BE <u>UP</u>.)

Parameter	Address	Value		Parameter	Address	Value n	Value exp	
N_B	3720			E_o	3771			
N_{R_1}	3721			ΔE_1	3772			
N_{R_2}	3722			ΔE_2	3773			
N_{R_3}	3723			ΔE_3	3774			
N_{R_4}	3724			ΔE_4	3775			
N_{R_5}	3725			ΔE_5	3776			
N_{R_6}	3726			ΔE_6	3777			
N_{R_7}	3727		INTEGERS	ΔE_7	3778			FLOATING POINT NUMBERS
N_{R_8}	3728			ΔE_8	3779			
N_{R_9}	3729			ΔE_9	3780			
$N_{R_{10}}$	3730			ΔE_{10}	3781			
$N_{R_{11}}$	3731			ΔE_{11}	3782			
$N_{R_{12}}$	3732			ΔE_{12}	3783			
$N_{R_{13}}$	3733			ΔE_{13}	3784			
$N_{R_{14}}$	3734			ΔE_{14}	3785			
$N_{R_{15}}$	3735			ΔE_{15}	3786			
$N_{R_{16}}$	3736			ΔE_{16}	3787			
$N_{R_{17}}$	3737			ΔE_{17}	3788			
$N_{R_{18}}$	3738			ΔE_{18}	3789			

NOTE: If additional N_{R_i} and ΔE_i (up to i = 50) are desired, attach another sheet. Addresses are consecutive.
Enter P_o as P on the FLEXIBLE FIVER Agendum.

APPENDIX II

OVERWRITE 2

Overwrite 2 is run by the program 280 B deck (sense switch 5 up). The methods used by this overwrite are described by M. M. Gordon and T. A. Welton in Computation Methods for AVF Cyclotron Design Studies, ORNL-2765.[1]

Overwrite 2 performs the following operations. The fixed point is determined by an iteration procedure employing both the exact and linearized equations of motion. When the iterative process yields x_f, p_{xf} to the accuracy specified by the user, the orbit having x_f, p_{xf} as initial conditions is integrated through one sector (n_{RK} steps). For subsequent calculations, this orbit is defined to be the equilibrium orbit (see page 38 for printing options). The program now calculates the parameters of the radial and vertical linear oscillations about the equilibrium orbit including the elements of the transformation matrix, $M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$, det M, $\cos\sigma$, σ/π, ν, α and β.

Equations (II-1) and (II-2) are exact for the motion of a particle in the median plane. Equations (II-3) and (II-4) are the linearized equations for the radial motion where ξ and p_ξ are the radial (first-order) displacement and corresponding momentum associated with the calculated orbit. Similarly, equations (II-5) and (II-6) represent the linear vertical motion.

---

[1] MURA Library, ORNL Reports, Book 9, Report 38.

EQUATIONS OF MOTION

$$x' = \frac{(1 + x)\, p_x}{\sqrt{P^2 - p_x^2}} \qquad \text{(II-1)}$$

$$p'_x = \sqrt{P^2 - p_x^2} + \lambda(1 + x)\, B_y \qquad \text{(II-2)}$$

$$\xi' = \left[\frac{p_x}{\sqrt{P^2 - p_x^2}}\right] \xi + \left[\frac{(1 + x)\, P^2}{(P^2 - p_x^2)^{3/2}}\right] p_\xi \qquad \text{(II-3)}$$

$$p'_\xi = \left[\frac{-\, p_x}{\sqrt{P^2 - p_x^2}}\right] p_\xi + \lambda X \xi \qquad \text{(II-4)}$$

$$\eta' = \left[\frac{1 + x}{\sqrt{P^2 - p_x^2}}\right] p_\eta \qquad \text{(II-5)}$$

$$p'_\eta = \lambda (Z - Y)\, \eta \qquad \text{(II-6)}$$

where

$$X = (1 + x)^k \left\{ \sum_m -(\mu_{o,m} \cos m\Phi + \nu_{o,m} \sin m\Phi) \right\} + Y$$

$$Y = (1 + x)^k \sum_m (\alpha_{1,m} \cos m\Phi + \beta_{1,m} \sin m\Phi) \qquad \text{(II-7)}$$

$$Z = \frac{p_x}{\sqrt{P^2 - p_x^2}} (1 + x)^k \sum_m (\gamma_{1,m} \cos m\Phi + \delta_{1,m} \sin m\Phi)$$

