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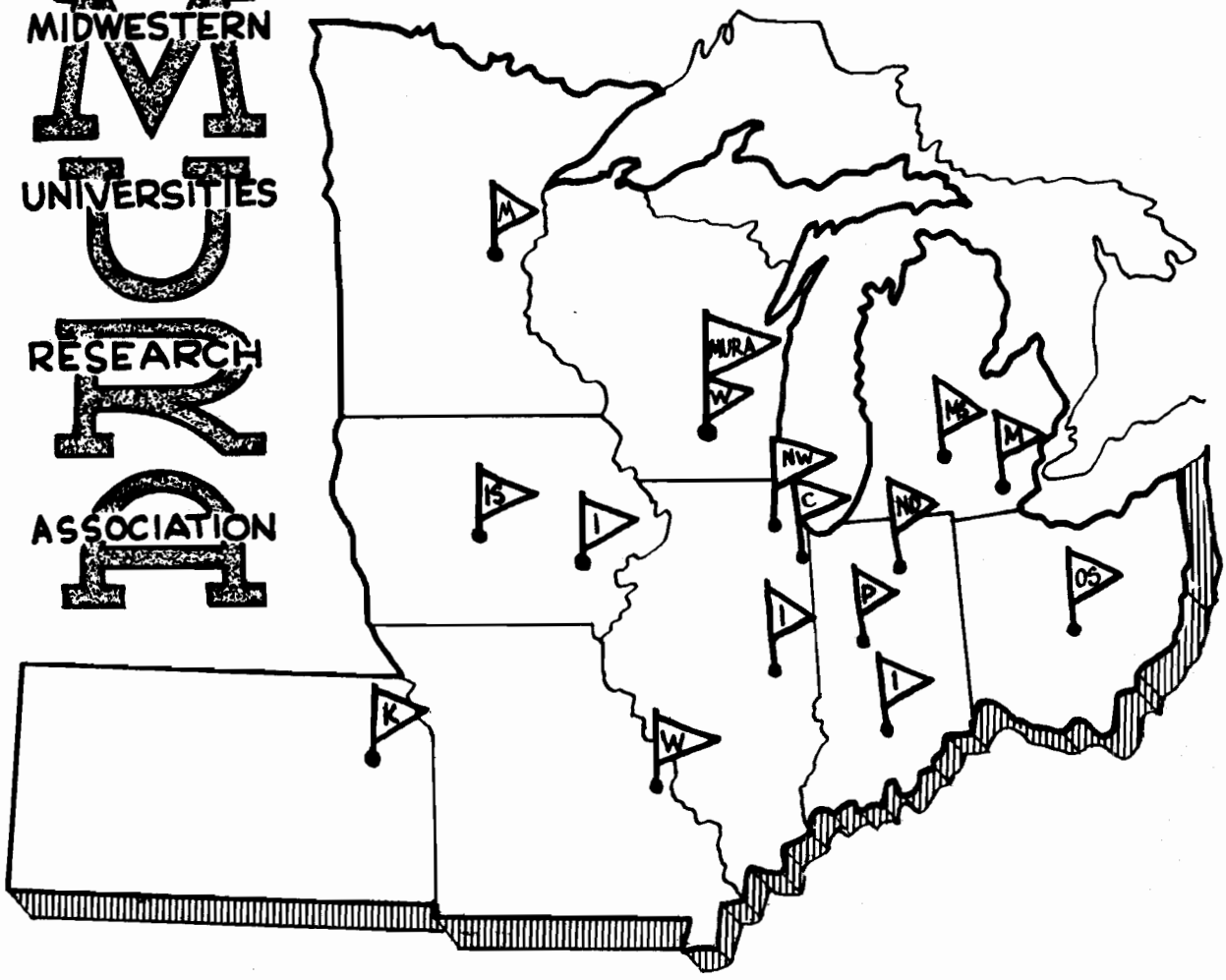

CONSTANT-AREA R-F BUCKETS IN THE IBM-704 PROGRAMME TTT

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ABSTRACT

A method is provided for using the polymonial "frequency" of TTT when R-F buckets of constant area are to be considered. The polymonial for the example $\Gamma = \frac{1}{2}$ is given.

