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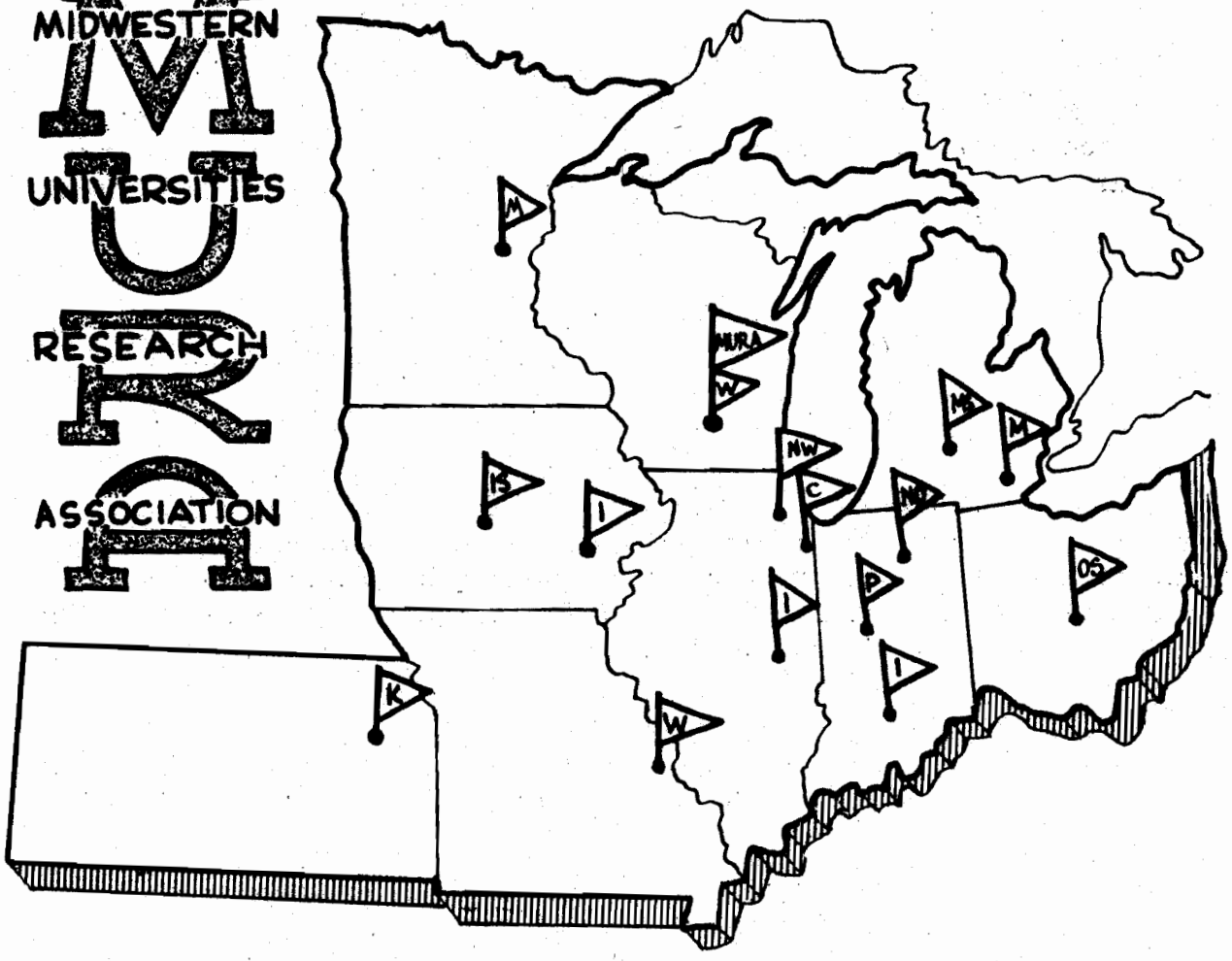

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Lawrence W. Jones

REPORT NUMBER 590

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A NEUTRINO BEAM DESIGN

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August 5, 1960

ABSTRACT

A neutrino beam is designed for use with accelerators such as the MURA FFAG using only quadrupoles. A very feasible system with 25 per cent momentum resolution is described which could produce one neutrino-induced reaction every 10 minutes in a large hydrogen bubble chamber.