The magnetic field used in these equations of motion (II-1) - (II-6) is given by the user in the same way as in a standard FLEXIBLE FIVER run. However, the following restrictions must also be satisfied:

a) i_{max} must be 1 (the program will not operate correctly if $i_{max} \neq 1$),

b) (j_{max} + 2) (no. of harmonics) $\leqslant$ 730.

FIXED POINT SEARCH

An iterative method is used to find the fixed point corresponding to the equilibrium orbit.

The program integrates equations (II-1) and (II-2) through one sector (n_{RK} steps) beginning with the initial conditions (x_o, p_{x_o}) entered on the agendum sheet. Simultaneously equations (II-3) and (II-4) are integrated with two sets of initial conditions:

$$\begin{aligned} \xi_o &= \rho\,, \quad & p_{\xi_o} &= \rho' \\ \overline{\xi_o} &= \rho'\,, \quad & \overline{p_{\xi_o}} &= \rho \end{aligned} \qquad \text{(II-8)}$$

where $\rho = 1$, $\rho' = 0$. From this integration we get the elements of the matrix, M_x, and the ϵ's as follows:

$$\begin{aligned} A_x &= \xi_q\,, \quad & B_x &= \overline{\xi}_q\,, \\ C_x &= p_{\xi q}\,, \quad & D_x &= \overline{p_{\xi q}}\,, \end{aligned} \qquad \text{(II-9)}$$

where q = n_{RK}-th integration steps,

$$\begin{aligned} \epsilon_1 &= x_q - x_o, \quad \epsilon_2 = p_{x_q} - p_{x_o} \\ \epsilon &= |\epsilon_1| + |\epsilon_2| \end{aligned} \qquad \text{(II-10)}$$

The calculated orbit, with (x_o, p_{x_o}) as initial conditions, is assumed to differ from the equilibrium orbit only in first-order effects. Therefore:

$$\begin{aligned} x &= x_f + a_1 \xi + a_2 \overline{\xi} \\ p_x &= p_{xf} + a_1 p_\xi + a_2 \overline{p_\xi} \end{aligned} \qquad \text{(II-11)}$$

Using equations (II-8), (II-9) and (II-10), we get the following equations for determining a_1 and a_2:

$$\begin{aligned} (A_x - 1)\, a_1 + B_x\, a_2 &= \epsilon_1 \\ C_x\, a_1 + (D_x - 1)\, a_2 &= \epsilon_2 \end{aligned} \qquad \text{(II-12)}$$

Solving for a_1 and a_2:

$$\begin{aligned} a_1 &= \frac{\epsilon_1 (D_x - 1) - \epsilon_2 B_x}{(A_x - 1)(D_x - 1) - B_x C_x} \\ a_2 &= \frac{\epsilon_2 (A_x - 1) - \epsilon_1 C_x}{(A_x - 1)(D_x - 1) - B_x C_x} \end{aligned} \qquad \text{(II-13)}$$

Using these values of a_1, a_2 and the initial conditions (x_o, p_{x_o}), the program calculates x_f, p_{xf}:

$$\begin{aligned} x_f &= x_o - a_1 \\ p_{xf} &= p_{x_o} - a_2 \,. \end{aligned} \qquad \text{(II-14)}$$

These values of x_f and p_{xf} are the new initial conditions for the next pass.

The above process is repeated until a sufficiently small value of ϵ is obtained. If ten passes are completed and no fixed point is found, the program proceeds to the next run of the series.

LINEAR MOTION PROPERTIES

The properties of the equilibrium orbit and the linear radial and vertical oscillations about it can now be calculated from a final integration.

