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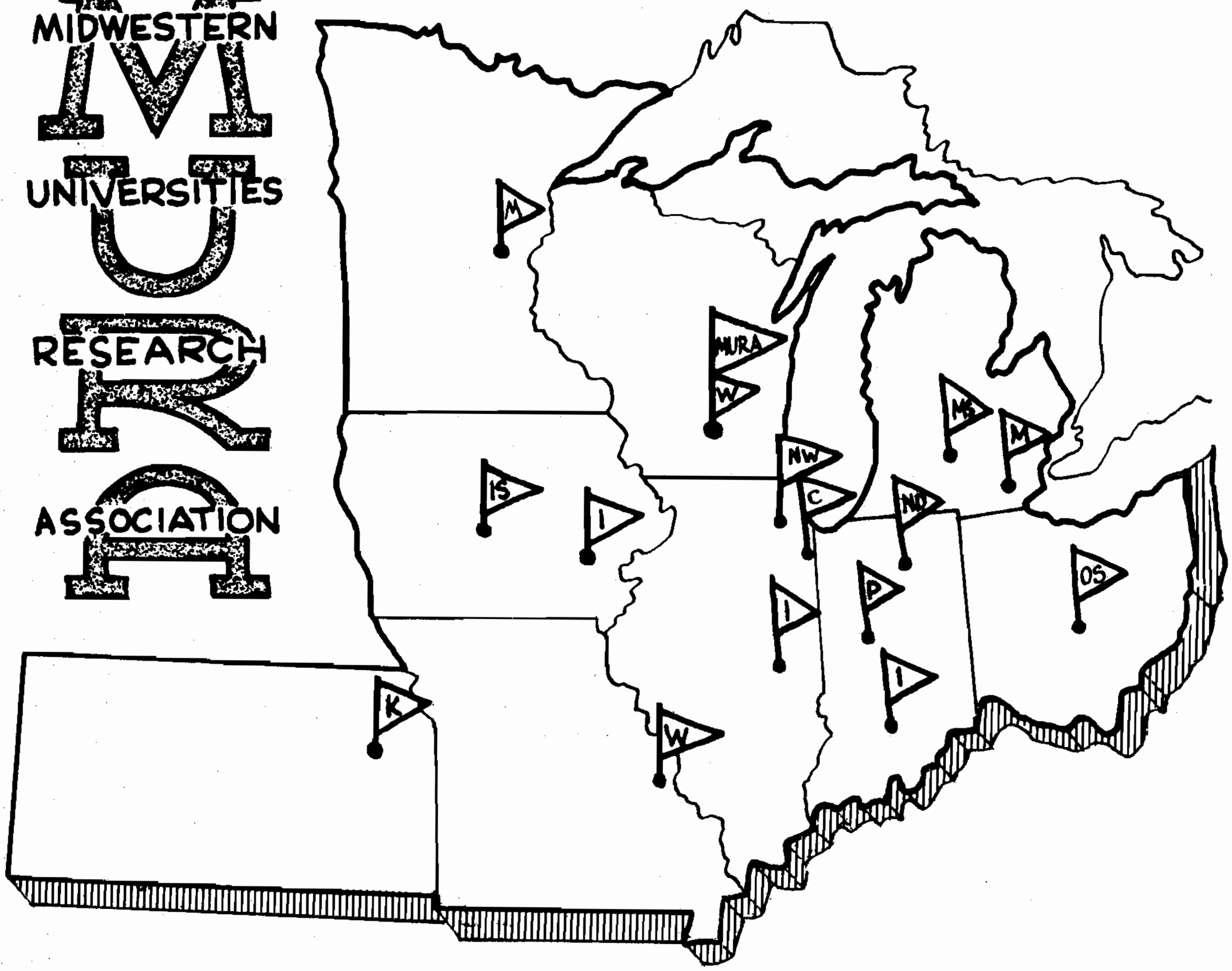

PERFORMANCE ESTIMATES FOR INJECTOR CYCLOTRONS

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PERFORMANCE ESTIMATES FOR INJECTOR CYCLOTRONS

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East Lansing, Michigan

Sept. 28, 1960

ABSTRACT

Available evidence on performance to be expected from injector cyclotrons is collected and reviewed. On the basis of this information, estimates are made of current, spot size, angular divergence, energy spread and duty cycle for the extracted beam of possible 40 Mev and 200 Mev injector cyclotrons.

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I. INTRODUCTION

Customarily large proton synchrotrons have utilized a linear accelerator to preaccelerate particles up to an energy appropriate for injection into the synchrotron. The choice of a linac in lieu of other possible devices has been dictated principally by the high density of the transverse x, p_x, y, p_y, phase space projections of the linac beam. Secondary factors have been: (1) the output energy of reasonably sized linacs matched moderately well with the desired injection energy, (2) the duty cycle of the synchrotron resulted in a linac duty cycle compatible with reasonable power consumption, and (3) the linac, though expensive compared with possible other devices, was still a moderately small fraction of the total synchrotron cost.

Results of recent developmental work on sector focused cyclotrons[1.-3.] has led to results indicating such a cyclotron

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1. T. A. Welton - Proceedings of CERN 1959 conference on High Energy Accelerators and Instrumentation, p. 366. Also private communications.
2. M. M. Gordon, Proc. of Sea Island Conf. NAS-NRC 656, 234 (1959)
3. Blosser, Gordon, and Arnette, manuscript in preparation.

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to be a possibly likely choice as an injection device with possible special relevance[4.] for FFAG type synchrotrons. Specific factors

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4. Lee C. Teng, R.S.I. 27, 106 (1956)

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are as follows:

(1) Detailed studies of orbit phenomena in such cyclotrons indicate rather conclusively that designs can be evolved to transmit the x, p_x, y, p_y, phase space density from the source to the exterior of the machine without appreciable dilution; if the cyclotron is provided with a source equivalent in emittance to that of the linac, it should then match the linac in transverse density of the extracted beam. (2) In the design of high performance synchrotrons there is an increasing trend to higher injection energies such that in some contemplated situations the large physical size of the linac has posed an awkward problem. (3) For FFAG synchrotrons, the possibility of enormously increasing the duty cycle with respect to that of pulsed field synchrotrons could lead to situations where the power requirements for the linac were an important economic consideration. In view of the above and of the established advantage of the cyclotron with respect to compactness and economy it appears appropriate to undertake a recomparison of the relative injection capabilities of cyclotrons and linacs. A further motivating factor for such a comparison arises from the distinct difference in the time profile of the output current for the two devices; if the cyclotron is established as a feasible injector, an additional

group of r-f acceleration schemes can be considered for the synchrotron with possible gains in efficiency in particular situations.

Objective evaluation of the injection capability of cyclotrons is unfortunately handicapped at present by a sparsity of written reports on a number of pertinent aspects of cyclotron information and secondly, by the absence of experimental testing of a number of design features indicated to be desirable. The present report is an effort to partially alleviate the first of these handicaps by collecting and reviewing available information, principally from unpublished notes and private communications, relative to the performance to be anticipated from injector cyclotrons.

It will be convenient to divide the discussion into three sections. Section II summarizes existing data on phase space characteristics of cyclotron sources. Sections III and IV view the cyclotron as a device for mapping source phase space characteristics into phase space characteristics of the extracted beam, section III considering specifically the transverse characteristics of the mapping x, p_x, y, p_y, and section IV considering longitudinal characteristics of the mapping (E, t).

As is obvious, but perhaps never-the less worthy of mention, the design of every cyclotron is in many respects a unique problem. Performance, even with optimized designs, may vary considerably depending on a number of boundary conditions, and detailed performance estimates can therefore only be made after considerable study of a particular situation.

The conclusions of this report will be drawn principally from results of detailed studies at Michigan State[3.] of the characteristics of a 40 Mev 3 sector low spiral cyclotron. The conclusions are then directly applicable to injector cyclotrons of energies up to perhaps 100 Mev. At energies of 200 Mev and higher a number of the problems discussed would be considerably different and would require additional detailed study before reliable performance estimates could be made. Lacking such detailed work, it is reasonable to assume that results comparable to the 40 Mev case can be achieved in the 200 Mev range; such an assumption is however quite qualitative and should be supplemented at an early stage with quantitative studies if active interest in such a device develops. (Rather extensive studies of an 800 Mev cyclotron have been performed[1., 2., 5., 6.]; problems encountered therein indicate a considerably lower final energy to be

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5. J. A. Martin, Proc. of CERN 1959 Conf. on High Energy Accelerators and Instrumentation, p. 205.
6. M. M. Gordon and H. G. Blosser, Progress Report on Beam Extraction Studies for Oak Ridge Cyclotron Analogue II (available from authors on request)

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more appropriate for injector cyclotrons).

II. THE CYCLOTRON SOURCE

Let the rectangular coordinates x and y designate displacement at right angles to the principal direction of motion of a beam of particles and p_x and p_y the corresponding conjugate momenta. The phase space density of some small element of the beam is then the current (particles per unit time) in the element divided by the product of the spreads in energy, in x, in p_x, in y, and in p_y of the element. For injection purposes the spread in energy of the particles leaving the source is usually of small consequence [7].

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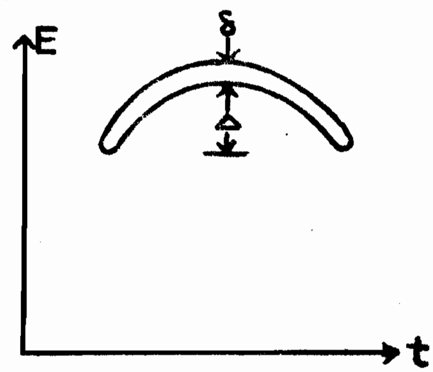
7. If the spread in energy of the output beam from either a cyclotron or a linac were measured as a function of time with extremely fast equipment, an area of the form shown in the sketch at right would be obtained (with similar figures following in sequence spaced in time at intervals of 1 r-f period.)



The energy spread δ arises from the source, the spread Δ from detailed features of the accelerating process. Due to matching difficulties between injector and synchrotron a phase figure of the form shown must normally be considered from the point of view of the synchrotron as having a spread $\Delta+\delta$ uniform in time. Since $\Delta \gg \delta$ in the usual circumstance, the precise value of δ is unimportant.

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and it is therefore adequate to consider the density in the x, p_x, y, p_y, t projection of the total phase space. Experimental determinations of this projection can be accomplished by allowing the beam to illuminate an aperture and measuring the current and angular divergence of the beam passing through the aperture. The x, y spread of the beam is defined by the aperture, the p_x, p_y spread by the angular divergence (for small angles $\theta_x = P_x/P$ etc.) Note that for given spread in p_x and p_y the angular divergence in each dimension will vary as p^{-1} as the particles are accelerated, and the solid angle as p^{-2}, where p is the total momentum.

Figure I gives the results of such a measurement on the beam of a Cockcroft-Walton accelerator of the Oak Ridge National Laboratory Biology Division[8.] The accelerator was equipped with a standard r-f

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8. The measurements were made by one of the authors (HGB) and results are briefly stated in Proc. of Sea Island Conf. NAS-NRC pub. 656 p. 203

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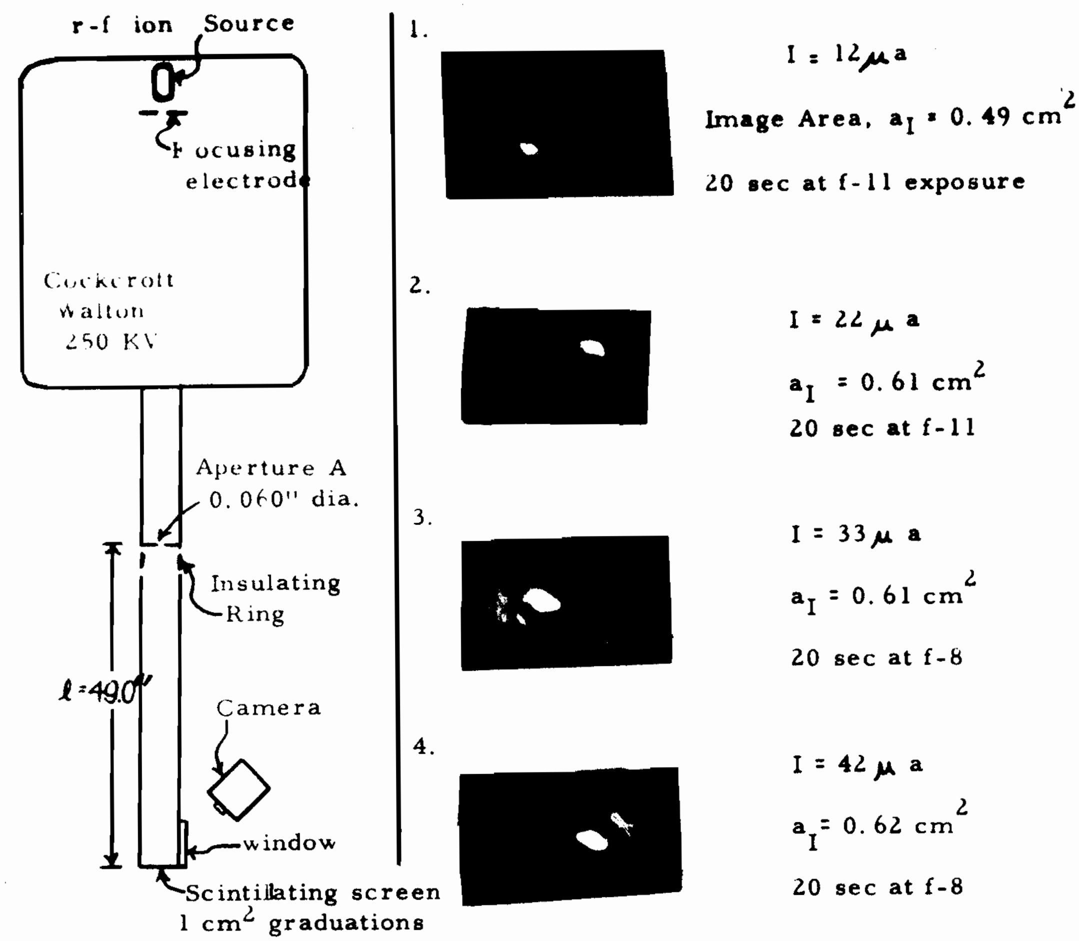
ion source of the type normally used in synchrotron injector pre-accelerators. The results of the measurements give values from 21 to 58 amp/cm^2 sterradian, the range of values corresponding to increasing voltage on the source extractor electrode up to approximately the limiting value for reliable operation. Assuming the maximum can be achieved routinely, the value of 60 amp/cm^2 sterradian for a DC instrument at 250 Kv is set as a standard of comparison.