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I. INTRODUCTION

A neutrino "beam" system for high-intensity accelerators such as MURA's proposed FFAG is detailed below. The concept here arose out of an attempt to formulate more critically a magnet system designed to produce the sort of neutrino beam conceived by Symon (MURA-554). The system (Fig. 1) is a quadrupole doublet with no separate bending magnet. The quadrupoles have rectangular apertures chosen so that particles of the wrong sign are deflected into the poles of the second magnet or stopped in a plug in the first magnet. The momentum resolution is achieved purely through the chromatic aberration of the system.

II. MAGNET SYSTEM

The rectangular quadrupoles are each three feet in length, spaced three feet (six feet between centers) with apertures (A) two feet by one foot and (B) 1.28 feet by 2.4 feet. The aperture of (B) was chosen to just accommodate the beam of 3 Bev pions through (A) of the correct sign focused on infinity in both dimensions. With the center of (A) 12 feet from the target in the proton beam, the maximum magnetic field strength in (A) is 16,000 gauss from the following simple formulae:

The focal length of a quadrupole element, regarded as a thin lens, is given by

$$\pm \frac{1}{f} = \frac{\ell}{B\rho} \frac{dB}{dx}$$

where ℓ is the length of the lens and $B\rho$ the magnetic rigidity of the particle.

The maximum magnetic field for a particular quadrupole lens is then

$$B_{max} = \left(\frac{a}{f}\right) \frac{B\rho}{\ell}$$

where a is the semi-aperture of the lens.

For two "thin" quadrupole lenses spaced a distance d with the first a distance s from the object point, the focal lengths of the two ($\pm f_1$, $\mp f_2$) are related by

$$\frac{|f_2| - |f_1|}{|f_1|} = \frac{d}{s}$$

and

$$ds = |f_1|\,|f_2|$$

for the image to be at infinity for both dimensions.

If we pick $s = 2\,d$, we get

$$d = 0.866\, f_1$$

$$s = 1.732\, f_1$$

$$f_2 = 1.5\, f_1$$

so that the lens f_1 is the stronger of the two. These thin lens relationships are expected to hold for real quadrupoles (thick lenses) to about 10% with separations like the above.

A plug in the aperture of lens A blocking the central one-fourth of the aperture radially would permit particles of the wrong sign of momentum only greater than 6 Bev to be transmitted; a plug occupying one-third of the aperture

would block all particles of the wrong sign. With no plug at all, the ratio of transmissions of the system to particles of the right and wrong sign is four or five to one, so a high percentage of rejection of the wrong kind of neutrinos can thus be simply achieved. A background of unwanted neutrinos will always exist due to decay in flight of pions of wrong sign before reaching the first magnet and from muon decay closer to the detector.

The momentum "resolution" of the system is estimated below. The beam is parallel for a particular momentum for both x and y focusing. At higher momenta it diverges, for lower momenta it converges, and in fact rather soon crosses (focuses) in front of the detector. The beam flux at the detector may be assumed as a first approximation to be strictly the pion flux, and to be given by the reciprocal of the beam area in the plane of the detector. This area is approximately given by the area of the second quadrupole aperture times the ratio of the distance from the final image point (real or virtual) to the detector to the final image distance, S" (distance from the second lens to the image). Since this ratio is different for x and y planes, a better expression for the illumination is

$$I = \frac{1}{XY} = \frac{1}{B_x B_y} \frac{S_x'' \, S_y''}{(q - S_x'')(q - S_y'')}$$

where B_x and B_y are x and y apertures of the second quadrupole and q is the distance from its center to the detector, here taken to be 90 feet, or 15 times d.

The expression for S_x'' and S_y'' are, again from the thin lens formulae,

$$S_x'' = -\frac{f_2 (S f_1 - ds + d f_1)}{s (f_1 - f_2) - d (S - f_1) + f_1 f_2}$$

$$S_y'' = -\frac{f_2 (S f_1 + dS + d f_1)}{S (f_2 - f_1) - d (S + f_1) + f_1 f_2} \quad .$$

For our case; $S = 2\,d$, $f_2 = 1.5\,f_1$, and setting $d/f_1 = \alpha$, we have

$$\frac{S_x''}{d} = \frac{9 - 6\alpha}{4\alpha^2 - 3} \quad \text{and} \quad \frac{S_y''}{d} = \frac{9 + 6\alpha}{4\alpha^2 - 3} \quad .$$

In Fig. 2 the illumination I is plotted for various momenta (proportional to f_1). If the detector area is, say two by two feet square, I should be considered to have a maximum value of about 0.75 in these units. The actual illumination of the target by neutrinos will be broader of course due to their spread in angle and energy, and the finite proton target dimensions.

