[redacted]
0 1160 0036617 3

IDENTIFICATION

Floating Point Runge-Kutta, MU RKY6
704 SAP Program
G. A. Westlund - October 19, 1959
Midwestern Universities Research Association, Madison, Wisconsin.

PURPOSE

To solve a set of N simultaneous, first order, ordinary differential equations in floating point arithmetic by the Gill* method.

RESTRICTIONS

No internal checks are made on overflow or underflow.

METHOD

Given the set of N differential equations

$$y_i' = f_i(y_1, y_2, y_3, \ldots, y_N), \qquad (i = 1, 2, 3, \ldots, N)$$

the process used in the integration is defined by the following equations:

$$k_{i1} = h f_i(y_1, y_2, y_3, \ldots, y_N)$$

$$y_{i1} = y_i + (1/2) k_{i1}$$

$$k_{i2} = h f_i(y_{11}, y_{21}, y_{31}, \ldots, y_{N1})$$

$$y_{i2} = y_i + (\sqrt{1/2} - 1/2) k_{i1} + (1 - \sqrt{1/2}) k_{i2}$$

$$k_{i3} = h f_i(y_{12}, y_{22}, y_{32}, \ldots, y_{N2})$$

$$y_{i3} = y_i + (-\sqrt{1/2}) k_{i2} + (1 + \sqrt{1/2}) k_{i3}$$

$$k_{i4} = h f_i(y_{13}, y_{23}, y_{33}, \ldots, y_{N3})$$

$$y_{i4} = y_i + (1/6) k_{i1} + (1/3)(1 - \sqrt{1/2}) k_{i2} + (1/3)(1 + \sqrt{1/2}) k_{i3} + (1/6) k_{i4}$$

where h is the integration step size.

*S. Gill: "A Process for the Step-by-Step Integration of Differential Equations in an Automatic Digital Computing Machine, "Proceedings of the Cambridge Philosophical Society, Vol. 47 (1951), pp. 96-108.

The f_i's must be evaluated by a closed subroutine provided by the user. During each pass through MURKY6, four entries are made into this subroutine to obtain the f_i's.

If the independent variable x occurs in the functions f_i, it must be obtained by integrating the equation $x' = 1$. The independent variable x is then treated as an additional dependent variable y, for which the auxiliary subroutine must provide an additional f: $x' = f = 1$. If the independent variable does not appear in any of the f_i's, but is merely wanted for indication purposes, it is quicker to use a simple counter in the main program.

ACCURACY

This is a fourth order process, hence the truncation error in one step is of the order of h^5. An approximation of the error is obtained from the expression

$$(1/15)(y_h - y_{h/2})$$

where y_h is the value of y obtained from using an interval of length h and $y_{h/2}$ is the value of y obtained from using an interval of length h/2.

CALLING SEQUENCES

To provide flexibility and speed in the integration step, two calling sequences are used. The first is made only to plant storage locations and necessary constants. The second is made for the actual integration step (s).

Location	Instruction
α	TSX RKY6, 4
$\alpha + 1$	HTR (location of h)
$\alpha + 2$	HTR (location of N)
$\alpha + 3$	HTR (location of y_1)
$\alpha + 4$	HTR (location of aux. subroutine)
$\alpha + 5$	return
β	TSX RKY6 + 56, 4
$\beta + 1$	return

h is the integration step size (a floating point number). N is the number of equations to be solved (an integer). $\alpha + 3$ contains the location of y_1. The values of $y_1, y_2, y_3, \ldots, y_N$ are located in the order of <u>increasing</u> absolute locations.

NOTE: The values of h and N must be stored at their locations before the first calling sequence is used.

AUXILIARY SUBROUTINE

The user must provide an auxiliary subroutine to compute the f_i's and store them at locations COMMON, COMMON + 1, . . . , COMMON + N - 1. It must preserve index registers, and return to MURKY6 via the instruction TRA 1,4.

CODING INFORMATION

RKY6 requires 122 words program space, plus 6N words temporary storage at COMMON, plus space for its auxiliary subroutine.

The COMMON locations may be used for temporary storage between integration steps, but must not be altered by the auxiliary subroutine (except, of course, COMMON through COMMON + N - 1 to store the f_i's).

Backward integration is achieved by making h negative.