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In the use of the IBM-704 programme TTT[1] for calculating particle orbits for certain beam-stacking radio-frequency programme problems, there arises a difficulty in that the revolution frequency f of the particle can be entered only as a third degree polynomial in the particle energy E.

In particular, since accelerating an incompletely filled bucket into or through an energy region in which particles have previously been deposited, causes phase-displacement deceleration of the particles; it is desirable to have the bucket area in the W, φ space of MURA-106[2], held constant.

1) $$A = 8\sqrt{2V/\pi ff'} = \text{const.}$$

where V is the maximum energy in electron-volts at the cavity in question and $f' = df/dE$. See figure 1.

Two other conditions enter this problem. The frequency ν_p of phase oscillation about the stable point $\varphi_s = h^{-1} \arcsin\Gamma$ given by

2) $$\nu_p = \sqrt{ff'V\ X^2/2\pi}$$

where $X^4 = 1 - \Gamma^2$; h is the number of the harmonic at which the bucket is being accelerated. This frequency is to be specified as being a given fraction of the revolution frequency f_o at injection:

3) $$\nu_p = f_o/\beta$$

The interval δW in W ($dW \equiv dE/f$) into which the bucket is deposited is to be a given fraction of the total interval $\Delta W = W \Big|_{f_0}^{f_1}$ where f_1 is the revolution frequency at which the beam is to be deposited.

4) $$\delta W = \Delta W/\alpha$$

By Liouville's theorem, $\delta W = 1/2\pi$ times the (constant) bucket area in 1).

If V is to be constant, one must have ff' = constant, or

5) $$f = f_o \, (E/E_o)^{1/2}$$

where $f = f_o$ at the injection energy E_o.

With $df/dW = \frac{dE}{dW} \frac{df}{dE} = ff'$ = constant, one may integrate to find f_1, and combine with 2) through 4):

$$f_1 = f_o + ff'\Delta W$$

6) $$f_1 = f_o \left(1 + \frac{8}{\pi X} \frac{\alpha}{\beta}\right) = \mu \, f_o$$

Then from 5),

7) $$E_1 = \mu^2 \, E_o$$

is the energy at the stacking frequency.

Without adding a "Frequency" of the square root type to TTT (the time required by such would average to about four times that required to evaluate the polynomial, for a range,

say, $0.01 \leq E/E_0 \leq 1.0$,) the use of the programme requires fitting the function

$$8) \quad f = f_0 \, (E/E_0)^{1/2} \qquad\qquad E_0 \leq E \leq \mu^2 \, E_0$$

by a polynomial.

Since the slope of polynomial approximations at the end points of an interval in which a root function is approximated deviates rather badly from that of the function, some care must be taken in choosing the points at which the function is fitted. Thus, fitting 8) in the region $(E_0, \; 6/5 \, \mu^2 \; E_0)$ keeps $\underline{ff'}$ much more nearly constant in the upper extremes of the interval $(E_0, \mu^2 \, E_0)$ than does fitting in this interval itself.

The Newton forward interpolation formula[3] provides a quick method of finding the coefficients in the interpolating polynomial. The function is evaluated at the points through which the polynomial is to pass, and a difference table is formed. If these points are equidistant the process is simplified, and to provide an $\underline{n}^{th}$ - degree polynomial, the function must be tabulated at $\underline{n + 1}$ points.

