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TYPICAL DESIGNS OF HIGH ENERGY FFAG ACCELERATORS

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(presented by K. R. Symon)

I. INTRODUCTION

FFAG accelerators afford the possibility of high energy beams of considerably higher intensity than are produced by existing accelerators. Because of the possibility of stacking, there is inherent much more flexibility in duty factor than in pulsed-field accelerators. The purpose of this paper is to review the MURA work as it affects multi-GeV accelerators and to summarize the designs which are of interest.

It is convenient to express the magnetic field in terms of quantities which are directly related to focusing. We write the median plane field in cylindrical coordinates as

$$\begin{cases} B_r = B_\theta = 0 \\ B_z = B_0\left(\dfrac{r}{r_0}\right)^k \sum_{n=0}^{\infty} (g_n \cos n\psi + f_n \sin n\psi) \\ \psi = K \ln \dfrac{r}{r_0} - N\theta \,. \end{cases} \tag{1}$$

The field off the median plane can be developed from Eq. (1), with the several components expressed as power series in z/r.

k is the relative gradient of the average field, or the " mean field index ". K, which has often been called $1/w$ in MURA work, is related to the angle ζ at which field spirals cut radial lines by

$$K = N \tan \zeta \,. \tag{2}$$

If the parameters k, K and the Fourier coefficients are constants (independent of radius), the field is scaling. A scaling field is the simplest means of insuring that ν_x and ν_y, the numbers of radial and vertical betatron oscillations per revolution, are independent of radius, so that resonances are not crossed during acceleration. In a scaling accelerator, the orbits of particles of different energy are geometrically similar and are related by

$$\left(\frac{r_1}{r_0}\right)^{k+1} = \frac{p_1}{p_0} \,, \tag{3}$$

where r_1 and r_0 are the radii of corresponding points on orbits of momentum p_1 and p_0, respectively.

Many of the orbit properties are determined to good approximation by the quantities

$$\begin{cases} F^2 = \sum_{n=1}^{\infty} (g_n^2 + f_n^2) \\ G^2 = \frac{1}{2} \sum_{n=1}^{\infty} \dfrac{g_n^2 + f_n^2}{n^2 N^2} \,. \end{cases} \tag{4}$$

F is called the " flutter ", while G is anonymous. Note that the scale of the Fourier coefficients f_n and g_n can be chosen arbitrarily by choice of the constant B_0. In cases where $g_0 \neq 0$, it is customary to fix the scale by taking $g_0 = 1$. For the Ohkawa two-way accelerator [1], the scale will always cancel.

F is a measure of the size of the azimuthally-varying field relative to the average field. For spiral sector accelerators, usually $F \simeq 1$ in units of g_0, while for radial sector accelerators, $F \gtrsim 6$. (Note that the definitions of k and flutter differ from those given by Symon et al. [4] in that here these quantities are specified with reference to a circle, rather than the equilibrium orbit.)

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II. REVIEW OF RESULTS

1. Magnet design

In the first two FFAG accelerators constructed by MURA, the radial gradient k was achieved by backwinding the forward energizing currents of the magnets across the pole-faces in order to terminate magnetomotive force surfaces appropriately. To make the flutter independent of radius, the vertical gap was constructed to be proportional to the radius. Since $k > 0$, the gap is larger at high field than at low, which increases the back wound current necessary to achieve a given k.

In higher field accelerators, this " scaling " pole is highly wasteful of gap and current. The radial variation desired can also be achieved by shaping the pole-face, as, for example, in an AG synchrotron. But in the FFAG accelerator the shaping must be three-dimensional in order that the flutter be independent of radius. In the MURA two-way electron accelerator, the curvature is critical only in the radial direction. In larger accelerators, where the vertical gap is smaller relative to the magnet length, so that f_n and g_n do not decrease as rapidly with n, pole-face shapes will be more critical.