Simultaneously equations (II-1) through (II-6) are integrated through one sector. Equations (II-3), (II-4) and (II-5), (II-6) each have two sets of initial conditions so that in all there are ten equations being integrated. The initial conditions are as follows:

$$
\begin{aligned}
x_o &= x_f \quad , & p_{x_o} &= p_{xf} \quad , \\
\xi_o &= \rho \quad , & p_{\xi o} &= \rho' \quad , \\
\overline{\xi_o} &= \rho' \quad , & \overline{p_{\xi o}} &= \rho \quad , \\
\eta_o &= \rho \quad , & p_{\eta o} &= \rho' \quad , \\
\overline{\eta_o} &= \rho' \quad , & \overline{p_{\eta o}} &= \rho \quad ,
\end{aligned}
\tag{II-15}
$$

where $\rho = 1$, $\rho' = 0$.

During this integration, a count is made of the number of times, R_x, the value of $\overline{\xi}$ changes sign. The initial value of $\overline{\xi}$ ($\overline{\xi_o} = 0$) is considered positive.

This final integration provides all the information necessary to calculate the following output data.

$$v = \frac{P}{E} \tag{II-16}$$

$$s_o = PN \int_o^{\frac{2\pi}{N}} \frac{(1 + x)\, d\,\theta}{\sqrt{P^2 - p_x^2}} \tag{II-17}$$

$$T = \frac{s_o}{v} \tag{II-18}$$

$$\langle x \rangle = \frac{1}{n_{RK}} \sum_{i=o}^{i=n_{RK}-1} x_i \tag{II-19}$$

As before the solutions of equations (II-3), (II-4) yield the elements of the transformation matrix, $M_x = \begin{pmatrix} A_x & B_x \\ C_x & D_x \end{pmatrix}$. Thus, we have

$$\cos \sigma_x = \frac{A_x + D_x}{2} \tag{II-20}$$

$$\sigma_x^* = 2\left[\frac{R_x + 1}{2}\right]\pi + (-1)^{R_x} \sigma_x \tag{II-21}$$

where $\left[\frac{R_x + 1}{2}\right]$ denotes the integer part of $\frac{R_x + 1}{2}$

$$\nu_x = \frac{N \sigma_x^*}{2\pi} \tag{II-22}$$

$$\det M_x = A_x D_x - B_x C_x \tag{II-23}$$

$$\beta_x = \frac{B_x}{\sin \sigma_x} \tag{II-24}$$

$$\alpha_x = \frac{\cos \sigma_x - D_x}{\sin \sigma_x} \tag{II-25}$$

Similarly, equations (II-5), (II-6) yield M_y, $\cos \sigma_y$, σ_y^*, ν_y, β_y and α_y.

PRINTING OPTIONS

The input data for Overwrite 2 is printed after the standard FLEXIBLE FIVER initial print in the following format:

Line Label	Col. 1	Col. 2	Col. 3	Col. 4
02222				
00000	ϵ	ρ	ρ'	E

During the fixed point search, successive approximations of the fixed point are printed in the following way:

Line Label	Col. 1	Col. 2	Col. 3
01000	x	p_x	$\frac{N\Theta}{2\pi}$
01001	x	p_x	$\frac{N\Theta}{2\pi}$
.	.		
.	.		
.	.		

The equilibrium orbit is printed next with the following format:

Line Label	Col. 1	Col. 2	Col. 3	Col. 4	Col. 5
02000	x_o	p_{xo}	$\left(\frac{N\Theta}{2\pi}\right)_o$	x'_o	$C_o = \left\lvert \frac{\lambda(1+x)B_y}{P} \right.$
02001	x_1	p_{x1}	$\left(\frac{N\Theta}{2\pi}\right)_1$	x'_1	C_1
.					
.					
.					
$02000+n_{RK}$	$x_{n_{RK}}$	$p_{x_{n_{RK}}}$	$\left(\frac{N\Theta}{2\pi}\right)_{n_{RK}}$	$x'_{n_{RK}}$	$C_{n_{RK}}$
$02000+n_{RK}+1$	v	s_o	T	$\langle x \rangle$	

If less frequent printing is desired, the quantities N_p and N_e on the standard agendum can be used. The program will print every step up to and including the N_eth step and thereafter every N_pth step will be printed. For a minimum of printing, sense switch 4 should be placed up. Then only lines 02000 and 02000 + n_{RK} will be printed. Line 02000 + n_{RK} + 1 is always printed.