As a cross check we note that with the above value for the density, the full width full angle product for a 50 Mev linac with 120° acceptance time and 5 ma average output during the pulse is inferred to be 1.4 milli-rad cm which is in accord with normal operating expectation for such a device.



Luminosity $L = \dfrac{I \ell^2}{a_I \, a_A}$

L_1 = 21 amp/cm^2 sterradian

L_2 = 31 amp/cm^2 sterradian

L_3 = 45 amp/cm^2 sterradian

L_4 = 58 amp/cm^2 sterradian

Focusing electrode used to approximately focus beam on Aperture A.. Note that L is independent of degree of focus so long as beam covers A.

FIG. 1: LUMINOSITY RESULTS FOR OAK RIDGE NATIONAL LABORATORY BIOLOGY DIVISION COCKCROFT-WALTON ACCELERATOR

Comparable data on cyclotron ion sources are difficult to obtain and are hence relatively scarce. (A cyclotron source is closely tailored to the characteristics of the center of a cyclotron and will operate only in closely similar environments.) The detailed studies of the central region of the Canberra cyclotron by Smith[9.] afford

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9. W. I. B. Smith, Sector Focused Cyclotrons, NAS-NRC pub. 656, 183 (1959)

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perhaps the best source of data from which estimates of the phase space density can be drawn. Figure 2 displays pertinent material from Smith's results. The experimental situation is considerably different from that of Figure 1 in that the ions are continually under the influence of focusing fields. In such a situation it is easily shown that the spot size divergence product Q is given by

$$Q = \frac{\pi}{4} \frac{\nu}{R} A(0) A(\pi/2) \tag{1}$$

where ν is focusing freq., R the radius of the orbit, A(0) and A (π/2) the full width of the beam at the reference angle (0) and at a second angle 1/4 cycle of the betatron wave length away, and where Q is then the product of full width and full divergence. It is unfortunately necessary in inferring Q's from Smith's results to refer to separate experiments to determine the radial and axial Q's respectively. Source and extractor geometries in the two situations were however identical so that the risk of error in inferring the two Q's from separate experiments appears slight.

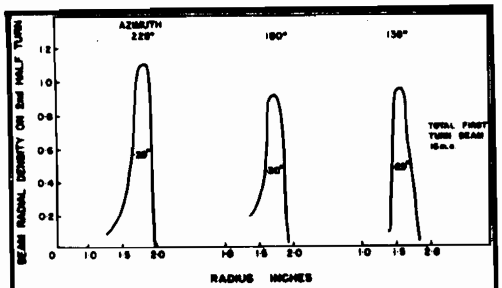

Fig. 175. Measured radial beam distribution on first turn.

(figure from Smith reference 9)

RADIAL APERTURE:

$\nu_r \simeq 1$, $R \simeq 1.7''$, $A(0) \simeq 0.29''$

$A(90) \simeq 0.29''$,

$$Q_r \simeq \frac{\pi}{4} \frac{\nu_r}{R} A(0) A(90)$$

$\simeq$ 100 milli-rad cm.

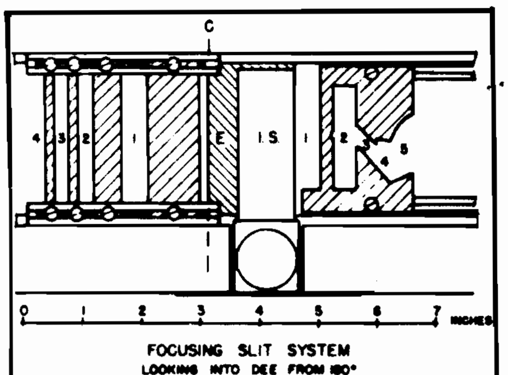

Fig. 186. Focusing grids on dee and dummy dee.

(figure from Smith reference 9)

AXIAL APERTURE:

Notches at digits 2, 3, 4, & 5 on right cut to fit beam. Notch pattern indicates $\nu_z \simeq 0.2$ and $A(90) \simeq 1$ cm.

$A(0) \approx 1$ cm. $Q_z \approx \frac{\pi}{4} \frac{\nu_R}{R} A(0) A(90)$

$\simeq$ 36 milli-rad cm.

Luminosity (150 Kev):

$$L \simeq \frac{I}{Q_r Q_z} \simeq 4.2 \text{ amp/cm}^2 \text{ sterradian}$$

Correct for 0.18 duty cycle (see text) and to 250 kev. energy equivalent:

$$L \text{ (DC, 250)} \approx 39 \text{ amp/cm}^2 \text{ sterradian}$$

FIG. 2: CYCLOTRON SOURCE LUMINOSITY INFERRED FROM DATA OF SMITH, REFERENCE 9.

(To the author's knowledge no data are available from which the two Q's can be inferred simultaneously). Since the cyclotron source is "on" for only a fraction of the time it is necessary in order to compare with the Cockcroft Walton data to correct for the duty cycle in addition to the correction for difference in momentum mentioned previously. For the duty cycle correction, Livingston and Jones[10.] report the

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10. R. S. Livingston & R. J. Jones, Rev. Sci. Inst. 25, 532 (1954)

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output of a source of the type used by Smith to be reduced by a factor 0.18 when r-f extraction voltage is substituted for D.C., other factors being held constant. Applying this correction and the p^{-2} correction gives an emittance (D.C., 250 Kv equivalent) for the Smith source of 39 amps/cm^2 sterradian.

The approximate equivalence of the result from the Smith data, and that from the Cockcroft Walton data is in accord with what would have been the most reasonable prior expectation, namely that the plasmas in the two sources are approximately equally effective as producers of collimated ions.

At the risk of belaboring the obvious it is perhaps worthwhile to note that equivalence of source emittance in no way necessarily implies equivalence in emittance in the extracted beams of Cockcroft-Walton and cyclotron. As an example, measurements on the extracted beam of the Oak Ridge 86" cyclotron gave emittance values a factor[8] of 5×10^3 lower than the Cockcroft Walton values, after correction for energy and r-f duty cycle (0.18), even though the cyclotron was equipped with a source of the same type used by Smith and with presumably comparable emittance. The result is indicative of the enormous dilution of filled phase space with unfilled which can occur in some circumstances. In the sector focused cyclotron it appears that such dilution can be avoided as is indicated in detail in sections III and IV which follow.

III. CYCLOTRON ORBIT DYNAMICS - TRANSVERSE CHARACTERISTICS

As mentioned in the introduction, this and the following section together view the cyclotron as a device for mapping the phase area of the source into a phase area of the extracted beam. In this section the transverse characteristics of the mapping are considered, by which is meant the behaviour of a phase area (i.e. beam of particles) leaving the source at a fixed instant in time and with uniform energy but with appropriate distributions in position and direction. At a later time some fraction of such an area or group of particles will have successfully traversed the cyclotron and extractor and arrived at the exterior. The objective of this section is to estimate the survival fraction and the phase space characteristics thereof.

Three distinct problems are encountered. (1) Near the center of the cyclotron, the operating point is at best extremely close to a non-linear resonance at $\nu_r = 1$, the effects of which can easily distort the phase area in a highly undesirable way. (2) In the intermediate radius region, though non-linear effects are inherently small, significant distortions can occur due to cumulative effects over the relatively large number of revolutions ($\simeq$ 90% of the total) which the particles make in the region. (3) To accomplish extraction, conventional electrostatic techniques are inadequate at energies of interest for injection, and it is therefore necessary to utilize resonant magnetic techniques which can again easily distort the phase area in an undesirable way. Each of these sources of difficulty are considered in turn.

A. Central Region Orbits.

Maxwell's equations require that the flutter field must approach zero near the center of a sector focused cyclotron as $(r/g)^N$ where g is proportional to the magnet gap, r is the distance from the center, and N is the number of sectors. As a consequence a region results in which axial focusing is more or less marginal depending on the number of sectors. To alleviate this difficulty the azimuthal average of the field, $\langle B \rangle$, is often designed to decrease with radius near the center to supplement the axial focusing; at the point where the sectors reach a strength sufficient to provide the desired focusing, $\langle B \rangle$ reverts to an increasing function of radius as is required for isochronism, and in the transition the radial tune passes through $\nu_r = 1$ resonance. If N = 3 the flutter rises sufficiently rapidly to also allow consideration of designs in which $\langle B \rangle$ is set for isochronism all the way to the center, and so no resonance transition is encountered.

An average field decreasing with radius has been successfully employed in the central region of the four sector Delft[11.] and Illinois cyclotrons[12.]

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11. Heyn and Khoe Kong Tat, Proc. Sea Island Conf. NAS-NRC 656. 29 (1959).
12. Allen, Chatterjee, Ernest and Yavin, R. S. I. 31, 813 (1960).

The operation, unfortunately, gives little detailed insight into the extent of possible distortions of the radial phase area in the transition through $\nu_r = 4/4$, but at least it is indicated that the transition is not disastrous.

For the three sector case, model magnet and computer studies of both field fall off[13.] and isochronus $\langle B \rangle$[14.] designs have

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13. J. E. Stover, Michigan State Univ. Master's thesis, 1960 (unpublished).
14. M. M. Gordon and H. G. Blosser, manuscript in preparation.

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been made at Michigan State. Results obtained show that for either type of field, with adequate accelerating voltage, source locations can be found such that the x, p_x, y, p_y, area from the source maps into the intermediate radius region of the cyclotron with negligible distortion. Field fall off designs would perhaps be preferred in an injector cyclotron since they result in acceptance of a somewhat larger axial phase area; we therefore consider fields of this type in further detail.

Typical performance to be expected are illustrated in Figures 3-6, which are results from ref. 13.

Fig. 3 shows the $\langle B \rangle$ employed in the studies, along with the corresponding radial and axial tunes, the tune data beginning at the radius of the first half turn for an assumed accelerating voltage of 280 Kv per turn. The strong ν_z, in conjunction with a relative absence of axial non-linearities, implies at once that the axial motion will be free of appreciable distortion. The large ν_z in addition enhances both the axial phase space acceptance given by Eq. (1), and the axial space charge limit, the latter given in the absence of neutralization by the approximate expression

$$I_{lim.} = A \epsilon_o \omega_o \nu_z^2 (\Delta\phi/2\pi)(\Delta E/e) \qquad (2)$$

where A is the full axial height of the beam, ϵ_o the permittivity of free space, ω_o the particles' rotational angular velocity, e the particles' charge, ΔE the energy gain per turn and $\Delta \phi$ the azimuthal spread of the accelerated beam, all in MKS units. The effect on the space charge limit is particularly important in view of the quadratic dependence of the limit on ν_z and the fact that the output of present cyclotrons appears from Eq. (2) to be directly determined by the space charge limit (increasing the limit should therefore directly increase the accelerated current as compared with presently operating cyclotrons).

