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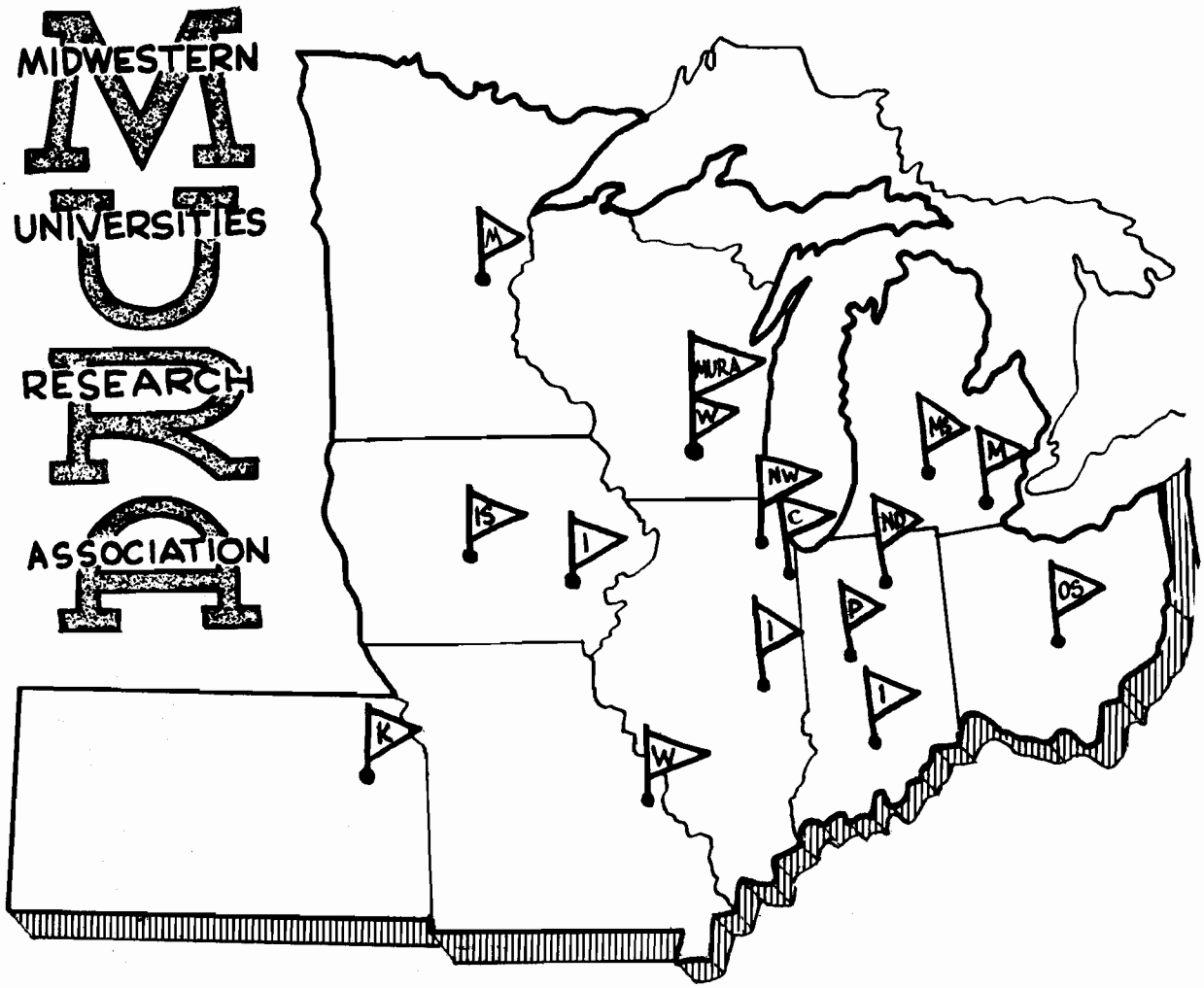

BEAM LIFETIME IN THE MURA ELECTRON FFAG ACCELERATOR

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ABSTRACT

The effects of radiation and multiple scattering on the amplitudes of the betatron oscillations of the electron beam in the MURA electron FFAG accelerator are considered. As a result of radiation anti-damping of the radial betatron oscillations, the amplitude of these oscillations grows and reaches the stability limit in about 6.5 seconds. A combination of multiple scattering and radiation damping leads to an equilibrium value for the amplitude of the vertical betatron oscillations which is not very much larger than its initial (adiabatically damped) value.