It does not appear to be feasible to attempt to achieve the radial gradient by pure pole-face shaping, because the large gap at low fields makes it exceedingly difficult to produce the desired flutter. It appears that some compromise between pole-face shaping and backwound current is the optimal way to build FFAG magnets. Pole-face shaping can be used to best advantage in saving winding and power at the high field end. For example, in the MURA two-way electron accelerator, the shaped pole-face at the high field end extends over only about 15 per cent of the radial aperture, but saves a factor of over 2 in backwound current.

In AG synchrotrons the large gradients limit the attainable guide field on the central orbit because they give rise to much larger fields at the chamber edges. There is an analogous effect in FFAG accelerators. The poles must extend about one vertical gap length beyond the maximum energy orbit in order that the field be correct on that orbit.

Analytical and digital calculations [2] show that short (of order 30 cm in a multi-GeV accelerator) radial straight sections can be introduced in spiral sector accelerators for RF cavities, backwinding return currents, etc., if some care is taken, without introducing too great changes in ν_x and ν_y. Present spiral and radial sector designs are similar in that the magnets are built up from radial slabs, whose thickness (about 15 cm) is chosen to be readily available from rolling-mills. The spiral field is achieved by spiraled pole-face grooves, forwardwound and backwound to increase the flutter. There appears to be a considerable saving of steel by placing the return yokes at the outside, near the high field region.

The volume of steel in the magnets can be estimated by calculating the flux carried by the poles and return yokes. The flux carried per unit azimuthal length from the return yoke to radius r is

$$\Phi(r) = \frac{1}{r_0}\int_0^r B_z r dr = \frac{r_0 B_0}{k+2}\left(\frac{r}{r_0}\right)^{k+2},$$

where B_0 and r_0 are as in Eq. (1). The pole thickness necessary to carry this flux with a density B_1 is

$$t_p(r) = \frac{\Phi(r)}{B_1}$$

and the volume of the poles is

$$V_p \simeq 4\pi \int_0^{r_{\max}} r t_p dr,$$

where $r_{\max}$ is the maximum good field radius. If the return yoke is to have a flux density B_2, its thickness is

$$t_r = \frac{\Phi(r_{\max})}{B_2}$$

and the total volume of the return yokes is

$$V_r \simeq 2\pi r_{\max} t_r^2 .$$

Then the total magnet volume is

$$V = V_r + V_p \simeq \frac{4\pi r_{\max}^3}{k^2}\frac{B(r_{\max})}{B_1}\left[1 + \frac{B(r_{\max})B_1}{B_2^2}\right]. \quad (5)$$

To this must be added extra steel for spirals (if any), for mechanical strength at the low field end and for extra return yoke to go around the forward winding. Even with these corrections, a rule of thumb result is that the magnet volume varies as $r_{\max}^3/k^2$.

2. Betatron oscillations

Particle orbits have been investigated by a combination of analytic and digital computer methods. A detailed discussion of our computer methods is not pertinent here, but we shall remark that most of our computing time is spent in direct numerical integration of the equations of motion, which we customarily write in terms of the relative deviations x and y from a circle of radius r_0 defined by

$$\begin{cases} r = r_0(1+x) \\ z = r_0 y \end{cases} . \tag{6}$$

The equations of motion of betatron oscillations, including the oscillations of the equilibrium orbit, are

$$\begin{cases} (x'Z)' = (1+x)Z - \dfrac{\alpha}{B_0}\left[(1+x)B_z - y'B_\theta\right] \\ (y'Z)' = \dfrac{\alpha}{B_0}\left[(1+x)B_r - x'B_\theta\right] , \end{cases} \tag{7}$$

where $Z = \left[(1+x)^2 + x'^2 + y'^2\right]^{-1/2}$, primes denote derivatives with respect to 0 and

$$\alpha = -\frac{er_0B_0}{cp} \tag{8}$$

is a dimensionless parameter which determines r_0.

Computer time of order some hundreds of hours has been spent on each of the three MURA accelerators and on each of several designs of high energy accelerators. We have also used algebraic transformations to investigate theoretical questions not specific to a particular accelerator [3].