Fig. 4 is a plot of stability limits for the radial motion at successive energy values corresponding approximately to successive full revolutions of the accelerated particles. The expected phase area of the beam from the source is shown at the bottom of the figure;

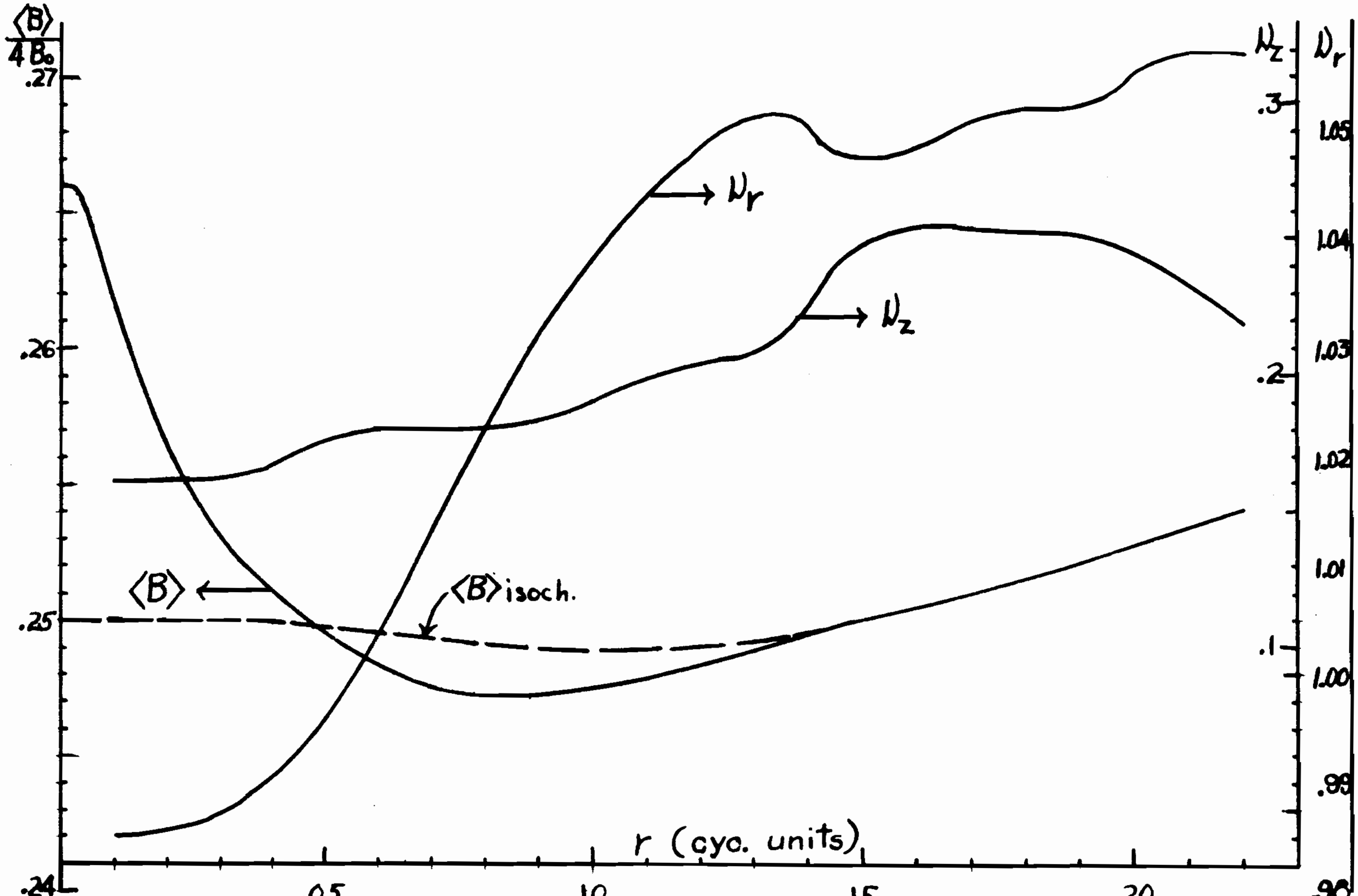

FIG. 3: CENTRAL REGION AVERAGE FIELD AND RESULTING TUNES USED IN STUDIES OF REFERENCE 13. FOR COMPARISON ISOCHRONUS AVERAGE FIELD IS ALSO SHOWN.

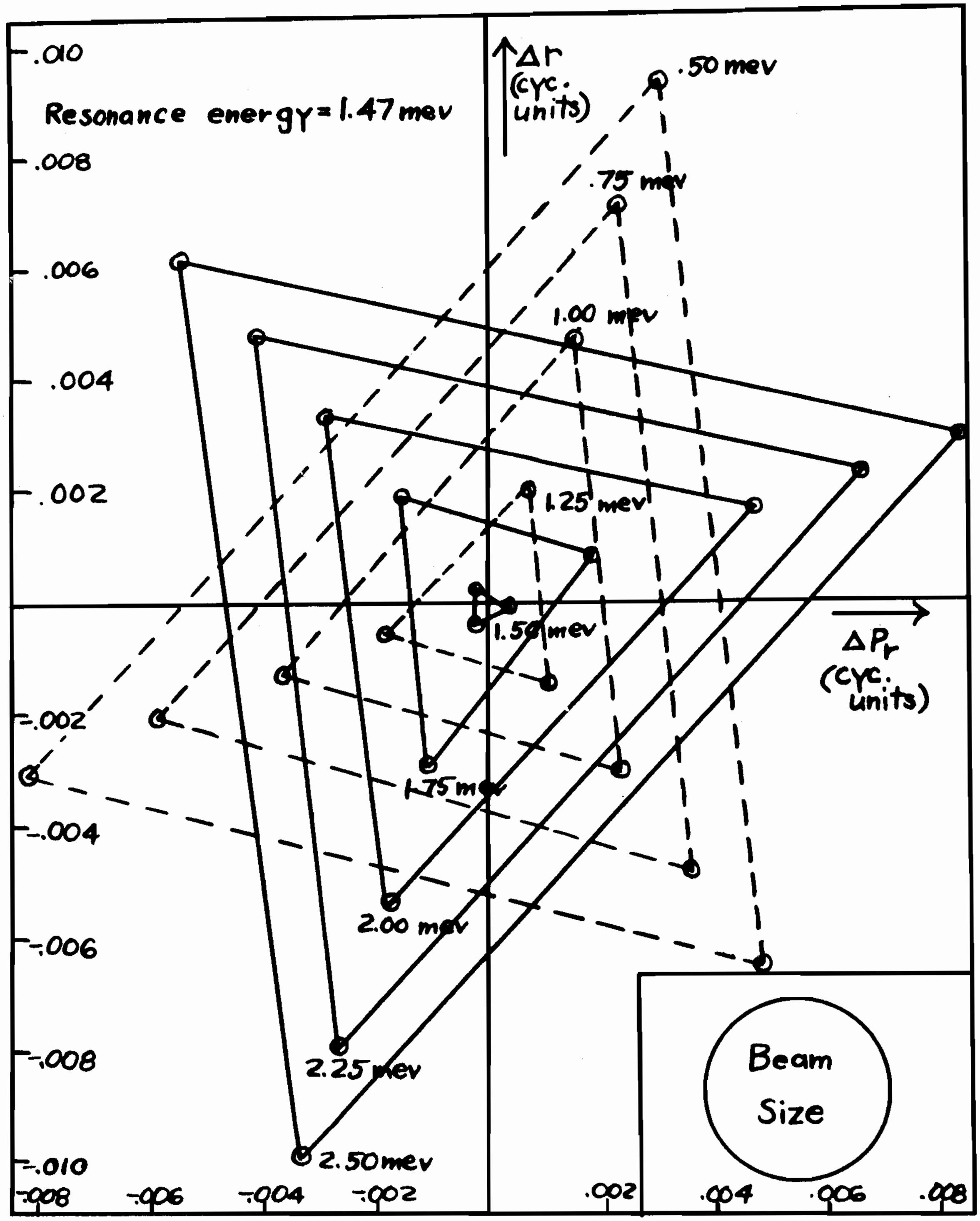

FIG. 4: RADIAL STABILITY LIMITS AT A SEQUENCE OF ENERGIES FOR FIELD OF FIG. 3 (FROM REFERENCE 13). STABILITY LIMIT POSITION INFERRED FROM TRACKING OF UNSTABLE FIXED POINTS

for several turns in the vicinity of the resonance, a substantial fraction of the beam must be outside the stability limit. The limited number of turns which must be accomplished in the unstable region in combination with the slow motion of the phase fluid illustrated in Fig. 5 results however, in a tolerable situation as can be seen in Fig. 6 which shows successive snapshots of a beam-sized array of particles accelerated upward from the source through the resonance and well into the intermediate radius region of the cyclotron. The geometry of the array of particles is well preserved, which implies transmission of the beam phase area without introduction of significant distortion.

The results shown in Figs. 3-6 will clearly be quite sensitive to reduction of the accelerating voltage, as is currently being explored in further detail. The 280 kv/turn used in the studies is however easily achievable in a fixed energy injector type cyclotron; hence for this application the results establish that a field of the type shown will allow acceleration of particles out of the central region with negligible distortion of beam optics.

In view of the described computational results for 3 sector fields, the successful operation of the Delft and Illinois cyclotrons, and the fact that $\nu_r = 4/4$ is a considerably less severe resonance than $\nu_r = 3/3$, it is reasonable to conclude that severe distortion of the phase area in the central region can be avoided for any of the normally considered choices of magnet geometry.

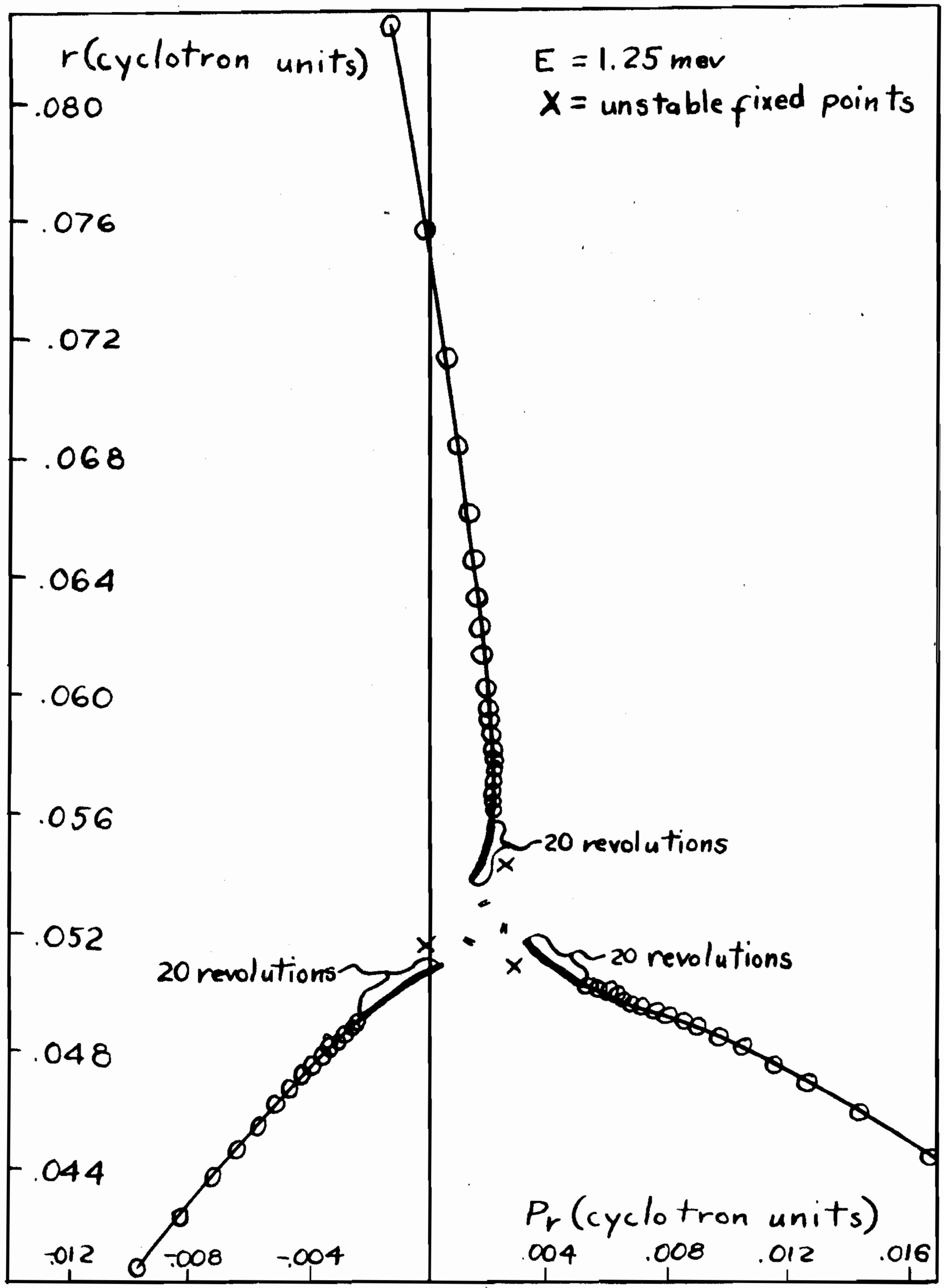

FIG. 5: STATIC PHASE PLOT AT 1.25 MEV FOR FIELD OF FIG. 3 (FROM REFERENCE 13).