The format for the printing of the properties of the linear radial and vertical oscillations is given below.

Line Label	Col. 1	Col. 2	Col. 3	Col. 4	Col. 5
03000	ξ_0	p_{ξ_0}	$\overline{\xi_0}$	$\overline{p_{\xi_0}}$	
03001	A_x	B_x	C_x	D_x	
03002	$\cos\sigma_x$	$\sin\sigma_x$	σ_x^*/π	ν_x	
03003	R_x	$\det M_x$			
03004	β_x	α_x	$1+\alpha_x^2$	$2\alpha_x\beta_x$	β_x^2
04000	η_0	p_{η_0}	$\overline{\eta_0}$	$\overline{p_{\eta_0}}$	
04001	A_y	B_y	C_y	D_y	
04002	$\cos\sigma_y$	$\sin\sigma_y$	σ_y^*/π	ν_y	
04003	R_y	$\det M_y$			
04004	β_y	α_y	$1+\alpha_y^2$	$2\alpha_y\beta_y$	β_y^2 END

If $\cos\sigma_x$ or $\cos\sigma_y$ is greater than 1, the program prints the line

COSINE SIGMA TOO LARGE

instead of line 03002 or 04002. Line 03004 or 04004 is also not printed.

A sample output page of an Overwrite 2 run is included on page 46.

Overwrite 2 input parameters are included on the standard FLEXIBLE FIVER agendum (page 8).

APPENDIX III

OVERWRITE 3

In Overwrite 3, the variables p_x, p_y, x and y are bumped or incremented at given integration steps by specified amounts.

The user specifies up to four sets of δ's beginning with $\delta_{1,px}$, $\delta_{1,py}$, $\delta_{1,x}$ and $\delta_{1,y}$. These δ's are added to the corresponding value of p_x, p_y, x, y at the end of the N_1-th integration step. At the N_2-th integration step, the second set of δ's is used, etc., until the N_{BP}-th integration step is reached. At this time, a counter is reset to zero and the entire process repeats.

The input data is printed after the standard initial print in the following format:

	Line Label	Col. 1	Col. 2	Col. 3	Col. 4
	03333				
I	00001	N_{BP}			
I	00002	N_4	N_3	N_2	N_1
	00003	δ_{1,p_x}	$\delta_{1,py}$	$\delta_{1,x}$	$\delta_{1,y}$
	00004	δ_{2,p_x}	δ_{2,p_y}	$\delta_{2,x}$	$\delta_{2,y}$
	00005	δ_{3,p_x}	δ_{3,p_y}	$\delta_{3,x}$	$\delta_{3,y}$
	00006	δ_{4,p_x}	δ_{4,p_y}	$\delta_{4,x}$	$\delta_{4,y}$

A sample agendum sheet is included on page 41.

FLEXIBLE FIVER

Overwrite 3 Agendum

PROGRAM 280A

To be attached by staples to the <u>front</u> of a series of FLEXIBLE FIVER runs.

Parameter	Address	Value		Remarks	
N_1	3255			Integration step for first bump.	FIXED POINT INTEGERS
N_2	3254			Integration step for second bump.	
N_3	3253			Integration step for third bump.	
N_4	3252			Integration step for fourth bump.	
N_{BP}	3198			Over-all periodicity.	
		n	exp		
$\delta_{1,px}$	3268				FLOATING POINT NUMBERS
$\delta_{1,py}$	3269				
$\delta_{1,x}$	3270				
$\delta_{1,y}$	3271				
$\delta_{2,px}$	3264				
$\delta_{2,py}$	3265				
$\delta_{2,x}$	3266				
$\delta_{2,y}$	3267				
$\delta_{3,px}$	3260				
$\delta_{3,py}$	3261				
$\delta_{3,x}$	3262				
$\delta_{3,y}$	3263				
$\delta_{4,px}$	3256				
$\delta_{4,py}$	3257				
$\delta_{4,x}$	3258				
$\delta_{4,y}$	3259				

