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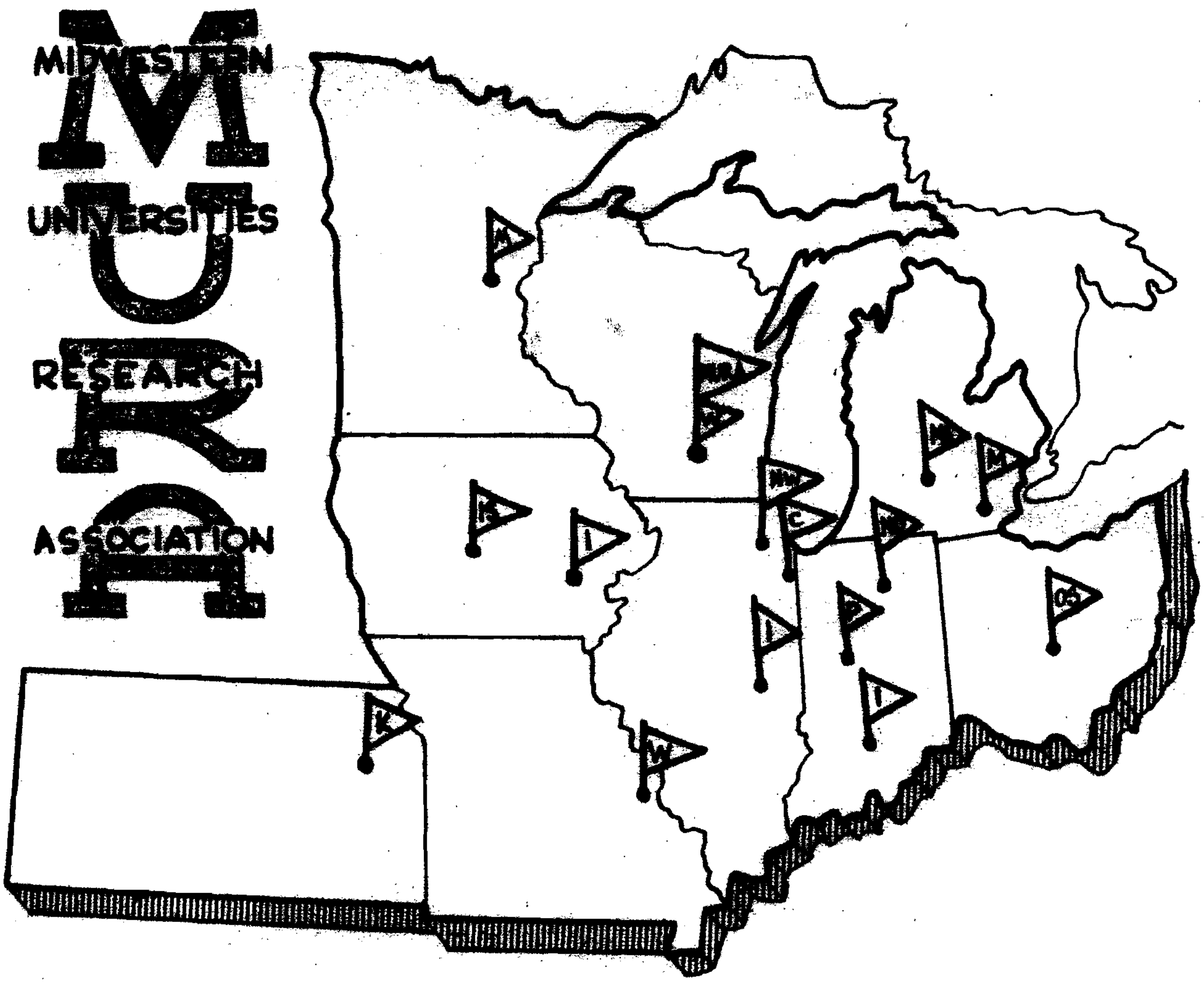


# MAGNETO-GRAVITATIONAL EFFECTS IN PARTICLE ACCELERATORS

L. W. Jones and A. M. Sessler

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2203 University Avenue, Madison, Wisconsin

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## ABSTRACT

That gravitational effects can be important in the design and operation of particle accelerators is generally believed not to be true. It has been pointed out[1] that the effect of gravitation on the trajectories of particles can, in certain cases, be compensated by suitably designed magnetic guide fields. It is the purpose of this report to call attention to a first order effect in the theory of general relativity, whose consequences for some accelerators can be pronounced--leading, in fact, if no corrections are made to the present designs, to breakdown of predicted behavior.

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**Lawrence Radiation Laboratory at present time but on leave from the University of Michigan.