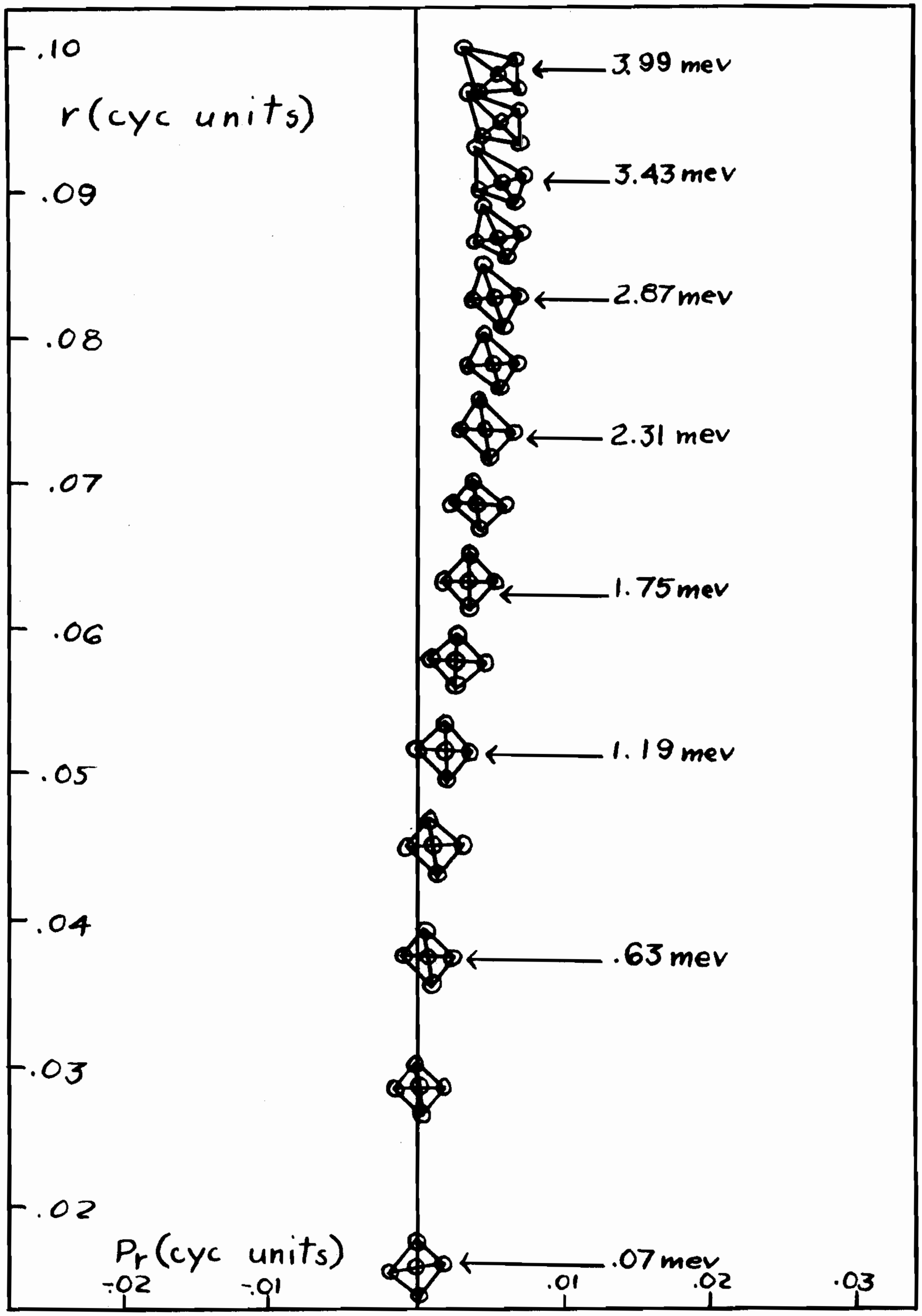

FIG. 6: RADIAL PHASE PLOT OF BEAM SIZED GROUP OF PARTICLES ACCELERATED FROM SOURCE IN FIELD OF FIG. 3 WITH ENERGY GAIN OF 280 KEV PER TURN (FROM REFERENCE 13)

B. Intermediate Radius Region Orbits

Orbit problems in the intermediate radius region of the cyclotron are minor in comparison with either the central region or the extraction region. Stability limits are much larger than the beam size, the radial stability limit for example typically corresponding to a factor of 10 to 40 larger amplitude (depending on the amount of spiral employed) than the amplitude expected from the source. One aspect of the motion in this region does however merit careful consideration. In the results shown in Fig. 6 for example, it was found that the beam area at the final energy was considerably displaced from the equilibrium orbit, or, in the usual terminology, a coherent betatron amplitude of approximately 20mm was superimposed on the random distribution of amplitudes from the source. Such a coherent amplitude is apparently a typical result of the optimum central region solution. Moreover, the substantial radial separation of turns in this region of the cyclotron implies that, averaged over a group of turns, there will always be an amount of coherent amplitude at least equal to half the radial separation of equilibrium orbits for successive half turns.[15.]

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15. One of the authors (H.G.B.) is indebted to Dr. K. M. Terwilliger for some very helpful discussions of this point.

In the intermediate radius region such a coherent amplitude could result in extensive distortion if the phase fluid rotation rate, given by ν_r varies appreciably with amplitude. The effect is sketched in Fig. 7, where the rotation rate is assumed to decrease with amplitude. Large amplitude particles rotate slower, hence lag behind those of somewhat smaller amplitude and stretch the phase figure into a long contorted spiral. If coherent amplitudes are present sufficient to place the beam near the stability limit for a moderately large number of revolutions, distortions of the sort shown in Fig. 7 will always result.

A limit on the amount of coherent amplitude tolerable without introduction of appreciable distortions may be estimated from the data of Fig. 8, which are numerical results for the shift in frequency with amplitude for the Michigan State magnet design. The shift in frequency is seen to be extremely small out to a relatively large fraction of the total stable amplitude. As a consequence, relatively large coherent amplitudes can be tolerated. As an example, let us arbitrarily define "excessive distortion" to be a precessional difference of 1/25 of 2π between inside and outside of the beam spot and compute the resulting limit on coherent amplitude in the intermediate radius region of Michigan State type magnets. For such a magnet stability limits are typically $\simeq$ 100mm, and making use of the data of Fig. 8, it is found that a source spot of 7.5 mm diameter (the expected radial phase area inferred from ref. 9) circulating for 200 turns can have a coherent amplitude of up to 40% of the limiting value (i.e. 40mm) before

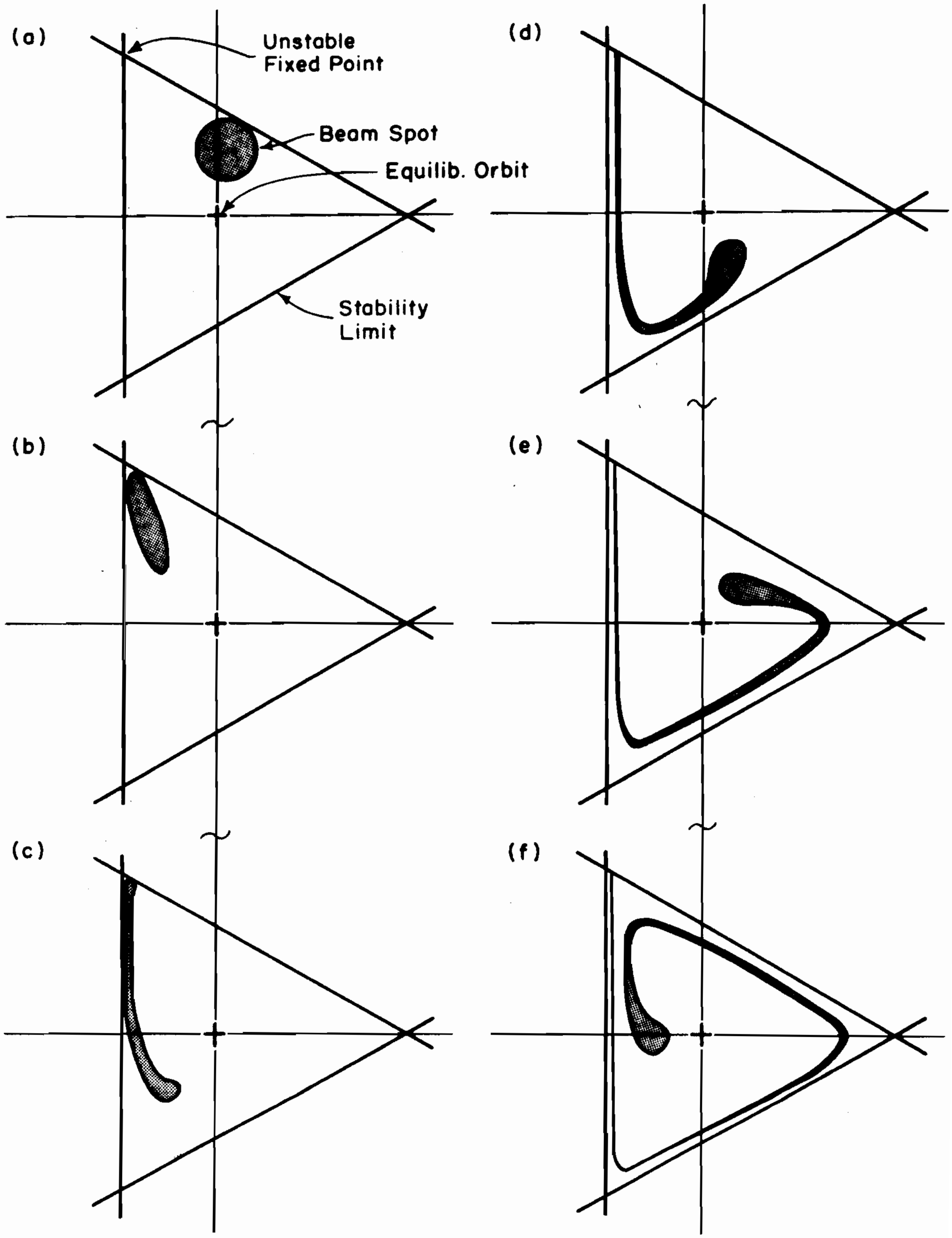

FIG. 7: SCHEMATIC RADIAL PHASE PLOTS ILLUSTRATING DISTORTION EFFECT TO BE EXPECTED IF BEAM SPOT CIRCULATES IN VICINITY OF STABILITY LIMIT FOR SIZABLE # OF REVOLUTIONS.

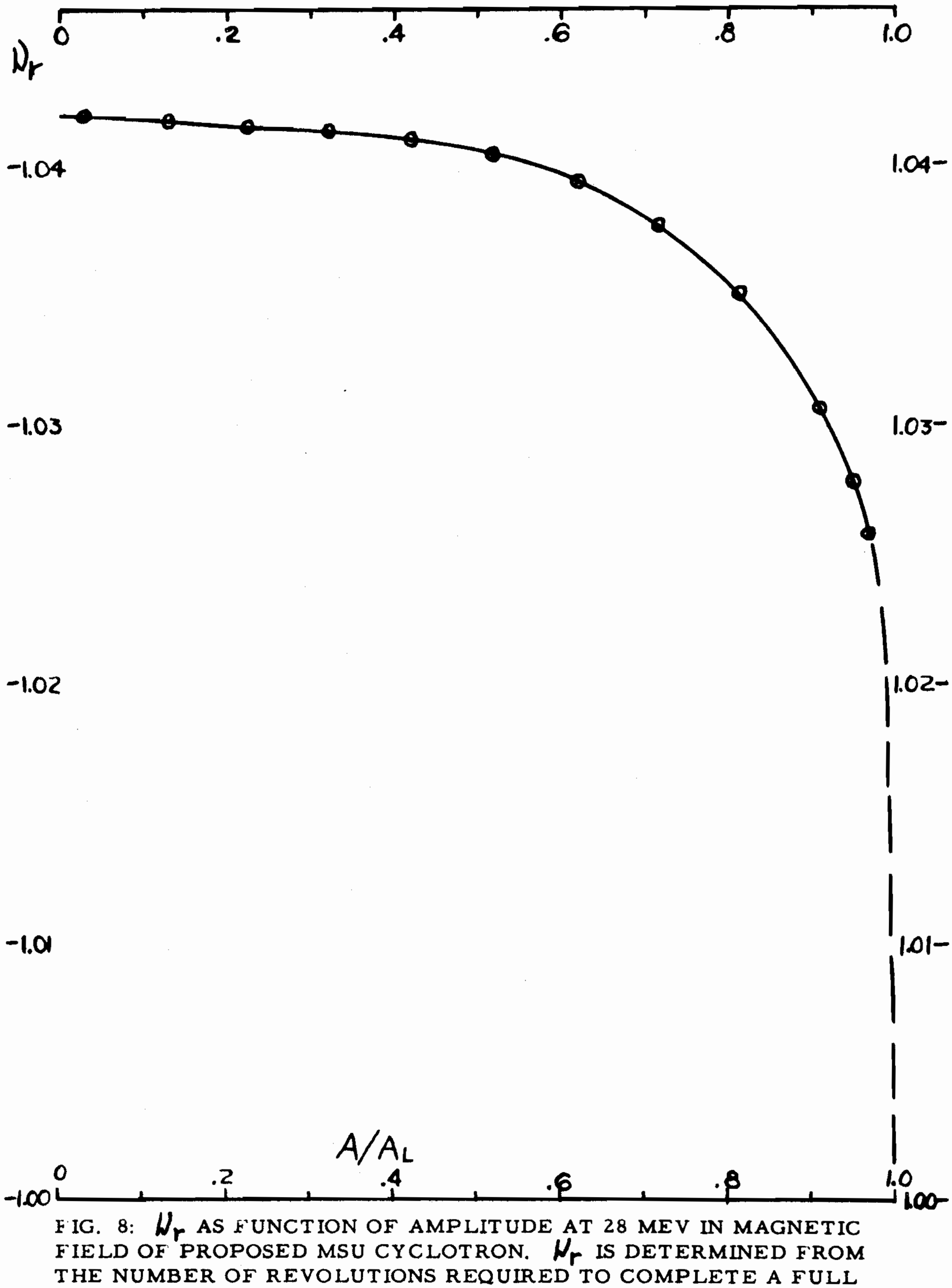

FIG. 8: ν_r AS FUNCTION OF AMPLITUDE AT 28 MEV IN MAGNETIC FIELD OF PROPOSED MSU CYCLOTRON. ν_r IS DETERMINED FROM THE NUMBER OF REVOLUTIONS REQUIRED TO COMPLETE A FULL PRECESSIONAL CYCLE ON THE PHASE DIAGRAM.

distortion becomes "excessive". Such a value for the coherent amplitude is well beyond the range which arises or is desirable in customary design situations. The results therefore indicate that the beam phase area will map through the intermediate radius region with negligible distortion.

C. Extraction Region Orbits

The cyclotron has with due cause earned a reputation as an efficient, straight forward device for producing intense internal beams; with equally valid cause it has earned a reputation as a poor device for producing well-collimated external beams. Typically 80 to 95% of the internal beam is lost in the extraction process and in addition the surviving beam tends to emerge in a broad fan. Manifestly, major improvement in the effectiveness of the extraction process is required if a cyclotron is to function as an effective injector.