One of the most striking phenomena occurring in work with a digital computer is the rapidity with which masses of data are accumulated. The interpretation of these data would be most difficult without two aids. The first is the appearance of regularities, the most noteworthy of which in our case is the existence for short times of invariant curves in one-dimensional motion. The rigorous existence of such curves over long times is moot, but for the number of revolutions usually of interest in an accelerator, they accurately represent the motion. They enable one to distinguish between stable and unstable motion with a small sampling of initial conditions.

The second aid is the guidance of analytic theory, which can give a physical understanding and predict phenomena of interest. In two-dimensional motion, we cannot plot invariant curves in four-dimensional phase space, but analytic theory leads us to study growth rates of the amplitude of one dimension as a function of the amplitude of the other, which is a systematic method of studying coupling effects.

The periodic solution which represents the equilibrium orbit can be found to arbitrary accuracy by variation-iteration procedures and the agreement between analytic theory and computer results is excellent. We customarily discuss separately the linear and non-linear motion about the equilibrium orbit. In the linear motion we are guided by the smooth approximation [4]. For one-way machines (with $g_0 = 1$), the radial and vertical betatron oscillation " frequencies " (number of oscillations per revolution) are given by

$$\begin{cases} \nu_x^2 = k + 1 + k^2G^2 \\ \nu_y^2 = -k + k^2G^2 + F^2\left(\dfrac{K^2}{N^2} + \dfrac{1}{2}\right) , \end{cases} \tag{9}$$

where the terms in k alone are the constant gradient focusing of the average field, the k^2G^2 terms are alternating gradient focusing, the $\frac{1}{2}F^2$ term is the Thomas (constant edge) focusing and the term depending on K is the spiral (alternating edge) focusing. The effects of Thomas and spiral focusing on the radial motion cancel because of the oscillations of the equilibrium orbit. In a spiral sector accelerator, the alternating gradient and Thomas focusing terms are small and are usually neglected. Then we must have $k > -1$ for radial stability.

For a two-way accelerator,

$$\begin{cases} \nu_x^2 = 2k \\ \nu_y^2 = \dfrac{F^2}{kG^2}\left(1 + \dfrac{2K^2}{N^2}\right) . \end{cases} \tag{10}$$

The radial focusing is all alternating gradient focusing, while the vertical focusing has only Thomas and spiral focusing. Neither radial nor vertical oscillations are stable for $k < 0$. It can also be shown that equilibrium orbits do not exist in the two-way accelerator with $k < 0$. The formof the smooth approximation used here is accurate for $\nu \lesssim \frac{1}{4}N$. It is well known that approximate formulas with wider ranges of validity have been developed by many workers.

Non-linear equations of motion are inherent in FFAG accelerators. Fields of the scaling type (Eq. (1)) clearly have non-linear terms in x, which in turn give rise to such terms in the equations of motion.

It has been shown [5] in the smooth approximation limit that no field linear in x can keep both ν_x and ν_y constant over a reasonable range of energies.

Many styles of analytic theory of non-linear effects in accelerators have been developed. They are all in agreement that non-linear forces give rise to stability limits — amplitudes beyond which the motion is rapidly unstable. For very small amplitudes, the frequencies of oscillation are determined by the linear forces. As the amplitude is increased, the frequencies of oscillation change because of the non-linear forces until a resonance relation of the form

$$m_x\nu_x + m_y\nu_y = m \tag{11}$$

is satisfied, with m_x, m_y and m integers. For larger amplitudes, in many cases the motion is unstable.

In practice, in FFAG accelerators, the radial motion is dominated by the $\nu_x = \frac{1}{3}N$ resonance. The largest stable amplitude is roughly

$$A = \frac{8}{3}\frac{N}{B}\left|\nu_x - \frac{1}{3}N\right|, \tag{12}$$

where for radial sector accelerators $B \simeq \frac{1}{2}g_1k^2$, while for spiral sector accelerators $B \simeq \frac{1}{2}g_1K^2$. Eq. (12) is accurate only for ν_x close to $\frac{1}{3}N$. Digital computation or a more accurate analytic treatment show that, farther from the resonance, A is considerably smaller above $\frac{1}{3}N$ than below.