***Lawrence Radiation Laboratory at present time but on leave from the Ohio State University.

## I. GENERAL THEORY AND AN ELECTROSTATIC EXAMPLE

The principle of equivalence, when combined with the Maxwell expression for the energy in an electromagnetic field, leads to an inertial as well as gravitational mass which may be associated with any particular field configuration. An immediate consequence of this is that the equilibrium distributions for electromagnetic fields (either magnetic or electric) in the presence of the earth's gravitational field, are altered from those computed soley on the basis of classical electromagnetic theory. Although this effect is not large, it is first order in $gL/c^2$ where g is the gravitational constant, and L is a dimension characteristic of the system. For particle accelerators, where high fields, large distances, and extremely high field tolerances are integral features of all designs, this magneto-gravitational effect can be significant.

We can illustrate the general features with an elementary example, which contains all of the essential physical concepts. Consider a condenser with large parallel plates and dimensions as indicated in Fig. 1, oriented vertically in the earth's gravitational field (assumed uniform). We are ignoring edge effects here, as a non-essential complication. The electric field in the gap of the condenser at height x is called $\mathcal{E}(x)$.

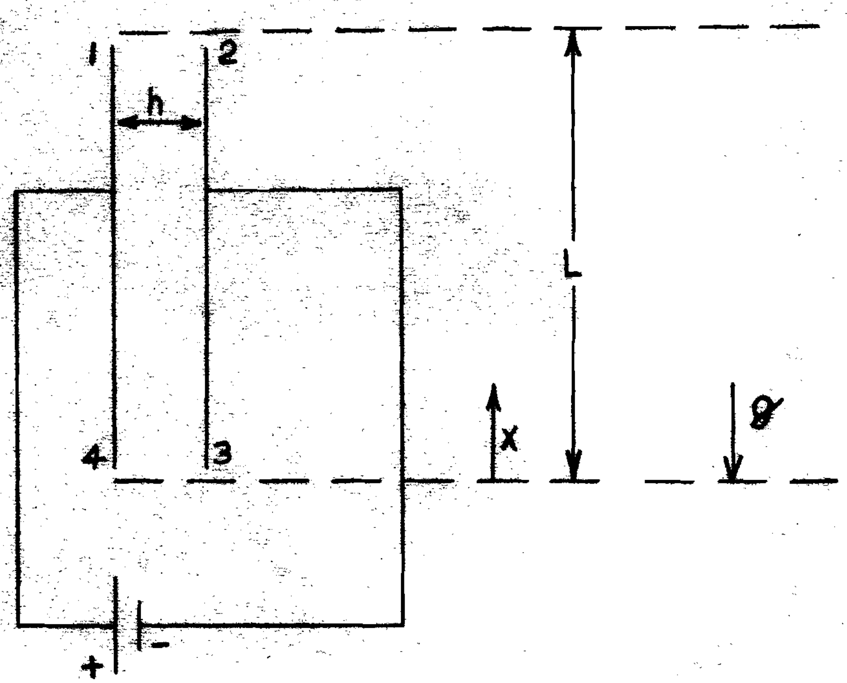


Fig. 1. Condenser oriented vertically in the earth's gravitational field.

It is then clear that if we take a test particle about a closed loop, no work must be done. If we cross at the height $x_1$ with a particle of mass m, and initial velocity zero, then after crossing, the energy of the particle is:

$$E_2 = E_1 + e\,\mathcal{E}(x_1)\,h \tag{1}$$

where $E_1 = m_1 c^2$. This particle now has a mass $m_2$ given by

$$m_2 = \frac{E_2}{c^2} = m_1 + \frac{e\,h\,\mathcal{E}(x_1)}{c^2} \tag{2}$$

so that if we take it down to height $x_2$ it gains gravitational energy $E_g$ where[2]

$$E_g \approx m_2\, g\,(x_1 - x_2) \quad . \tag{3}$$

The energy of the particle is now $E_3$ where

$$E_3 = \left(\frac{E_1 + e\,h\,\mathcal{E}(x_1)}{c^2}\right)\left[c^2 + g\,(x_1 - x_2)\right] \cdot \tag{4}$$

Upon crossing the gap, the energy of the particle is

$$E_4 = E_3 - e\,\mathcal{E}(x_2)\,h \cdot \tag{5}$$

If we bring the particle back to the original position, the energy of the particle is

$$E = E_4 - \frac{E_4}{c^2}\,g\,(x_1 - x_2) \cdot \tag{6}$$

Evaluating E, equating it to $E_1$, and ignoring higher order terms, one obtains that

$$\mathcal{E}(x_1) = \mathcal{E}(x_2) - \frac{g\,(x_1 - x_2)}{c^2}\,\mathcal{E}(x_2) \tag{7}$$

Thus the electric field across the condenser is not uniform, but larger at the bottom. This result might have been expected by elementary considerations based upon the principle of equivalence.[3]

## II. MAGNETOSTATIC CASE WITH APPLICATIONS TO THE ARGONNE ZGS

It is clear that the considerations given in Section I apply equally to a static magnetic field. Thus the magnetic field will be larger at the bottom of a magnet gap than at the top. Since accelerator operation is crucially dependent upon accurate fields, this effect requires investigation.