APPENDIX IV

OVERWRITE 4

In Overwrite 4, the field components, B_x, B_y and B_θ, are incremented each time they are computed with the following increments:

$$\Delta B_x = -\sum_{i',m',j'} \rho_{i',m',j'}\, x^{j'} y^{i'} \cos m'\theta$$

$$\Delta B_y = \sum_{i',m',j'} \zeta_{i',m',j'}\, x^{j'} y^{i'} \cos m'\theta \qquad \text{(IV-1)}$$

$$\Delta B_\theta = -\sum_{i',m',j'} \psi_{i',m',j'}\, x^{j'} y^{i'} \sin m'\theta \; .$$

The sum on i' runs from o to i'_{max}, the sum on j' from o to j'_{max}, and the sum on m' over harmonics which the user lists on the special agendum (page 45).

The recursion relations which follow from Maxwell's equations can be written:

$$\zeta_{i'+1,m',j'} = -\zeta_{i'+1,m',j'-1} + \frac{1}{i'+1}\left[m'\,\psi_{i',m',j'} + (j'+1)(\rho_{i',m',j'} + \rho_{i',m',j'+1})\right]$$

$$\psi_{i'+1,m',j'} = -\psi_{i'+1,m',j'-1} + \frac{m'\,\zeta_{i',m',j'}}{i'+1}$$

$$\rho_{i'+1,m',j'} = -\frac{j'+1}{i'+1}\,\zeta_{i',m',j'+1} \qquad \text{(IV-2)}$$

$$\psi_{o,m',j'+1} = -\psi_{o,m',j'} - \frac{m'}{j'+1}\,\rho_{o,m',j'} \; .$$

The user specifies the $\zeta_{o,m',j'}$, $\rho_{o,m',j'}$ and $\psi_{o,m',o}$ for each non-zero harmonic. The user must also enter on the agendum (page) the addresses for these parameters.

PARAMETER	ADDRESS
$\zeta_{o,m'_1,o}$	4215
$\zeta_{o,m'_1,1}$	4214
$\vdots$	$\vdots$
$\zeta_{o,m'_1,j'max}$	4215 - j'max
$\zeta_{o,m'_2,o}$	4215 -(j'$_{max}$+1)
$\zeta_{o,m'_2,1}$	4215 -(j'$_{max}$+1) - 1
$\vdots$	$\vdots$
$\zeta_{o,m'_3,o}$	4215 - 2(j'$_{max}$+1)
$\vdots$	$\vdots$
$\rho_{o,m'_1,o}$	4215 - (No. of m') (j'$_{max}$+1)
$\rho_{o,m'_1,1}$	4215 - (No. of m') (j'$_{max}$+1) - 1
$\vdots$	$\vdots$
$\rho_{o,m'_2,o}$	4215 - (No. of m' +1) (j$_{max}$+1)
$\vdots$	$\vdots$
$\psi_{o,m'_1,o}$	4215 - 2(No. of m') (j'$_{max}$+1)
$\psi_{o,m'_2,o}$	4215 - 2(No. of m') (j'$_{max}$+1) - 1
$\psi_{o,m'_3,o}$	4215 - 2(No. of m') (j'$_{max}$+1) - 2
$\vdots$	$\vdots$

For the correct operation of Overwrite 4, i'_{max}, j'_{max} and the number of harmonics must satisfy

$$3\,(i'_{max}+1)\,(\text{No. of } m')\,(j'_{max}+1) \leq 504,$$

$$i'_{max} \leq 6, \qquad \text{(IV-3)}$$

and $\quad$ no. of $m' \leq 10$.