III. INTENSITY

This system subtends a solid angle from the target in the proton beam of $\Omega = \frac{1 \times 2}{(12)^2} = \frac{1}{72} \cong 0.014$ steradians. The momentum acceptance is approximately 25 per cent. The pion flux may be then estimated in two ways.

First, assume each proton produces pions with a multiplicity given by the values observed in antiproton annihilation; e.g., 6. These are one-third pions of the "desired" sign, and half of these go in the forward cone (in the center of mass). Thus one pion per interacting proton may be assumed. If the lab angle corresponding to 90° in the center-of-mass system is estimated from the pion lab momentum over the average pion transverse momentum, an angle of one-tenth radian is obtained. We thus estimate one pion per inter-

action of each sign in a solid angle of $\pi/100$ steradians. Now if pions are distributed from half a Bev to several Bev such that the number in each momentum interval is proportional to $\Delta P/P$ (as approximately observed at Berkeley), we should have

$$N \int_{.5}^{5} \frac{dP}{P} = 1$$

where N is a normalization, equal to about $\frac{1}{2.3}$. Thus, roughly, the number of pions in an interval $\Delta P/P$ is roughly $\Delta P/2.3\,P$ of the total. The above angle and momentum estimates suggest then the number of pions per millisteradian per percent momentum interval should be

$$N = \frac{10^{-2}}{2.3} \times \frac{100 \times 10^{-3}}{\pi} \cong 1.4 \times 10^{-4} .$$

For our numbers above, this would yield $14 \times 25 \times 1.4 \times 10^{-4} \times \frac{1}{2} = 2.5 \times 10^{-2}$ pions into a solid angle of 14 millisteradians and 25 per cent momentum bite. We assume only half of the protons entering the target result in pions which leave the target.

The second, and perhaps more reliable, estimate of pion flux comes from the Bevatron experience. S. Goldhaber (Bev-493, March, 1960) finds typically 1 to 5×10^3 pions per one per cent $\Delta P/P$ per millisteradian per 10^{10} protons in the Bevatron. This gives an N of about 3×10^{-6} in contrast to our 1.4×10^{-4} above, although of course targeting efficiency is also an important factor here, and the proton energy is lower. A reasonable flux of pions to assume, then, might be twice the Bevatron figure multiplied by our momentum interval and solid angle: (14 x 125).

We might get $6 \times 14 \times 25 \times 10^{-6}$, or 2.1×10^{-3} pions per proton into our system. The plug in quadrupole A would drop this to 1.5×10^{-3}. For 30 microamperes, or 2×10^{14} protons per second, we would have 3×10^{11} pions per second available for neutrino production.

Following Symon, the neutrino energy for 3 Bev pions is 1320 Mev (lab) and they come off with an angle of 1/40 of a radian. If the detector area is two by two feet and the average decay is 20 meters (60 feet) from the detector, the detector half angle at the average point of pion decay is 1/60 radian, so that perhaps one-third to one-half of neutrinos from pions heading toward the detector pass through the detector. For a 20-meter flight path ($\gamma = 20$, $\tau = 2 \times 10^{-8}$ sec, $\gamma\tau = 4 \times 10^{-7}$ sec, $\lambda = 120$ meters decay mean free path) one-fifth of the pions decay.

The neutrino flux through the target, ϕ, is then

$$\phi = \frac{1}{6} \times \frac{1}{5} \times 3 \times 10^{11} = 10^{10} \frac{\text{neutrinos}}{\text{second}} .$$

Since the target was assumed two by two feet (60 x 60 cm), the flux per unit area is about 3×10^{6} ν/cm^2 sec.

For a cross section, $\sigma = 2 \times 10^{-38}$ cm^2 and a six-foot hydrogen detector (bubble chamber) the rate of events is one every 10 minutes, with the same numbers as in Symon's calculation. With a heavy liquid chamber or a dense spark chamber the rate would be correspondingly higher in direct proportion to the mass of the detector. The hydrogen mass in a two by two by six foot bubble chamber is 50 kg; filling the same chamber with density two material raises it to 1400 kg or 1.4 metric tons. A ten-metric ton spark

chamber could be made using 100 gold plates one meter square by one centimeter thick. (We leave as an exercise the cost of the gold.) The above-rate figures give two events per minute per metric ton of detector, so that the spark chamber could record a neutrino interaction every three seconds.