Low energy cyclotrons normally accomplish beam extraction by using an electrostatic channel, the region of strong electric field being separated from the region of normal orbits by a thin electrode called a septum. In principle the septum is placed between successive turns so that particles on one turn move just inside the septum and on the next just outside and the electric field is made sufficiently strong to bend the latter turn out of the cyclotron. As the energy of the cyclotron is raised several effects act to markedly reduce the effectiveness of such devices.

(1) The radial separation of successive equilibrium orbits decreases, due to the r^2 dependence of the energy and quickly becomes comparable with the thickness of feasible septums; more particles then strike the septum, reducing the extraction efficiency and enhancing the engineering problems of septum design. (2) The radial separation of successive equilibrium orbits becomes small with respect to the spread in radius of the particles from a particular turn; particles from a number of turns with a corresponding broad distribution in energy enter the channel. (3) As the velocity increases, successively stronger electric fields are required to obtain a given angular deflection (the electric force must balance a given fraction of the magnetic force and the magnetic force increases linearly with v); the required field strengths quickly approach technological limits.

The first of the above difficulties is set purely by the turn separation and may be alleviated by introduction of features to appropriately magnify the separation. The second is related but somewhat different in that magnification of the separation without corresponding magnification of the radial spread of a given turn is required. The third is basically also a function of turn separation in that the choice of E field is to begin with dictated by small turn separation (electrodes can be made thinner than magnetic shields); if the turn separation is adequate, magnetic shields can be employed at all energies, hence eliminating the velocity difficulty.

The Tuck, Teng, LeCouteur extraction system[16.] normally

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16. J. L. Tuck and L. C. Teng, Univ. of Chicago 170 in. Synchrocyclotron Progress Report III, Chap. VIII (1950).

    J. L. Tuck and L. C. Teng, Phys. Rev. 81, 305 (1951)

    K. J. LeCouteur, Proc. Roy. Soc. (London) B64, 1073 (1951)

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employed in high energy synchrocyclotrons amplifies orbit separation by introducing magnetic field perturbations of a type such as to reasonantly excite the radial betatron oscillation. The usual perturbation strengthens the field over a narrow angular region near the maximum energy orbit and in some cases an accompanying weakened field region is introduced 60° to 90° away. Fourier analysis of the perturbation results in a variety of harmonics; in most cases the $\cos\theta$ and $\cos 2\theta$ components appear to be the dominant perturbing factors in the motion. Fig. 9 is a series of sketches depicting qualitatively the features of radial phase plots at a sequence of energies near the maximum energy.[17.]

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17. See for example, N. F. Verster, Proc. of Sea Island Conf. NAS-NRC 656, (1959), p. 199.

The initially large stable region corresponding to centered orbits shrinks under the influence of the perturbation, finally disappearing completely. The shaded area in the figure depicts the behavior of a group of particles being slowly accelerated. As the particle energy increases the separatrix defining the stable region pulls gradually

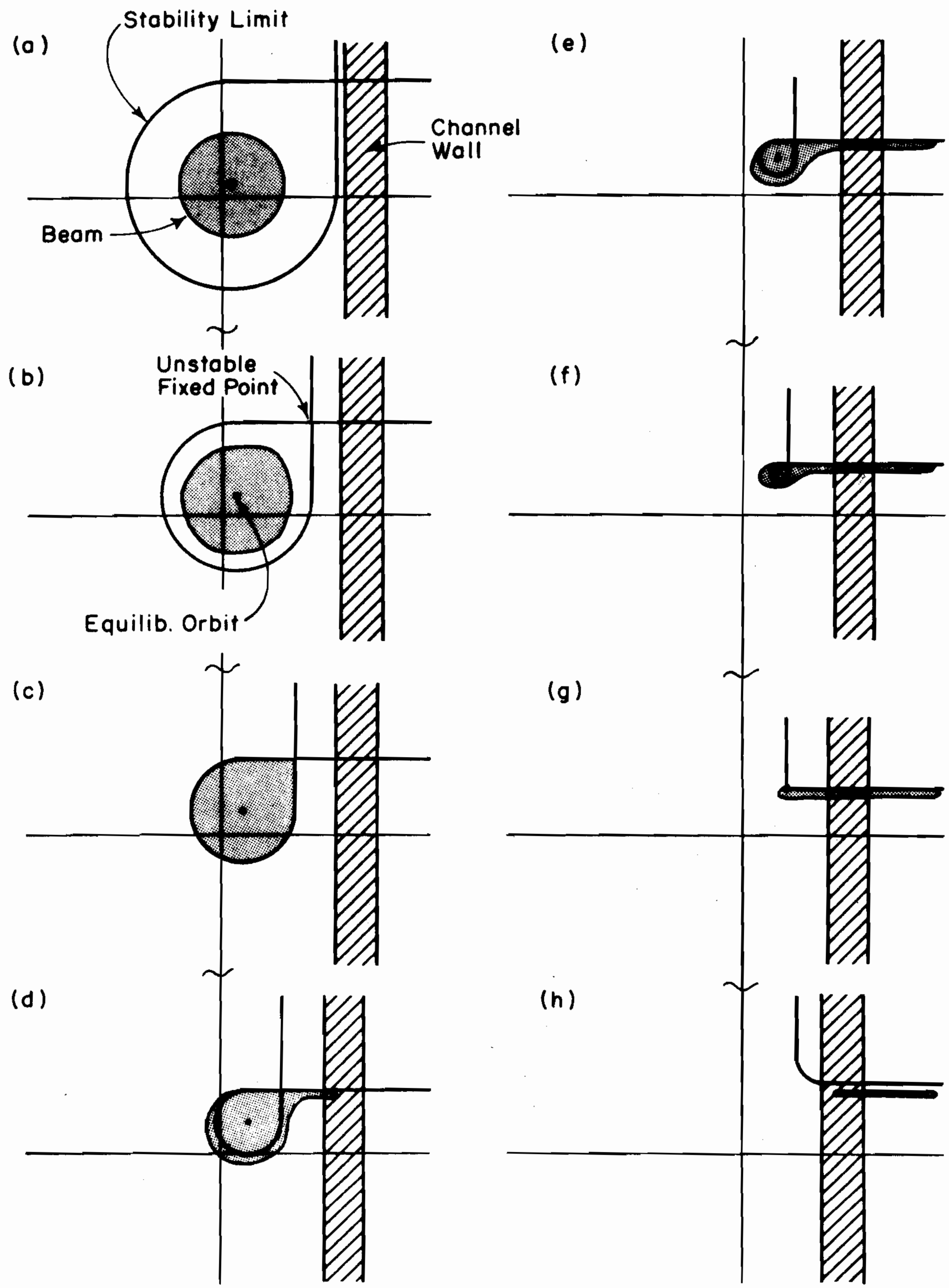

FIG. 9: SCHEMATIC RADIAL PHASE PLOTS ILLUSTRATING OPERATION OF SYNCHROCYCLOTRON REGENERATIVE EXTRACTORS.

in through the beam spot, dumping particles into the unstable region; the particles in the unstable region move out along the asymptote until they reach the channel radius, at which point they either enter the channel or are lost depending on whether their position coincides with the channel aperture. Viewed as a mechanism for getting particles into a channel, the procedure is quite effective. If the inner wall of the channel has radial width a, and the channel aperature radial width b, channel entry efficiencies of up to $\frac{b}{a + b}$ can be achieved, the maximum efficiency occuring if the channel is placed at a radius where the growth in radial oscillation amplitude per turn is just a + b. Entry efficiencies of 50% or more are often achieved.

Unfortunately, other aspects of the process are not similarly effective. First of all, particles entering the magnetic channel will have an energy, spread corresponding approximately to the energy difference between sketches (c) and (g) of Fig. 9, i.e. between the energy at which the stable region is just equal to the beam spot area and the energy at which the stable region has just vanished. In addition to the energy spread itself, the asymptote position shifts in position on the phase diagram as the stable region shrinks, thereby imparting an additional spread in angle (coherent with the energy spread) into the beam entering the channel.

The combination of energy spread, angular spread, and a distorted object shape combine to make design of the channel an extremely difficult problem.

The customary channel design appears in fact to be a rather highly selective energy monochrometer; the result is that most beam is lost in the channel but the output beam is highly monochromatic. Typically 50% of the internal beam will enter the channel and of this, approximately 1/10 or 5% of the original beam, will be transmitted. The beam entering the channel will have an energy spread of about 1% whereas the exit beam will have spread of only a few tenths of a percent.

For sector focused cyclotrons, a situation exactly analogous to conventional cyclotrons is found; at low energies an electrostatic peeler can be effectively employed, but as the energy is increased the same difficulties listed previously are encountered. In a sector focused cyclotron the operating point is, however, never far from some linear or non-linear resonance and hence the first and third of the aforementioned difficulties can be avoided with almost trivial ease by taking the operating point onto a resonance. Oscillations will be excited sufficient to swing the beam into a magnetic channel in a manner quite analogous to the Tuck, Teng, LeCouteur system.

There is, however, one crucially important difference between the sector focused and synchrocyclotron. The sector focused cyclotron is a fixed frequency device and can hence have an r-f system with enormously higher volts/turn than can be achieved in the synchrocyclotron. Rather than slowly shrinking the stable area thru the beam giving the difusing effect shown in Fig. 9, it is possible (as is shown in more detail in following paragraphs), with the large energy gain to effectively snap the stable region instantaneously out from under the beam whereupon the beam walks as a spot out to the channel radius. Since the beam is a spot rather than a streamer, 100% channel entrance efficiency is easily achievable.[18.] In addition, the energy spread of the beam entering

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18. It is perhaps worthwhile to recall to the reader's attention that throughout this section we are discussing a group of particles leaving the source at fixed time and with no energy spread. While 100% efficiency is achievable for such particles, a considerable reduction in efficiency will result when averaged over time as is discussed in Section IV.

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the channel should be extremely small and the angular divergence essentially no larger than the damped divergence from the source. With such a beam, the problem of channel design is almost trivially easy compared with the synchro-cyclotron case and it is therefore quite probable that 100% channel transmission could also be achieved.

The principal features of such a system, as applied to the 3 sector 40 Mev cyclotron, are sketched qualitatively in Figs. 10 and 11. In this case the most convenient resonance to employ is at $\nu_r = 1 (=3/3)$ which is reached of necessity,[19.] as shown in Fig. 10 when the

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19. $\nu_r = 1$ will of course always occur near the edge of any isochronous cyclotron at the transition from increasing to

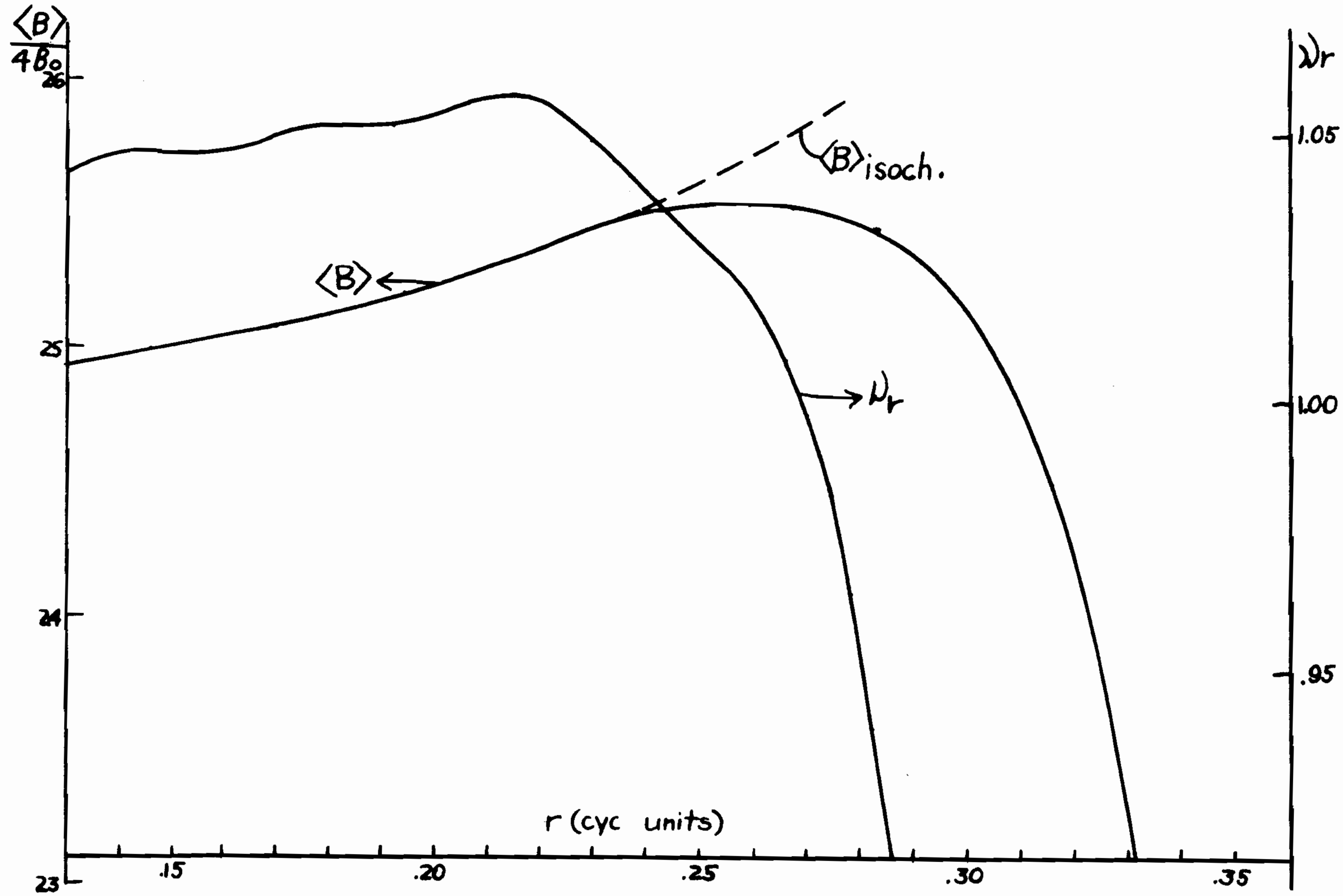

FIG. 10: EXTRACTION REGION AVERAGE FIELD (AND RESULTING RADIAL TUNE) USED IN MSU EXTRACTION STUDIES.