The input data is printed in the following format after the standard initial print:

Line Lable	Col. 1	Col. 2	Col. 3	Col. 4	Col. 5
04444					
00000	i'_{max}	no. of m'	j'_{max}		
Space					
0000m'*	$\zeta_{o,m',j'_{max}}$	$\zeta_{o,m',j'_{max}-1}$	$\zeta_{o,m',j'_{max}-2}$	$\zeta_{o,m',j'_{max}-3}$	$\zeta_{o,m',j'_{max}-4}$
⋮	⋮				
0000m'	$\zeta_{o,m',4}$	$\zeta_{o,m',3}$	$\zeta_{o,m',2}$	$\zeta_{o,m',1}$	$\zeta_{o,m',o}$
Space					
1000m'*	$\rho_{o,m',j'_{max}}$	$\rho_{o,m',j'_{max}-1}$	$\rho_{o,m',j'_{max}-2}$	$\rho_{o,m',j'_{max}-3}$	$\rho_{o,m',j'_{max}-4}$
⋮	⋮				
1000m'	$\rho_{o,m',4}$	$\rho_{o,m',3}$	$\rho_{o,m',2}$	$\rho_{o,m',1}$	$\rho_{o,m',o}$
Space					
2000m'*	$\psi_{o,m',o}$				
⋮	⋮				

*$m' = m_1, m_2, \ldots, m_{10}$.

A sample agendum sheet is included on page 45.

FLEXIBLE FIVER

Overwrite 4 Agendum

PROGRAM 280A

To be attached by staples to the front of a series of FLEXIBLE FIVER runs.

Parameter	Address	Value n	Value exp	Remarks
i'_{max}	3696			Last term in power series for ΔB_x, ΔB_y, ΔB_θ $\leqslant 6$
No. of m'	3697			No. of non-zero bumped field harmonics, $\leqslant 10$
j'_{max}	3698			Last term in power series for $\rho_{i',m'}$, $\zeta_{i',m'}$, $\psi_{i',m'}$

List of Harmonic Numbers

m'_1	3711			m'_6	3706		
m'_2	3710			m'_7	3705		
m'_3	3709			m'_8	3704		
m'_4	3708			m'_9	3703		
m'_5	3707			m'_{10}	3702		

Fourier Coefficients of the Median Plane Bumped Field

	4215						

FLOATING POINT NUMBERS

01000	30920000- 4	-14400000- 3	00000000000		
01001	31015361- 4	-13425065- 3	00000000000		
01002	31015596- 4	-13423997- 3	00000000000		

02000	31015596- 4	-13423997- 3	00000000000	-13428173- 3	20253108+ 0
02001	29117016- 4	-63893515- 3	31250000- 2	-63913423- 3	20236392+ 0
02002	24760377- 4	-11316612- 2	62500000- 2	-11320139- 2	19758524+ 0
02003	18061248- 4	-15891015- 2	93750000- 2	-15895892- 2	18796440+ 0
02004	92512256- 5	-19876275- 2	12500000- 1	-19882041- 2	17362125+ 0
02005	-13250114- 5	-23051997- 2	15625000- 1	-23057818- 2	15509220+ 0
02006	-13222588- 4	-25234524- 2	18750000- 1	-25239224- 2	13333487+ 0
02007	-25919395- 4	-26296178- 2	21875000- 1	-26298456- 2	10965604+ 0
02008	-38847407- 4	-26179141- 2	25000000- 1	-26177943- 2	85568236- 1
⋮	⋮	⋮	⋮	⋮	⋮
02030	27517954- 4	82771179- 3	93750000- 1	82796789- 3	19109658+ 0
02031	30451151- 4	36089975- 3	96875000- 1	36101200- 3	19856745+ 0
02032	31015590- 4	-13424006- 3	10000000+ 0	-13428181- 3	
02033	10000000+ 0	62821757+ 0	62821757+ 0	-31787801- 4	

03000	10000000+ 0	00000000000	00000000000	10000000+ 0	
03001	-91156506- 1	-17317360+ 1	14887979- 1	17313140+ 0	
03002	40987447- 1	91214194- 1	36557250- 1	73114500+ 0	
03003	00000000000	99999952- 1			
03004	16321998- 1	-14487214+ 0	30987936+ 0	-47292054- 1	26640761- 2

04000	10000000+ 0	00000000000	00000000000	10000000+ 0	
04001	19442822+ 0	-12986191+ 1	13912867- 1	-41493551- 1	
04002	76467335- 1	64441809- 1	22290034- 1	44580067+ 0	
04003	00000000000	99999965- 1			
04004	21589814- 1	18305024+ 0	43507390+ 0	79040412- 1	46612007- 2