Coupling resonances, where both m_x and m_y are different from zero, are more important in practice than vertical resonances. Such coupling resonances introduce thresholds. There is a radial amplitude below which the two-dimensional motion is stable, but above which the vertical amplitude grows exponentially from any initial value different from zero. The general effect is thus to decrease the radial stability limit.

The properties of the linear motion are determined in an ideal accelerator by the phase changes per sector $2\pi\nu_x/N$ and $2\pi\nu_y/N$. The non-linear stability limits in an ideal accelerator are determined by resonances where m is a multiple of N — "essential" resonances. If we vary the parameters k, K and N in such a manner as to keep ν_x/N and ν_y/N constant (thus keeping the same distance from the essential resonances), then from Eq. (9) or (10) we must keep k/N^2 and K/N^2 constant. Then the non-linear stability limits vary as N^{-2}.

The same integral and half-integral resonances exist in FFAG accelerators as in AG synchrotrons. In addition, there are non-linear effects which are not yet well-explored. The deleterious effects of misalignments in the linear approximation vary in general as $k/\nu N^{1/2}$ in radial and $K/\nu N^{1/2}$ in spiral sector accelerators. If we vary parameters again to keep ν_x/N and ν_y/N constant, the misalignments and field errors which can be tolerated vary as $N^{-1/2}$. The more rapid variation of the non-linear stability limits usually halts the increase of parameters before misalignment and error effects. The general magnitude of tolerances in FFAG accelerators is similar to that in AG synchrotrons.

3. Acceleration

There is remarkably little interaction between the design of the RF accelerating system of an FFAG accelerator and that of the rest of the accelerator, which is just a reflection of the great flexibility in acceleration possible in an FFAG accelerator [6]. Aside from the problem, trivial in principle, but sometimes non-trivial in practice, of providing enough straight section room for the desired number of cavities, there are few problems which reflect at all on the rest of the design. The only important problem is the transition energy given by

$$E_t = (k+1)^{1/2}E_0 \tag{13}$$

in a scaling accelerator, where E_0 is the rest energy. For the values of k practical for betatron oscillations in a large proton accelerator, the transition energy is usually between about 7 and 12 GeV. Acceleration over the transition energy is not too difficult, but stacking close to it is not easy, because it becomes increasingly difficult to keep processes adiabatic as the transition energy is approached and the rate of change of frequency with energy approaches zero.

4. Intensity considerations

It is obviously desirable in most cases to accelerate as many particles per acceleration cycle as possible. At the same time, it is desirable to accelerate a beam with as small an energy spread as possible, since a larger energy spread requires larger RF voltage for phase stability and, in the case of stacking, reduces the current density attainable. In other words, the injector should fill betatron phase space as efficiently as possible before the RF is turned on. Present-day

injectors have emittances which are small compared with accelerator admittances, so that many turns must be injected to fill betatron phase space. Liouville's theorem puts an upper limit on the number of turns which can be injected, since the phase space density in the accelerator cannot be greater than that in the injector. (It can be worse because empty space can be mixed in.)

All the systems to accomplish multi-turn injection which have been studied involve time-dependent field perturbations. In most such schemes, the equilibrium orbit is moved adiabatically away from the inflector during injection. Laslett and Symon [7] have studied schemes which employ non-linear instabilities to move the beam radially. If the acceptance of the radial (vertical) betatron phase space is n_x (n_y) times the emittance of the injector, a total of $n_x n_y$ turns can theoretically be injected and contained in the accelerator. At the present time, it is certainly feasible to inject about $\frac{1}{3}n_x$ turns into the radial betatron phase space and to gain further a factor of about 3 turns in vertical phase space, so that, in effect, about n_x turns can be injected.