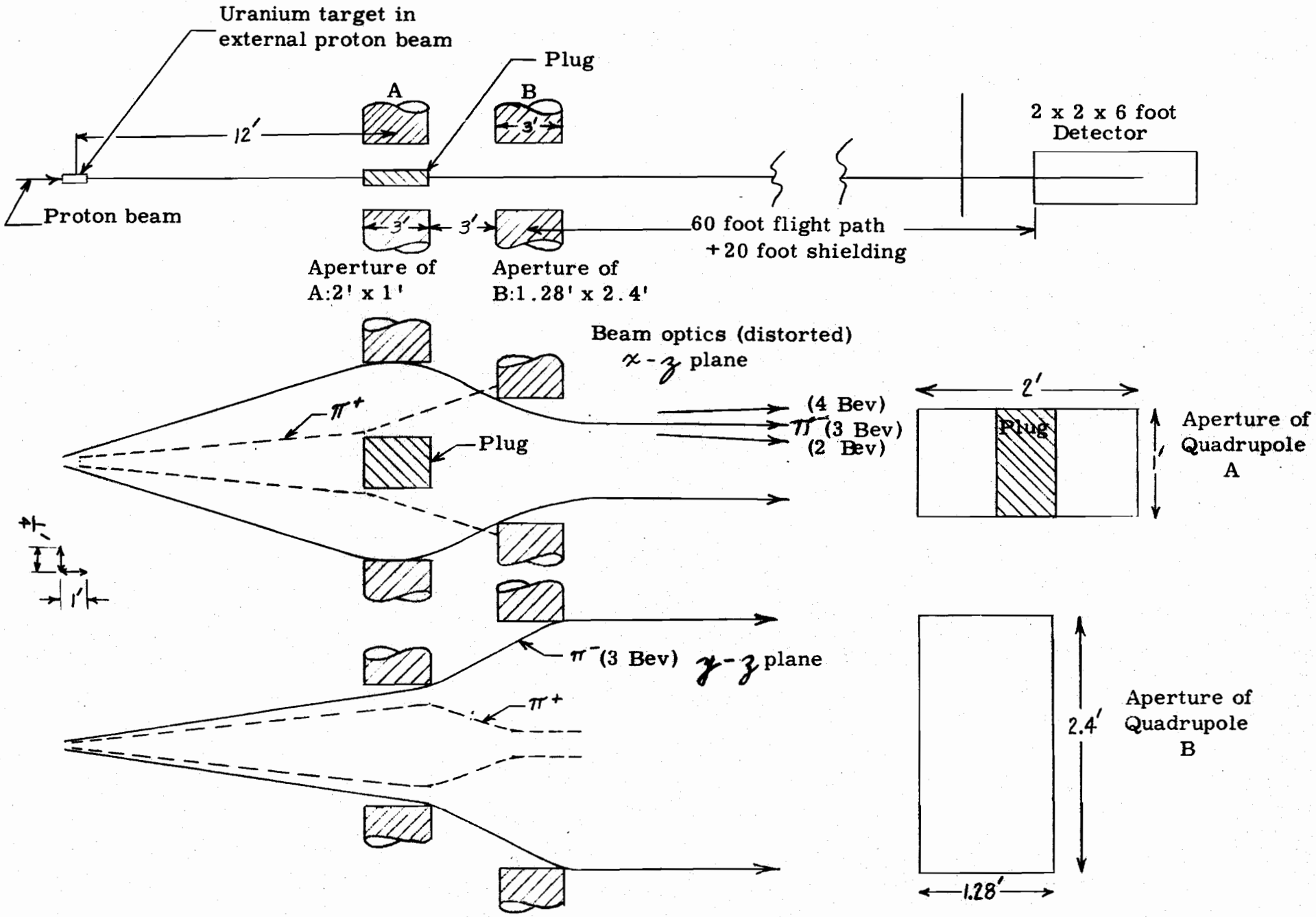
Uranium target in
external proton beam
Plug
A
B
12'
3'
2 x 2 x 6 foot
Detector
Proton beam
3'
3'
60 foot flight path
+20 foot shielding
Aperture of
A:2' x 1'
Aperture of
B:1.28' x 2.4'
Beam optics (distorted)
x - z plane
π+
(4 Bev)
π (3 Bev)
(2 Bev)
Plug
2'
Plug
1'
Aperture of
Quadrupole
A
1'
π-(3 Bev)
y - z plane
π+
2.4'
Aperture of
Quadrupole
B
1.28'