$E \quad E_0$	$f \quad f_0$	Δ_1	Δ_2	Δ_3	$\ldots$
1	$1 = \Delta_0^0$				
		$\Delta_0^1 - \Delta_0^0 = \Delta_1^0$			
$1+a$	$\sqrt{1+a} = \Delta_0^1$		$\Delta_1^1 - \Delta_1^0 = \Delta_2^0$		
		$\Delta_0^2 - \Delta_0^1 = \Delta_1^1$		$\Delta_2^1 - \Delta_2^0 = \Delta_3^0$	
$1+2a$	$\sqrt{1+2a} = \Delta_0^2$		$\Delta_1^2 - \Delta_1^1 = \Delta_2^1$		$\ldots$
		$\ldots$		$\ldots$	
$\ldots$	$\ldots$		$\ldots$		
		$\Delta_0^n - \Delta_0^{n-1} = \Delta_1^{n-1}$			
$1+na$	$\sqrt{1+na} = \Delta_0^n$				

The polynomial then is:

9) $$f/f_0 \simeq \Delta_0^0 + \Delta_1^0 u + \Delta_2^0 \frac{u(u-1)}{2!} + \Delta_3^0 \frac{u(u-1)(u-2)}{3!} + \ldots$$

where $u = (E-E_0)/a.$

In the case where $n = 3$, one may fit the points

$$E_0,\ E_0 + \tfrac{2}{5}(\mu^2-1)E_0,\ E_0 + \tfrac{4}{5}(\mu^2-1)E_0,\ E_0 + \tfrac{6}{5}(\mu^2-1)E_0$$

to get the polynomial

10) $$f\ f_0 \simeq \sqrt{\tfrac{1}{5}}\left[b + cu + du^2 + eu^3\right] \qquad \text{where } u = \frac{5}{2}\,\frac{E-E_0}{(\mu^2-1)E_0}$$

where, with $S_j = \sqrt{5+2j(\mu^2-1)}$ $\qquad j = 0, 1, 2, 3$

11)
$$b = S_0$$
$$c = \tfrac{1}{6}\left[2S_3 - 9S_2 + 18S_1 - 11S_0\right]$$
$$d = -\tfrac{1}{2}\left[S_3 - 4S_2 + 5S_1 - 2S_0\right]$$
$$e = \tfrac{1}{6}\left[S_3 - 3S_2 + 3S_1 - S_0\right]$$

The S_j are $\sqrt{5}$ f/f_0 evaluated at four points.

In the very particular case that μ = 3.718 ($\alpha = \beta$, Γ = ½,) ff' is constant to within about 8% in the interval $(E_0 + \frac{1}{5}(E_1 - E_0), E_1 = \mu^2 E_0)$ although it tails off by about 24% in the usually unimportant region (E_0, E_0 + 1/5 E_1). Figure 2 shows f and ff' for these parameters.

The 24% change in ff' near E_0 causes a 12% decrease in the bucket area as E increases from E_0. This is acceptable since buckets filled at injection will remain filled, although some particles will be lost. the 4% increase in bucket area in the intermediate part of the energy range is an inescapable result of using a polynomial frequency; it should not have a large effect on the problem.

TTT Senario number 4200 1001 gives the coefficients for the case

$$\alpha = \beta \qquad \Gamma = 1/2$$

with E_0 and f_0 chosen arbitrarily to be 10^9 ev and 10^6 cps. The polynomial used is:

$$f/f_0 \simeq 1 + 1.91025\, u - 0.50270\, u^2 + 0.06821\, u^3;$$

$$0 \leq u \leq \frac{E - E_0}{\epsilon} \leq 2.5; \qquad \epsilon = 5.1294\, E_0.$$

1. F. E. Mills and J. N. Snyder, TTT (Programme 39), MURA-225 (February-March, 1957).

2. K. R. Symon and A. M. Sessler, Methods of Radio Frequency Acceleration in Fixed Field Accelerators, MURA-106 (April 16, 1956).