The current which can be injected into an accelerator is also limited by space charge, which lowers the frequencies of betatron oscillation, so that a resonance can be reached. The maximum circulating current which can be contained in a toroidal tube of major radius r and minor radius a is

$$I = I_0 \nu \Delta\nu \left(\frac{a}{r}\right)^2 (\beta\gamma)^3 , \tag{14}$$

where $I_0 = mc^3/e = 3.129 \times 10^7$ A for protons, ν is the relevant betatron oscillation "frequency" (usually the smaller), $\Delta\nu$ is the allowable "frequency" change, $\beta = v/c$ and $\gamma = E/E_0$. A beam which is bunched has a space charge limit smaller than that of Eq. (14) because the local current is larger at the position of a bunch. A beam which is neutralized by charges of opposite sign which are not synchronous with the beam has a space charge limited circulating current

$$I = I_0 \nu \Delta\nu \left(\frac{a}{r}\right)^2 \beta\gamma , \tag{15}$$

because the relativistic cancellation of electric and magnetic forces no longer takes place. The current given by Eq. (15) is smaller than that of Eq. (14) at relativistic energies. It is therefore usually desirable to sweep out the trapped charges with clearing fields. Image charges and currents in the conducting surroundings of the beam also affect the space charge limit, particularly of a stacked beam. These image effects can be largely cancelled by suitable currents above and below the stacked beam.

There are also space charge effects on the synchrotron oscillations. For a stationary distribution [8], the longitudinal space charge limit is usually greater than the transverse limit given by Eq. (14).

Lioville's theorem also limits the current density which can be stacked. One can increase the current density by overlapping betatron oscillations up to the radial width of the stack. If we assume that this is done with groups which are space charge limited at injection and adiabatically damped, the maximum current density is

$$J \simeq I_0 \frac{ak\nu\Delta\nu\gamma^2}{Pr^3} f , \tag{16}$$

where f describes the lowering of the space charge limit at injection due to bunching, P is the relative energy spread at injection, and a, γ and r are evaluated at the stacking energy. In the derivation of Eq. (16), it has been assumed that the injection energy is non-relativistic, the stacking energy ultra-relativistic and $k \gg 1$.

Terwilliger has shown [9] that perturbations can be introduced to overlap equilibrium orbits of different energies (thus making the accelerator locally non-scaling) to increase the current densities attainable.

III. A TYPICAL RADIAL SECTOR DESIGN

In the design of radial sector accelerators the overriding consideration is magnet weight. Raising k reduces magnet weight in three ways: magnet weight varies as k^{-2}, the scalloping contribution to the net guide field on the equilibrium orbit increases with k, so that the circumference factor and radius are reduced for a given peak field and the radial aperture necessary for a given momentum spread decreases rapidly as k increases, so that the steel need for mechanical strength at the low field end decreases.

As k is increased, the average field and alternating gradient terms of ν_y^2 in Eq. (9) approach each other in magnitude. The vertical focusing is due mostly to the Thomas term, as in a radial sector two-way accelerator. Thus a large radial sector accelerator can be a two-way accelerator at little extra cost.

In Table I we give parameters and dimensions of a 10 GeV two-way radial sector proton accelerator, together with those of a 10 GeV spiral sector proton accelerator which we discuss in the next section. Both accelerators use as an injector a 50 MeV linear accelerator which is assumed to give 5 mA current for one-half millisecond with an emittance of 10^{-3} cm × rad per betatron oscillation mode and an energy spread of 50 keV. The repetition rate is chosen to be 10 c/s, easily achievable by conventional RF systems. It may be noted that fewer volts per turn are required in a spiral than in a radial sector accelerator for the same repetition rate because there volution frequency is higher.