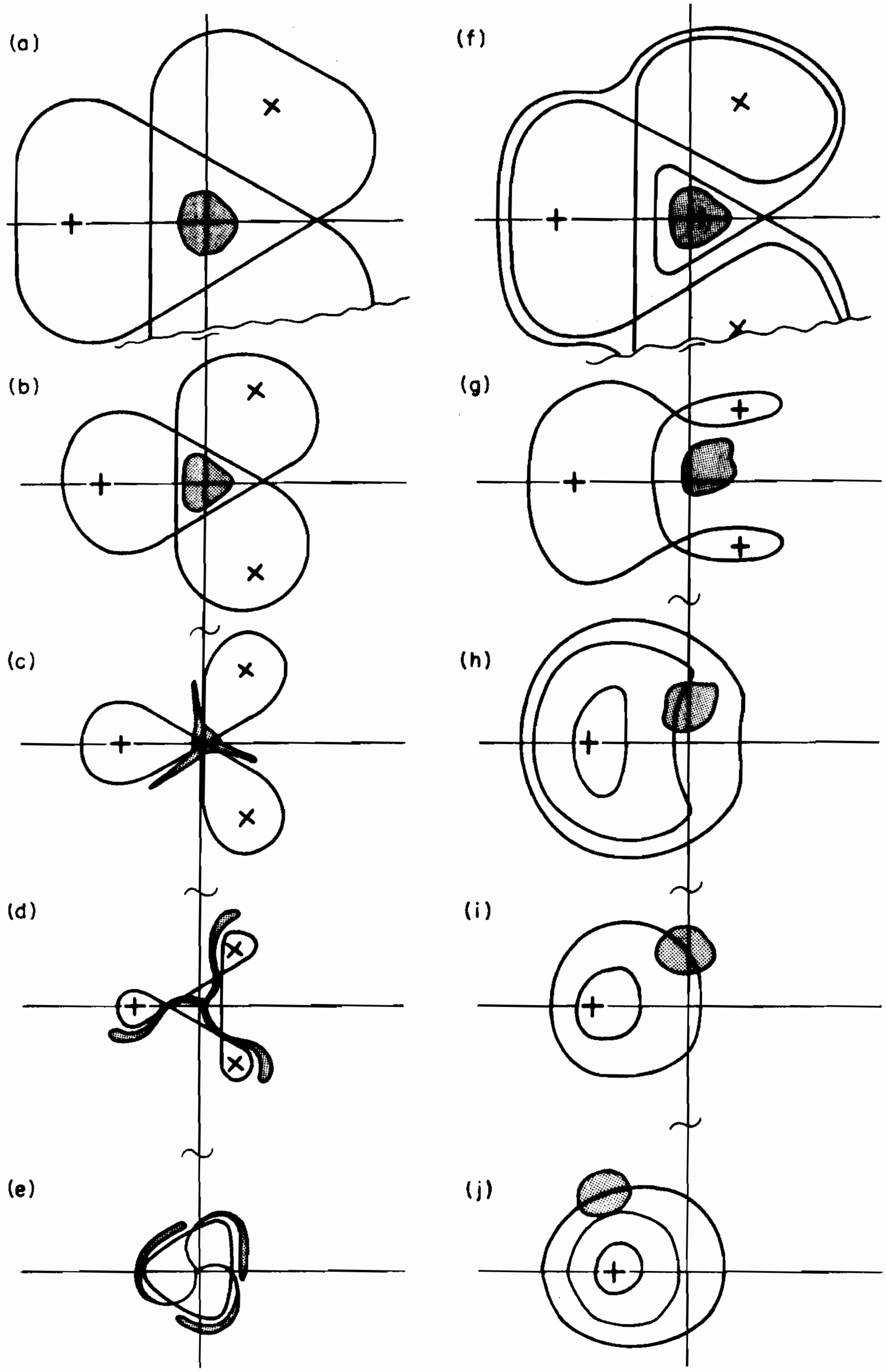

FIG. 11: SCHEMATIC RADIAL PHASE PLOTS ILLUSTRATING BEHAVIOUR AT ν_r = 3/3 WITHOUT (LEFT) AND WITH (RIGHT) COS θ FIELD PERTURBATION. SEQUENCE FROM TOP TO BOTTOM DEPICTS TRANSITION FROM ABOVE TO BELOW RESONANCE.

decreasing $\langle B \rangle$ vs. radius. Immediately preceding the resonance is a region of non-isochronous orbits and the accelerating voltage must hence be high enough to limit the phase slip to an acceptable value. In view of this it may be more convenient in many circumstances to use a different resonance, a likely choice being simply the resonance closest to the isochronous tune at the maximum energy.

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isochronous field region terminates and changes into the usual edge region fall off. The left hand side of Fig. 11 depicts the behaviour of the phase space as the resonance is traversed assuming a pure 3 sector field. If the beam spot were approximately on center prior to the resonance, the beam would diffuse out in 3 places as shown by the gray area in the figure, the non-linearity giving an undesirable stretching effect quite similar to the stretching caused by the slow acceleration in Fig. 9.

If a field component with cos Θ angular dependence is included, a number of helpful changes occur as are indicated at the right of Fig. 11. Pairs of fixed points move together and vanish corresponding to the disappearance of the original center stable region followed by two of the outer stable regions leaving, as shown in 7 (j), a phase plot corresponding to approximately linear motion about a displaced stable fixed point. If the accelerating voltage is high enough to shift the particle energy from (f) to (i) of Fig. 7 in a few turns, the beam effectively jumps from linear motion about a nearby fixed point to linear motion about a considerably distant fixed point, hence yielding turn separation of magnitude determined by the amplitude and effective ν_r of the oscillation about the final fixed point and with a minimum of distortion, since for almost the entire process the beam spot is in a reasonably linear region. The gray spot in the right hand sketches of Fig. 11 indicates qualitatively the behaviour of a beam spot in such circumstances.

The rapidity of the transition from 7 (f) to 7 (i) is approximately proportional to d ν_r/dn, the change in ν_r per turn in the extraction region. To enhance d ν_r/dn requires high volts (large change in E per turn) and/or a small magnet gap (small gap allows large d ν_r/dE).

Actual beam behaviour in such a process is illustrated in Fig. 12. The figure shows results of orbit integrations in which a grid of particles (again in the magnetic field of the proposed 3 sector 40 Mev MSU cyclotron) are accelerated into the resonance with an energy gain of 280 Kev/turn. The grid is initially positioned near the center of the stable region of the r, p_r phase space as can be seen in Fig. 13 which shows the initial grid on its static phase plot as well as corresponding displays at the end of 1st and 2nd full turns. At the end of the third and following turns no stable region exists. The second turn is then the only one on which the grid overlaps the stability boundary and even on this turn the overlap is on a well behaved portion of the boundary, well removed from the proximity of the unstable fixed point. Referring again to Fig. 12 it is seen that on the initial turns the grid positions fairly fully superimpose corresponding to motion about a nearby stable fixed point (for clarity results from the

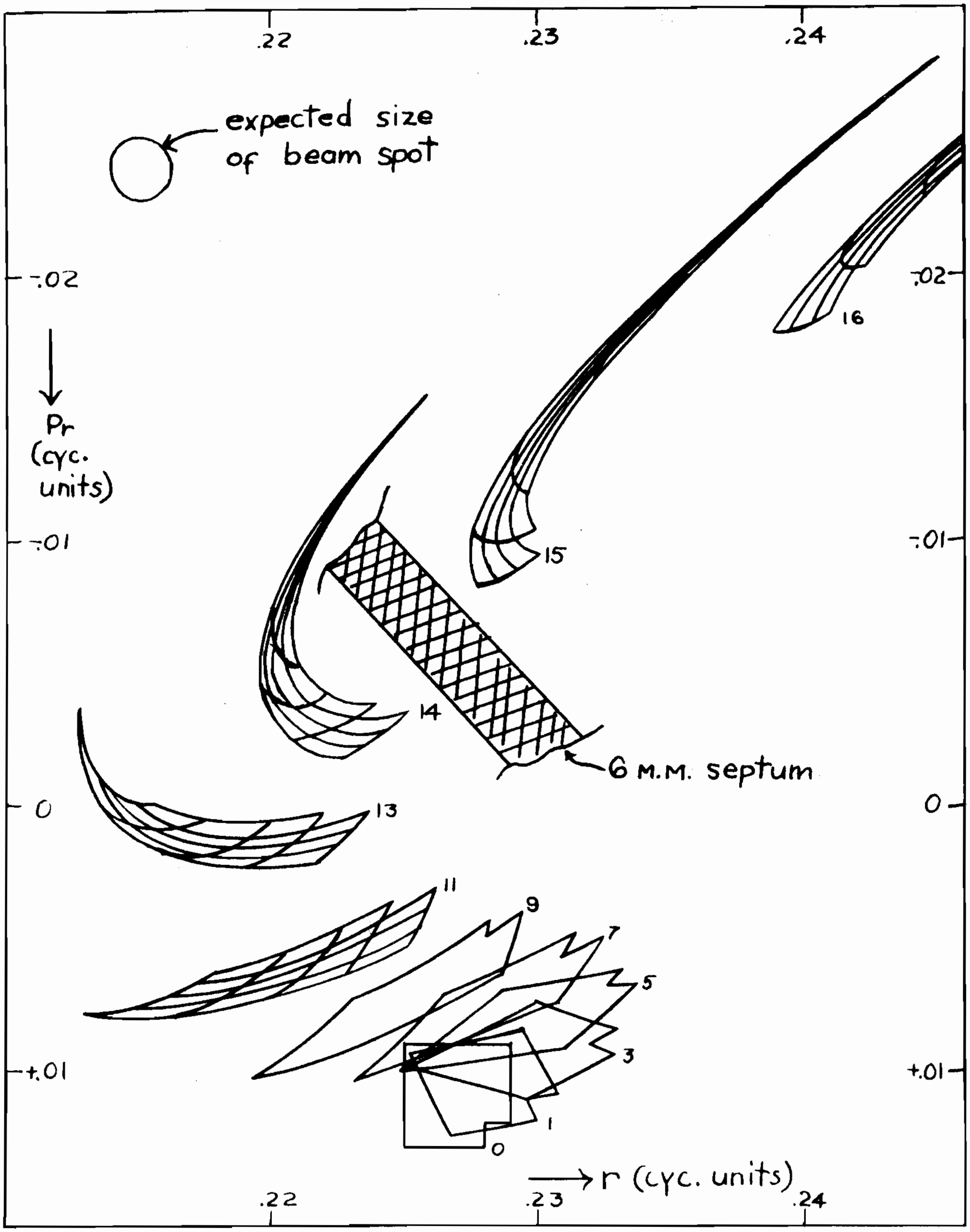

FIG. 12: COMPUTATIONAL RESULTS FOR GRID OF PARTICLES ACCELERATED WITH 280 KEV/TURN FROM INITIAL ENERGY OF 30 MEV IN FIELD OF PROPOSED MSU CYCLOTRON. FIELD HAS COSθ COMPONENT OF AMPLITUDE 1%. CIRCLE AT UPPER LEFT CORRESPONDS TO RADIAL PHASE ACCEPTANCE OF 76 MILLI-RADIAN CM AT 70 KEV