Fig. 1

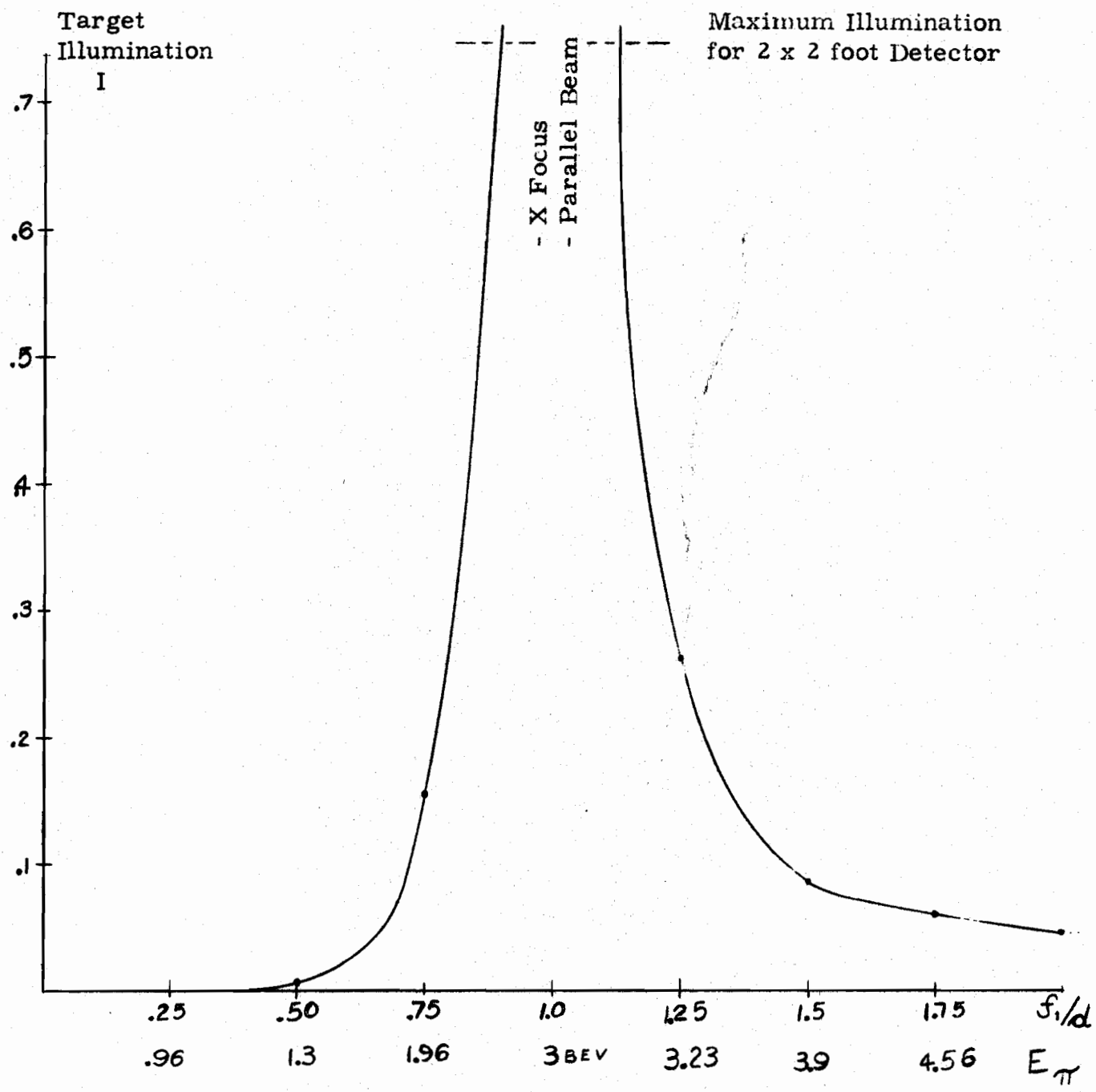
Target
Illumination
I
.7
.6
.5
.4
.3
.2
.1
- X Focus
- Parallel Beam
Maximum Illumination
for 2 x 2 foot Detector
.25
.50
.75
1.0
1.25
1.5
1.75
f_1/d
.96
1.3
1.96
3 BEV
3.23
3.9
4.56
E_π

Fig. 2