TABLE I

Possible parameters and dimensions of 10 GeV radial and spiral sector proton accelerators

	Radial	*Spiral*	*Unit*
Maximum orbit radius	126	50	m
Peak field	17.5	14.4	kG
Circumference factor	6.0	2.0	
Radial aperture	3.25	4.0	m
N	62	30	
k	212	53	
ζ	0	84.3°	
ν_x	24.75	8.4	
ν_y	4.3	7.2	
Radial stability limit	± 16	± 5	cm
Vertical stability limit	± 12	± 3.5	cm
Minimum vertical gap	15	15	cm
Magnet weight	20 000	13 300	tons
Copper weight	1 330	1 140	tons
Magnet power	20	17	MW
Peak RF power	4.3	2.3	MW

These data enable performance figures to be estimated. We give in Table II n_x and n_y, the number of turns which can be accommodated in radial and vertical betatron oscillations phase spaces by Liouville's theorem (assuming the given injector), I_{sc}, the space charge limited circulating current which can be injected, assuming $\Delta\nu = \frac{1}{4}$ and the azimuthal bunching factor $f = \frac{1}{4}$, P_i, the number of particles per pulse and $\langle I \rangle_{av}$, the time average accelerated current, both assuming the space charge limited current at injection and J, the maximum attainable stacked current density with Terwilliger's equilibrium orbit superposition, which determines the interaction rate density of colliding beams.

TABLE II

Estimated performance data for 10 GeV radial and spiral sector accelerators

	Radial	*Spiral*	*Unit*
n_x	400	40	
n_y	0.25	15	
I_{sc} (*)	70	0.25	A
P_i	1.2×10^{13}	5×10^{12}	
$\langle I \rangle_{av}$	19	8	μA
J	190	470	A/cm^2

(*) Both accelerators require 50 turns to be injected to reach the space charge limit.

IV. A TYPICAL SPIRAL SECTOR DESIGN

The spiral sector design is dominated by the influence of non-linear restoring forces, which force the adoption of small N, k and K. The flutter is unity (in units of g_0). If it were lowered, the circumference factor and the radius would decrease, but K would have to be increased to keep the same ν_y and the stability limits would be smaller.

It is important to realize that large stability limits are desirable at injection, but perhaps not at high energy, because adiabatic damping decreases the betatron oscillation amplitudes as the energy is increased, while circumference factor is important at high energy, but not at injection, because the fields at injection can be increased, while the fields at high energy are presumably already as high as is practical. A non-scaling spiral sector accelerator whose flutter decreases and whose K increases as functions of radius will have both virtues — large stability limits at injection and small circumference factor at high energy. It must still be true that ν_x and ν_y remain constant, so that resonances will not be crossed. Study of such non-scaling accelerators has begun at MURA and preliminary results encourage us to believe that ν_x and ν_y can be kept constant. It certainly seems clear that the spiral sector design is open to improvement.

The spiral sector accelerator cannot provide colliding beams by itself, as can the two-way accelerator. It appears that the best way to produce colliding beams is to transfer the accelerated beam from the FFAG accelerator to a pair of storage rings in which oppositely circulating beams collide. The concentric storage ring of O'Neill [10] is well suited for this purpose.

The combination of FFAG accelerators and storage rings has some advantages. With an FFAG accelerator one would stack an intense beam before transferring it to the storage ring. No RF acceleration would be required in the storage ring. This would reduce the radial aperture of the ring. Stacking in the storage ring, as is necessary with a pulsed field accelerator, requires extreme precision in the frequency of the RF at pickup in order that phase density should not be lowered by mixing in empty phase space. The precision required has been estimated to be about one part in 10^5, which might be done by beam control of the radio frequency. In contrast, in the FFAG accelerator, the control needed at the injector to preserve RF phase density is one part in 10^3. The small aperture of the storage ring offers easy access to the colliding beam region for experiments from all sides and eases the problems of maintaining the ultra-high vacuum desirable for colliding beam experiments.

Designs of concentric storage rings have been given by O'Neill [10] and are similar in many respects to AG synchrotron designs.

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(*) see note on reports, p. 696.

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