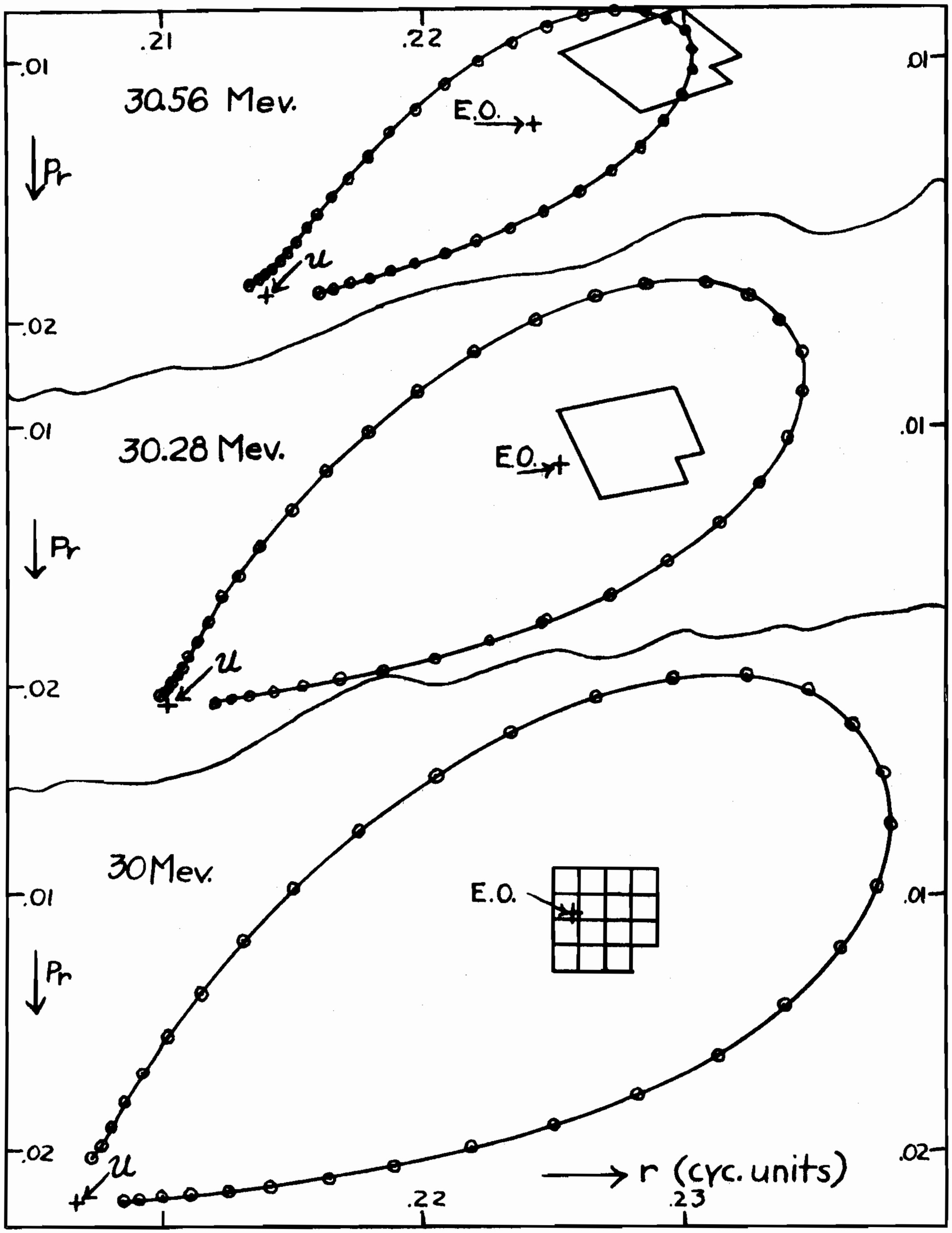

FIG. 13: GRIDS FROM ZEROTH FIRST AND SECOND TURNS OF FIG. 12 SUPERIMPOSED ON CORRESPONDING STATIC PHASE PLOTS.

2nd, 4th, 6th, 8th, 10th and 12th turns have not been plotted and, in addition, on the early turns only the outline of the grid is shown.) After the normal stable region has vanished the grid moves off, corresponding to motion about a considerably displaced stable fixed point. The separation of the 14th and 15th turns is 15mm, that of the 15th and 16th 35mm. With careful design a magnetic septum could probably be inserted between the 14th and 15th turns. The cross hatched area on the figure corresponds approximately for example to a 6mm thick septum.[20.] Between the 15th and 16th turns a 20mm thick septum could easily be inserted. In either case, since the turn

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20. The optimum azimuth for the septum is clearly not that at which the phase plot in Fig. 12 is made. The slanting of the septum area in the phase space arises from projecting the septum area from the optimum septum azimuth to the azimuth of the phase plot.

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separation is considerably larger than the septum width, 100% channel entry efficiency would result for the particles here considered.

The grid as a whole is seen to distort rather severely, particularly on the later turns where the separation is large. Sub-areas of the grid are, however, much better behaved. The circle at the upper left of Fig. 12 corresponds to a beam with spot size-divergence at the source (70 kev) of 76 milli-radian cm. (This area is somewhat smaller than the area from an unapertured source which from Fig. 2 would be 190 milli-radian cm at this energy, but also somewhat larger than the area which would probably be used in most actual designs due to space charge effects, as will be discussed in Sec. V.) Sub areas of the grid comparable in size to the beam spot are seen to have much less distortion than the grid as a whole. The area in the vicinity of the notched corner is particularly well behaved. If a septum were successfully fitted between the 14th and 15th turns, distortion of a beam of area corresponding to the circle in the figure would be negligible, when considered from the practical viewpoint of design of subsequent optical elements.

The orbit integrations in Fig. 12 and 13 start with the beam centered in the phase space whereas in subsections A and B of this section it was indicated that the beam spot was likely to have a coherent amplitude as it reaches the extraction region. Further numerical studies indicate actually that a properly phased coherent amplitude eases the distortion problem. In addition to possible coherent amplitudes arising in the central region, a controlled coherent amplitude of up to $\simeq$ 50 mm may be induced in the intermediate radius region by proper manipulation of the radial profile of the cos θ magnetic field component. In view of these factors the results of Fig. 12 may be regarded as a minimal performance expectation.

As in the Tuck, Teng, LeCouteur extraction process, a considerable tendency toward axial instability is found in extraction processes of the type considered here. The origin of this instability is quite easy to understand as are the steps necessary to alleviate the difficulty. In the extraction process the beam is moving off center rather rapidly, particularly in the final two or three revolutions and hence on one side of the magnet successive orbits move further and further into the strong field fall off at the edge of the magnet. The fall off gives a strong axial focusing impulse to the particles, much stronger in fact than any other axial force encountered on the revolution. In the extreme situation axial motion of the type shown in Fig. 14 often results; the particles are sharply focused once per revolution, crossing the median plane shortly thereafter and proceeding to drift off the opposite side of the median plane for the remainder of the revolution until they again return to the focusing region whereupon the cycle is repeated. The motion repeats once each 2 revolutions and, since $\nu_r=1$, the amplitude buildup corresponds to the $\nu_r = 2\nu_z$ coupling resonance.

To alleviate the difficulty it is necessary to reduce the field fall-off in the vicinity of the protruding orbits. Two rather different approaches can be employed in the design of magnet structures to accomplish this objective. Simplest from the magnet design viewpoint is to extend the field on one side of the magnet via a cam-like structure designed to give a gentle slope to the field for a distance of 15 or so cm beyond the main field, such that the focusing impulse in the region is weak enough not to cause trouble. A more complicated but probably more economical technique is to attempt "terracing" of the edge of the field in the vicinity of the last two turns by bringing iron close to the median plane. The last two turns are sufficiently separated from previous turns and from each other to make such terracing feasible and in most circumstances these are in addition the only turns on which the instability is a serious problem. The dees can terminate at an earlier radius since acceleration is not really important on the last several turns, thereby permitting iron to be brought close to the beam space, as is required to accomplish the terracing. While neither the cam-sided nor the

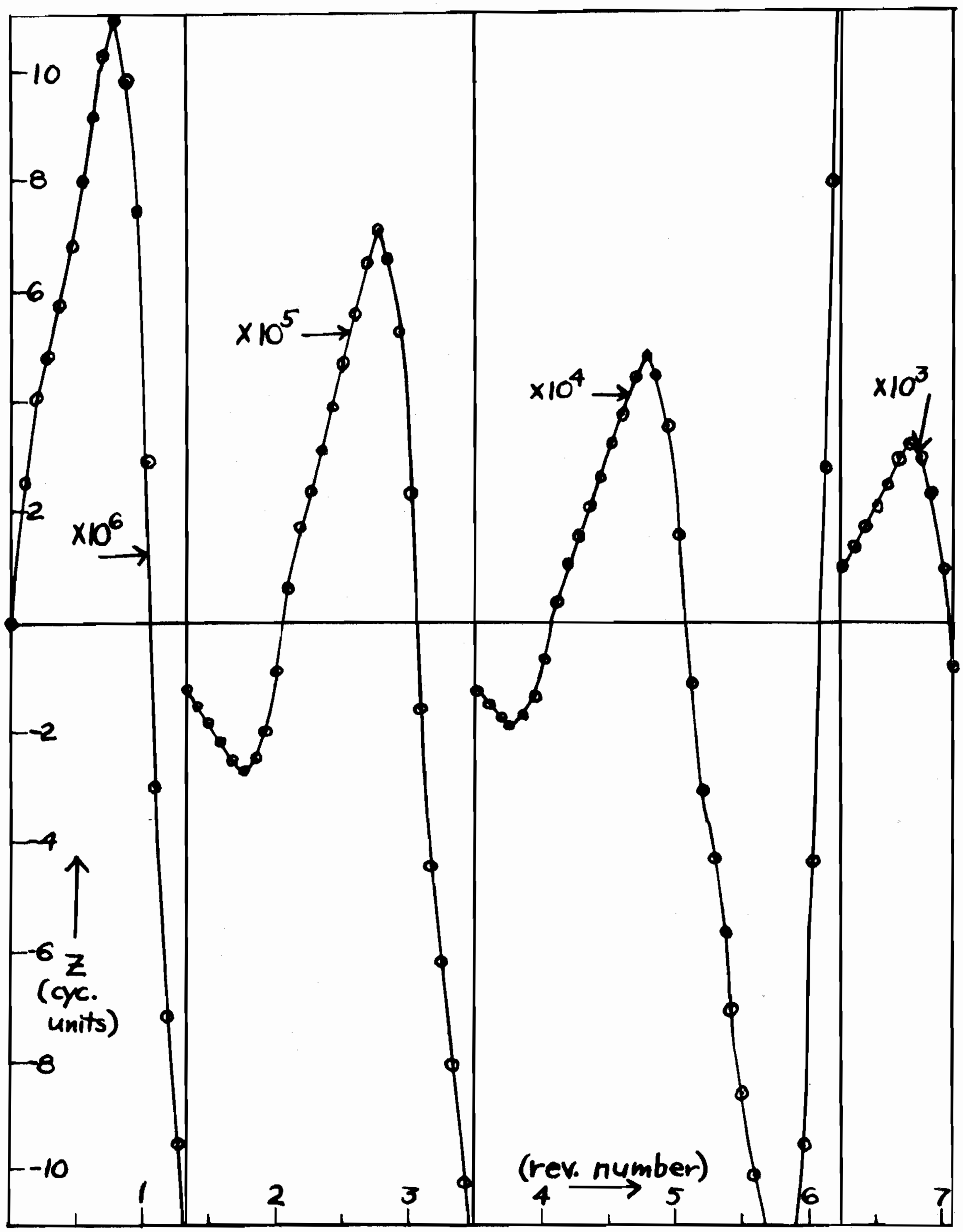

FIG. 14: VERTICAL MOTION OF BEAM IN THE VICINITY OF $\omega_r = 2\,\omega_z$ RESONANCE.

terraced field have been developed in detail at the present time, both are sufficiently straightforward as to make it extremely likely that at least one can be developed, and probably both, without undue difficulty.

The problem of magnetic channel design for such a beam, while also not yet treated in detail, should clearly be vastly easier than the comparable synchrocyclotron problem. Since by assumption the beam has left the source at fixed time and with fixed energy the only energy spread comes from a phenomenon which is effectively a slight coupling between betatron and synchrotron oscillations, principally in the central region of the cyclotron. For the 40 Mev case this coupling gives an energy spread of roughly 1 in 2,000 so that to all intents and purposes the instantaneous beam is monochromatic. In addition both radial and axial phase areas are compact approximately elliptical figures. In view of these factors it appears quite reasonable to assume that channel designs can be evolved which will transmit the instantaneous beam with negligible additional distortion and with essentially 100% efficiency.

Before proceeding to consideration of the E, t phase space characteristics, it is perhaps worthwhile to summarize the results of this section. Combining the conclusions of subsections A, B, and C it is seen that the instantaneous beam can be expected to negotiate the entire acceleration-extraction process with negligible x, p_x, y, p_y distortion and 100% efficiency, or at least for the 3 sector 40 Mev MSU design such performance is clearly indicated.

Qualitative considerations lead to the conclusion that similar results can be achieved for other sector numbers and for final resonances other than $\nu_r = 1$. We conclude that an injector cyclotron can be expected to map the transverse phase space characteristics from source to extracted beam with essentially 100% efficiency and negligible distortion.

IV. CYCLOTRON ORBIT DYNAMICS - LONGITUDINAL CHARACTERISTICS

We now remove the restriction of the previous section and consider the behaviour of a beam of particles leaving the source over a finite time interval. For particles leaving the source at different times the principal different physical factor is the accelerating voltage; possible time variations in the magnetic field are readily reducible to a negligible level with presently available techniques. Time variations in the accelerating voltage experienced by the particles are of two types, one due to time variations of the amplitude of the r-f voltage and the other arising from differences in the starting phase of the particles with respect to the r-f voltage. The first of these can in principle be reduced to a negligible level although techniques comparable to those available for control of the magnetic field are not as yet available. Considerable progress[21.] has been made, however, and in view of

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21. H. Fauska, J. W. Orth, and F. H. Schmidt (Private communication)

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this it seems reasonable to assume in considering injector cyclotrons that developmental work will have been done sufficient to reduce amplitude time variations of the r-f to a negligible level (say to less than ± 1/40n of the total voltage where n is the number of turns).We are left then with the effect of variations in the initial phase. The phase at which particles depart from the source is of course not a controllable factor. Particles leaving at certain phases will be extracted, those at other phases will not. Our objective is to estimate the fractional time during which particles can be accepted from the source and still be extracted.

We note first of all that in an exactly isochronous cyclotron, the particle's initial phase is maintained throughout the acceleration process so that from the point of view of the particle an initial phase off from the peak voltage value is completely equivalent to acceleration on the peak but with reduced volts per turn. Looking again at Fig. 12 we note that the r, p_r areas of the 0th and 1st turns largely overlap, yet 14 turns later for each, i. e. on the areas labeled 14 and 15 in the figure, the overlap has completely disappeared. The factor which determines whether a particular point in the overlapping region of 0 and 1 arrives, after 14 turns, in area 14 or in area 15 must then be the particle energy since this is the only distinguishing coordinate in the overlap region. Consider then a particle in the overlapping region but with energy half way between the energies of 0 and 1. After 14 turns this particle will arrive spatially half way between 14 and 15 of the figure, or right where the septum is positioned, and will hence be lost.