Specifically, we can set

$$B_z = B_{z_0}\,e^{-gz/c^2} \cong B_{z_0}\,(1 - \frac{gz}{c^2}) \tag{8}$$

in an accelerator where $B_{z_0}$ is the (vertical component of) field on the median plane. Thus

$$\frac{\partial B_z}{\partial z} = - B_z \, g/c^2 \cong - B_{z_0} \, g/c^2 \qquad (9)$$

and, from $\nabla \cdot B = 0$ (assumed for simplicity to be valid in the gravitational field)

$$\frac{\partial B_x}{\partial x} = B_z \, g/c^2 \cong B_{z_0} \, g/c^2 \, . \qquad (10)$$

In general, these gradients may be small compared to other deliberate gradients introduced into the guide field of an accelerator. However, this may not be so in the so-called (otherwise) Zero Gradient synchrotron. It should be noted that here the gradients are not of the usual sort. Normally, in an alternating gradient structure,

$$\frac{\partial B_z}{\partial x} = \frac{\partial B_x}{\partial z} = n_1, \text{ a constant} \, . \qquad (11)$$

In poles of the Siegergeirsen type

$$\frac{\partial B_z}{\partial z} = - \frac{\partial B_x}{\partial x} = n_2, \text{ a constant} \, . \qquad (12)$$

The present case is thus related to the latter, in which the orthogonal planes of betatron oscillation are inclined at $45^0$ to the median plane of the accelerator. Such gradients may be expected to introduce coupling between radial and vertical oscillations when superposed on the conventional focusing structure.

In the absence of other magnetic focusing forces, let us explore the equation for vertical motion of a particle.

$$\ddot{z} = B_x \frac{ev}{mc} \tag{13}$$

with

$$B_x = B_o \frac{g(x - x_o)}{c^2} \tag{14}$$

where $x_o$ is the radial position at which $B_x = 0$. Also, in an azimuthally uniform field,

$$B_o = \frac{mvc}{eR} . \tag{15}$$

Thus

$$\ddot{z} = \frac{g(x - x_o)}{Rc^2} v^2 , \tag{16}$$

and

$$z = 1/2\, gt^2 \frac{(x - x_o)\beta^2}{R} . \tag{17}$$

Since the weight of the particle should not be neglected, to this should be added the usual $z = - 1/2\, gt^2$ so that

$$z = 1/2\, gt^2 \left[\frac{(x - x_o)}{R} \beta^2 - 1\right] . \tag{18}$$

For a completely relativistic particle ($\beta = 1$) in a cyclotron where $x_o = 0$ and $x = R$, it is seen that this effect just cancels the normal gravitational force! To express it another way, the effective weight is reduced by $1/\gamma^2$.

The magnitude of the effects under consideration may be inferred from the above z equation or from the vertical lapse distance of the field $\zeta = c^2/g \approx 10^{18}$ cm; or approximately one light-year. Across the 14.6 cm gap of the Argonne accelerator, the field may be expected to decrease by $3.15 \times 10^{-13}$ gauss at full field (21, 500 gauss). Thus it is seen that without compensating windings on the top pole, the Argonne accelerator magnets will not have zero gradient. As an alternative to correction coils, these effects could be minimized by locating the accelerator at the point of zero gravitational field between the earth and the moon.

## ACKNOWLEDGEMENTS

The authors have profited greatly from stimulating discussions with Dr. D. L. Judd, in which their attention was first drawn to this class of phenomena. They are also indebted to Dr. T. A. Welton for an illuminating discussion of the electromagnetic stress tensor.

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## REFERENCES


1. J. Livingood, private communication.

2. This expression is first order. The correct expression is

$$E_g = mc^2 \left[ 1 - e^{-g/c^2 (x_2 - x_1)} \right] ,$$

which can easily be seen to reduce to the formula given in the text if only the lowest order term in $g(x_2 - x_1)/c^2$ is retained. The higher order terms are of course completely negligible for any terrestial apparatus. It should be noted that they might warrant investigation in the case of accelerators located on the sun or even on Jupiter.

3. It might appear as if the variation of electric field with height might be easily derived from a weight density balancing pressure argument. This is relatively complicated however, since the variation of horizontal field with height leads to a variation of vertical field with horizontal distance--namely, a bending of field lines under gravitational stress, and this must be included in a correct evaluation of the pressure. It might be noted that the correct expression to replace Eq. (7) is

$$\mathcal{E}(x_1) = \mathcal{E}(x_2)\, e^{-g(x_1 - x_2)/c^2} .$$

This is valid to all orders in $gx/c^2$.