3. K. L. Nielsen, Methods in Numerical Analysis, The Macmillan Co., New York (1957).

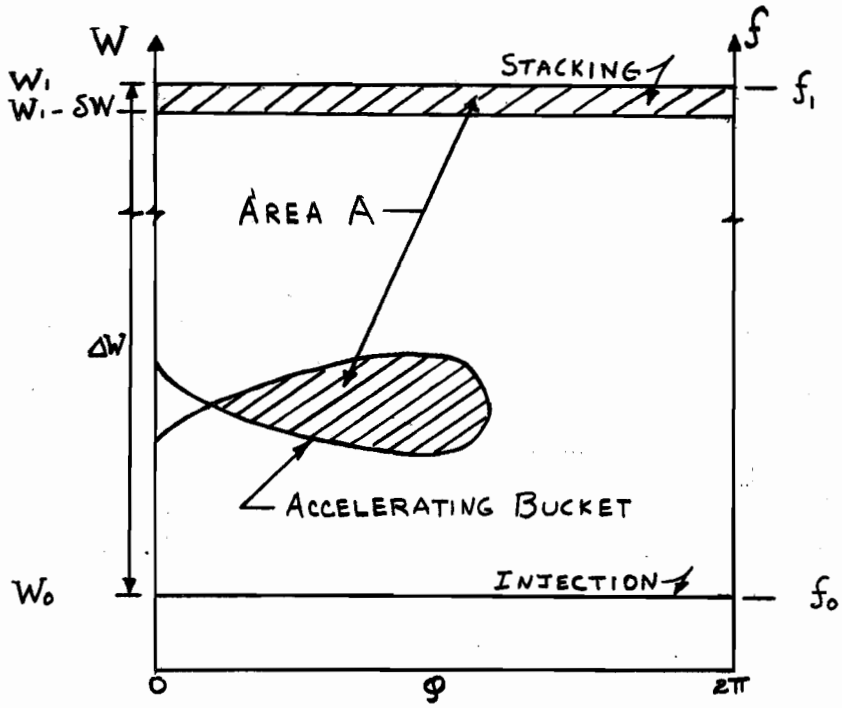

FIGURE 1.

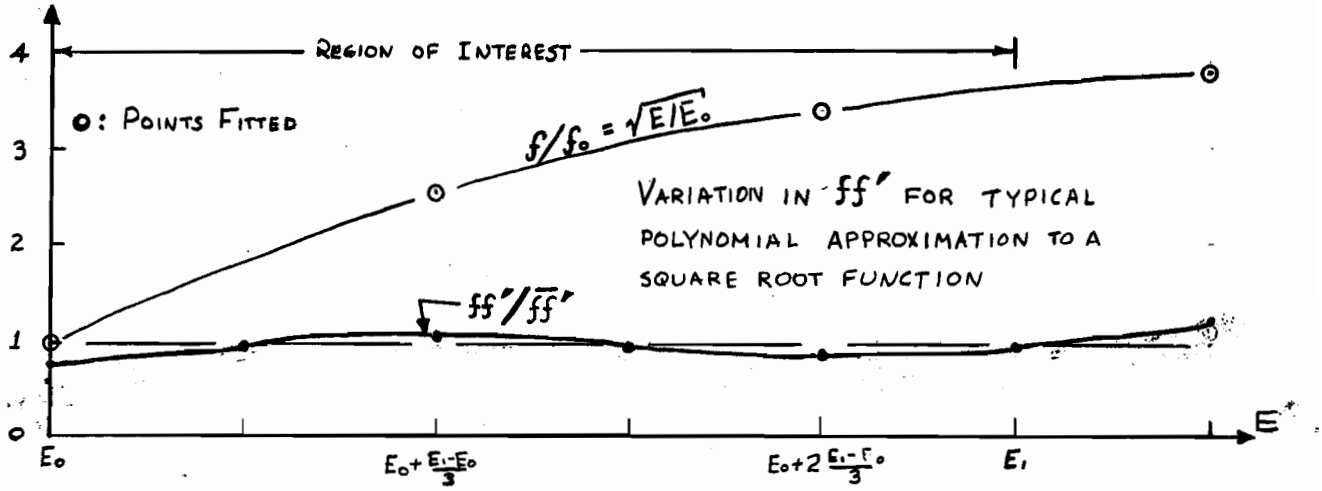

FIGURE 2.