Fig. 15 depicts the effect quantitatively. The particle at the notched corner of the Fig. 12 grid has been traced backward with deacceleration for 85 revolutions from turn 15 of Fig. 12, i. e. 70 turns back from the beginning of Fig. 12. The final point of this backward deacceleration run has then been used as initial condition for a set of 4 forward accelerations runs, each forward run starting at the same position and with the same energy particle but with accelerating voltages in the ratio of 85/85.5, 85/86, 85/86.5 and 85/87 respectively to that of the deaccelerated run. Hence after 85 and 86 turns the first and third respectively of the accelerated particles have energies differing from that of the reference notch particle by 1/2 the energy gain per turn. As is seen in the figure, these particles fall approximately half way between the 14th and 15th grid positions and will collide with the septum as expected. The second and fourth of the accelerated particles after 86 and 87 turns respectively have identical energies to that of the reference notch particle and on the basis of the qualitative argument would be expected to arrive at the same position. From Fig. 15 it is seen however, that they fail to do so by a considerable amount such that even though they successfully entered the channel, their direction would be different and the probability of surviving the channel consequently small.

From the above results a safe criterion as to fractional "on time" of the cyclotron is to assert that only those particles survive which depart from the source in a phase interval in which the voltage is

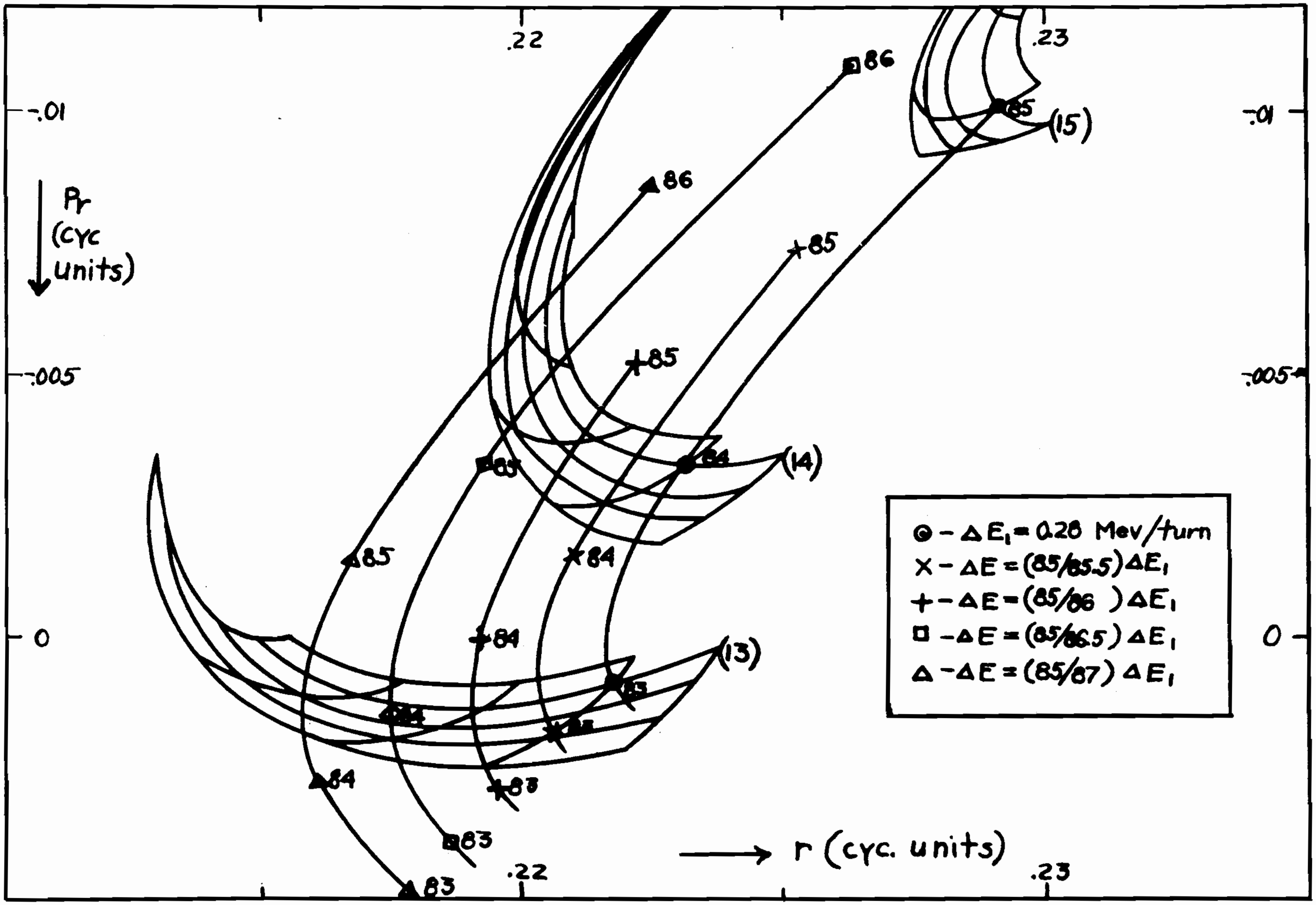

FIG. 15: ACCELERATED ORBITS WITH VARYING VOLTAGES AS SHOWN.

constant to ± 1/10n where n is the total number of turns in the cyclotron. Within such limits the composite r, p_r phase area would be enlarged due to the time wandering of the beam by no more than a factor 2. The turns would continue to be spatially separated, the energy spread would be essentially ± 1/10n (since spreads arising from other effects are negligibly smaller) and the beam would still have compact radial and axial phase areas; in view of these facts a near perfect channel remains a reasonable expectation.

With the above criterion, and with a cyclotron of 200 turns, which is a reasonable value for an injector, one obtains an acceptable phase interval of ± 2.6° or 5.2° total for a sinusoidal r-f voltage, yielding an "on time" of 1/70 of D. C. If the r-f voltage is flat topped by addition of a third harmonic (as has been suggested by Rossi[22.] and independently suggested and experimentally tested by Goodman[23.]) the above "on time" can be extended to ± 36.5° or 1/10 of

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22. G. B. Rossi, Cyclotron Square Wave R-F System
    U. S. patent #2,778,937
23. C. D. Goodman, Oak Ridge National Laboratory
    Report ORNL-2403 (unpublished)

D. C. with optimum third harmonic amplitude (11.9%)

Before passing to other matters it should be noted that a longitudinal space charge effect has been pointed out by Welton[24.]

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24. T. A. Welton, Proc. of Sea Island Conf. NAS-NRC 656, 192 (1959)

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which could, if ignored, considerably increase the energy spread in the beam and decrease the efficiency of the extraction process. Methods for alleviating this difficulty, which appear straightforward, have also been pointed out by Welton and so we will not consider the effect further.

V. CONCLUSIONS

We now proceed to combine the results of the previous three sections to obtain composite performance estimates for the complete cyclotron. In section II it was concluded that cyclotron type sources could furnish beams with D. C. luminosity at 250 Kev of at least 40amp/cm^2 sterradian. From sections III and IV together it is concluded that the source beam should be transmitted through the machine 1/70 of the time for pure sinusoidal accelerating voltage and 1/10 of the time if optimum third harmonic is included with the accelerating voltage. For normal choice of field strengths and r-f frequency, the output would consist of a continous train of pulses, the time separation between pulses corresponding to a frequency of approximately 20 mc/s. The transmitted beam would have an energy spread of $\simeq$ 1 in 1000 for the 200 turn case, the major part of the energy spread arising from a coherent variation of energy[25.] with time of the sort sketched in

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25. The coherent energy spread could of course be eliminated by a properly phased accelerating cavity acting on the extracted beam. The residual energy spread in such case would be difficult to estimate.

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footnote 7. A coherent spatial oscillation also accompanies the energy spread, the oscillation as described in Section IV, being such as to effectively double the radial phase area of the beam. Little mention has been made of axial motion in the report except to point out that possible instabilities arising in the extraction process appear amenable to solution. Throughout the remainder of the cyclotron the axial motion is well behaved, the principal responsible factor being the relatively large distance between the magnet poles and the beam space in cyclotron structures of normal type. The choice of axial aperture arises principally from economic factors; apertures in the range from 25 to 50 mm would appear likely choices. With central fields in the vicinity of 14 Kg and shaped to decrease with radius as discussed in section IIA, an initial ν_z of 0.2 appears easily achievable. Assuming an energy of 70 Kev as a typical value for beam leaving the source extractor slit (the point at which the data of Smith[9.] are applicable), an axial phase acceptance of 470 and 1900 milli-rad cm results for 25 and 50 mm aperture respectively. By appropriate use of apertures near the center, the radial phase area can be set for any value from near zero up to ~ 200 milli-rad cm (the 70 kev equivalent of Smith's results[9]). In some situations depending on the energy gain, the combination of radial and axial acceptances listed will yield currents in excess of the space charge limit. In such case for optimum design either radial or axial acceptance should be restricted to reduce the initial current to conform with the limit.

Combining the above statements, detailed estimates of the performance of a given cyclotron can be obtained. As an example, Table I is a summary of properties to be expected from the 40 Mev cyclotron for the two choices of axial aperture mentioned above. In the table, effective phase acceptance at injection is reduced in a self consistent way to allow for the reduction in ν_z due to the space charge repulsion. Table II is a somewhat more hazardous summary of properties of a possible 200 Mev cyclotron.

Finally, it is perhaps appropriate to hazard a few remarks as to relative merits of such a beam for synchrotron injection. It is at once clear, as mentioned in the introduction that the optimum radio-frequency system in the synchrotron is considerably different when a cyclotron is used as the injector. The cyclotron beam is rather tightly bunched in time and one is at once led to consideration of synchronous synchrotron r-f systems in which each cyclotron pulse at injection comes into the approximate center of an already turned on bucket. Very interesting possibilites for achieving extremely high intensities ensue, although apparently at the expense of considerably enhanced fabricational problems in the synchrotron r-f system. The cyclotron beam is also novel with respect to large transverse phase space area of the beam (in conjunction with high density this means very large currents indeed during the cyclotron pulse) principally in the axial direction. The large area means that the phase area of a synchrotron can be filled in fewer turns and since on every turn some empty space, corresponding to the inflector, is always injected, filling in fewer turns results in more efficient filling. Cyclotrons can be constructed with approximately equal ease in either horizontal or vertical orientations so that the large axial phase area in the cyclotron can be either axial or radial in the synchrotron.

Axial Aperture	25 mm	50 mm
Volts/turn	2.8×10^5	2.8×10^5
Number of turns	140	140
Central Magnetic Field (kilogauss)	14	14
Duty cycle:		
$\sin \omega t$	1.7%	1.7%
$\sin \omega t + a \sin 3\omega t$ voltage	11.1%	11.1%
Repetition Rate (pulse/sec)	21×10^6	21×10^6
Source Extraction Energy	70 Kev	70 Kev
Phase Space Acceptance from Source (full width-full angle):		
Radial (milli-radian cm)	45	23
Axial (milli-radian cm)	470	1880
Output Beam Current (time average):		
$\sin \omega t$ voltage	2.8 ma	5.6 ma
$\sin \omega t + a \sin 3\omega t$ voltage	18.3 ma	36.6 ma
Output Beam Spot Size-Divergence (full width-full angle):		
Radial (milli-radian cm)	3.8	1.9
Axial (milli-radian cm)	20	80
Output Beam Energy Spread	±30 Kev	±30 Kev

TABLE I: ESTIMATED PERFORMANCE CHARACTERISTICS OF 40 MEV INJECTOR CYCLOTRON FOR TWO CHOICES OF AXIAL APERTURE. IN THE TABLE THE RADIAL APERTURE AT THE SOURCE IS ASSUMED TO BE RESTRICTED TO YIELD A NET ACCELERATED CURRENT OF 1/2 THE SPACE CHARGE LIMIT. SOURCE LUMINOSITY AT 250 KEV AND D. C. IS ASSUMED TO BE 40 AMP/CM^2 STERRADIAN. RADIAL SPOT-SIZE DIVERGENCE OF OUTPUT BEAM IS ENLARGED BY x2 TO ACCOUNT FOR TIME WANDERING OF BEAM.

Axial Aperture	25 mm	50 mm
Volts/turn	5×10^5	5×10^5
Number of turns	400	400
Central Magnetic Field (kilogauss)	14	14
Duty cycle:		
$\sin \omega t$	1.0%	1.0%
$\sin \omega t + a \sin 3\omega t$ voltage	8.5%	8.5%
Repetition Rate (pulse/sec)	21×10^6	21×10^6
Source Extraction Energy	125 Kev	125 Kev
Phase Space Acceptance from Source (full width-full angle):		
Radial (milli-radian cm)	60	30
Axial (milli-radian cm)	350	1400
Output Beam Current (time average):		
$\sin \omega t$ voltage	3 ma	6 ma
$\sin \omega t + a \sin 3\omega t$ voltage	25 ma	50 ma
Output Beam Spot Size-Divergence (full width-full angle):		
Radial (milli-radian cm)	3	1.5
Axial (milli-radian cm)	9	35
Output Beam Energy Spread	±50 Kev	±50 Kev

TABLE II: ESTIMATED PERFORMANCE CHARACTERISTICS OF 200 MEV INJECTOR CYCLOTRON FOR TWO CHOICES OF AXIAL APERTURE. RADIAL APERTURE OF SOURCE ASSUMED TO BE RESTRICTED AS DESCRIBED IN TABLE I CAPTION.