Timing: 0.336 + 3.168N milliseconds.

```
      REM RKY6 MURA FLOATING POINT RUNGE-KUTTA                          MU  RKY6
RKY6  CLA 1,4          LOC OF H                                        RKY6  0001
      STA *+1                                                          RKY6  0002
      CAL **                                                           RKY6  0003
      SLW H            H                                               RKY6  0004
      SUB ONE          1 IN CHARACTERISTIC                             RKY6  0005
      SLW H2           H/2                                             RKY6  0006
      LDQ H            H                                               RKY6  0007
      FMP SQRT         SQRT(1/2)                                       RKY6  0008
      STO HSQRT        H*SQRT(1/2)                                     RKY6  0009
      CLA 2,4          LOC OF N                                        RKY6  0010
      STA *+1                                                          RKY6  0011
      CAL **                                                           RKY6  0012
      STO N            N                                               RKY6  0013
      ADD 3,4          LOC OF Y BLOCK                                  RKY6  0014
      STA AA           FOR Y                                           RKY6  0015
      STA AI                                                           RKY6  0016
      STA AS                                                           RKY6  0017
      STA AZ                                                           RKY6  0018
      STA BH                                                           RKY6  0019
      CLA 4,4          LOC OF AUX. SUBROUTINE                          RKY6  0020
      STA AC                                                           RKY6  0021
      STA AJ                                                           RKY6  0022
      STA AT                                                           RKY6  0023
      STA BA                                                           RKY6  0024
      CLA CMN          LOC OF COMMON                                   RKY6  0025
      ADD N                                                            RKY6  0026
      STA AD           FOR F                                           RKY6  0027
      STA AF                                                           RKY6  0028
      STA AK                                                           RKY6  0029
      STA AM                                                           RKY6  0030
      STA AV                                                           RKY6  0031
      STA BB                                                           RKY6  0032
      ADD N                                                            RKY6  0033
      STA AB           FOR Y0                                          RKY6  0034
      STA AH                                                           RKY6  0035
      STA AR                                                           RKY6  0036
      STA AY                                                           RKY6  0037
      STA BG                                                           RKY6  0038
      ADD N                                                            RKY6  0039
      STA AG           FOR (H/2)F1                                     RKY6  0040
      STA AP                                                           RKY6  0041
      STA BF                                                           RKY6  0042
      ADD N                                                            RKY6  0043
      STA AE           FOR G(F1) OR G1(F3)                             RKY6  0044
      STA AQ                                                           RKY6  0045
      STA AW                                                           RKY6  0046
      STA BC                                                           RKY6  0047
      ADD N                                                            RKY6  0048
      STA AL           FOR G(F2)                                       RKY6  0049
      STA AO                                                           RKY6  0050
      STA AX                                                           RKY6  0051
      STA BD                                                           RKY6  0052
      ADD N                                                            RKY6  0053
      STA AN           FOR (H)F2                                       RKY6  0054
      STA BE                                                           RKY6  0055
      TRA 5,4                                                          RKY6  0056
      SXD CMN,1                                                        RKY6  0057
      SXD N,4                                                          RKY6  0058
      LXA N,1          N                                               RKY6  0059
```

```
AA     CLA **,1          Y                                  RKY6
AB     STO **,1          Y0                                 RKY6
       TIX AA,1,1                                           RKY6
       TSX **,4          F SUBROUTINE                       RKY6
       LXA N,1           N                                  RKY6
AD     LDQ **,1          F                                  RKY6
       FMP HSQRT         H*SQRT(1/2)                        RKY6
AE     STO **,1          H*SQRT(1/2)*F1                     RKY6
       LDQ H2            H/2                                RKY6
AF     FMP **,1          F                                  RKY6
AG     STO **,1          (H/2)F1                            RKY6
AH     FAD **,1          Y0                                 RKY6
AI     STO **,1          Y                                  RKY6
       TIX AD,1,1                                           RKY6
AJ     TSX **,4          F SUBROUTINE                       RKY6
       LXA N,1           N                                  RKY6
AK     LDQ **,1          F                                  RKY6
       FMP HSQRT         H*SQRT(1/2)                        RKY6
AL     STO **,1          H*SQRT(1/2)*F2                     RKY6
       LDQ H             H                                  RKY6
AM     FMP **,1          F                                  RKY6
AN     STO **,1          HF2                                RKY6
AO     FSB **,1          H*SQRT(1/2)*F2                     RKY6
AP     FSB **,1          (H/2)F1                            RKY6
AQ     FAD **,1          H*SQRT(1/2)*F1                     RKY6
AR     FAD **,1          Y0                                 RKY6
AS     STO **,1          Y                                  RKY6
       TIX AK,1,1                                           RKY6
AT     TSX **,4          F SUBROUTINE                       RKY6
       LXA N,1           N                                  RKY6
AU     CLA H             H                                  RKY6
       FAD HSQRT         H*SQRT(1/2)                        RKY6
       LRS 35                                               RKY6
AV     FMP **,1          F                                  RKY6
AW     STO **,1          (H+H*SQRT(1/2))F3                  RKY6
AX     FSB **,1          H*SQRT(1/2)*F2                     RKY6
AY     FAD **,1          Y0                                 RKY6
AZ     STO **,1          Y                                  RKY6
       TIX AU,1,1                                           RKY6
BA     TSX **,4          F SUBROUTINE                       RKY6
       LXA N,1                                              RKY6
BB     LDQ **,1          F                                  RKY6
       FMP H2            H/2                                RKY6
BC     FAD **,1          (H+H*SQRT(1/2))F3                  RKY6
BD     FSB **,1          H*SQRT(1/2)*F2                     RKY6
BE     FAD **,1          HF2                                RKY6
BF     FAD **,1          (H/2)F1                            RKY6
       LRS 35                                               RKY6
       FMP THIRD         1/3                                RKY6
BG     FAD **,1          Y0                                 RKY6
BH     STO **,1          Y                                  RKY6
       TIX BB,1,1                                           RKY6
       LXD CMN,1                                            RKY6
       LXD N,4                                              RKY6
       TRA 1,4                                              RKY6
[illegible]  OCT 001000000000  1 IN CHARACTERISTIC          RKY6
CMN    HTR COMMON        LOC OF COMMON                      RKY6
H      HTR **            H=STEP SIZE                        RKY6
H2     HTR **            H/2                                RKY6
HSQRT  HTR **            H*SQRT(1/2)                        RKY6
N      HTR **            N=NO. OF EQUATIONS                 RKY6
```

SQRT OCT 2005520236332 SQRT(1/2) RKY6 0121
THIRD OCT 177525252525 1/3 RKY6 0122