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I. INTRODUCTION

At injection, the radial and vertical betatron oscillations acquire amplitudes $A_{ri} \cong 0.6 - 1.0$ cm. and $A_{vi} \cong 0.5 - 1.5$ cm., respectively. During the acceleration time, the betatron oscillations are adiabatically damped. Since the acceleration time is only a few milliseconds, changes in the betatron oscillation amplitudes which occur during this time due to effects other than adiabatic damping can be neglected. Thus, when the beam reaches full energy, the radial and vertical betatron oscillations have amplitudes A_{ro} and A_{vo} given by

$$A_{ro} = \sqrt{\frac{B_i}{B_f}} \; A_{ri}$$
$$A_{vo} = \sqrt{\frac{B_i}{B_f}} \; A_{vi} \qquad (1)$$

where B_i and B_f are the values of the magnetic field at the initial and final orbit radii, respectively. When the beam reaches full energy, the amplitudes of the betatron oscillations, consequently, will be $\leq$ 1 mm. However, because of radiation anti-damping and multiple scattering of beam electrons on residual gas atoms, the amplitude of the radial betatron oscillations increases rather rapidly with time. This increase continues until the radial stability limit is reached and the electrons are lost from the beam.

II. INCOHERENT RADIATION OF THE ELECTRON BEAM

The effects of radiation on the betatron oscillation are determined by the rate of radiation of the electrons in the beam. Only the incoherent radiation will be considered.

The radiation loss per turn in general spiral sector and radial sector accelerators has been evaluated by Parzen.[1] We shall specialize his results to the radiation loss in the MURA model.

The rate of radiation of an electron moving in the median plane is given by

$$P_\gamma = \frac{2c}{3}\left(\frac{e^2}{mc^2}\right)^2 \beta^2 \gamma^2 B_z^2 \tag{2}$$

where B_z is the magnetic field in the median plane, and $\gamma = E/mc^2 = (1 - \beta^2)^{-1/2}$. Assuming that $\left(\frac{1}{r}\frac{dr}{d\theta}\right)^2 << 1$, the radiation per turn is

$$W_1 = \frac{2}{3}\left(\frac{e^2}{mc^2}\right)^2 \beta \gamma^2 \int_0^{2\pi} B_z^2 \, r \, d\theta \tag{3}$$

The magnetic field and the radius may be written as follows:

$$B_z = B_m \left(\frac{r}{r_m}\right)^k \sum_n g_n \cos n N \theta \tag{4}$$

$$r = r_o (1 + x) \tag{5}$$

where B_m is the maximum value of the magnetic field (at the center of a positive magnet), and r_o and r_m are the average and maximum radii of the equilibrium orbit, respectively. In Eq. (4) the Fourier coefficients, g_n, are normalized so that

$$\sum_n g_n = 1 \tag{6}$$

Using the expansions given by Eqs. (4) and (5) in Eq. (3), the radiation per turn is given by

$$W_1 = \frac{4\pi}{3} \frac{e^2}{r_o} \frac{C^2}{2} \left(\frac{r_o}{r_m}\right)^{2k+2} \left(\sum_n g_n^2\right) F \tag{7}$$

where C is the circumference factor, and

$$F = \frac{2}{\sum_n g_n^2} \left\langle (1 + x)^{2k+1} \left(\sum_n g_n \cos n N \theta\right)^2 \right\rangle_{\text{Av. over 1 turn}}$$

$$= 1 + \frac{3}{4} k (2k + 1) \left(\frac{r_m - r_o}{r_o}\right)^2 \tag{8}$$

The various factors which occur in Eq. (7) have the following interpretation: The quantity

$$\frac{4\pi}{3} \frac{e^2}{r_o} \frac{C^2}{2} \left(\frac{r_o}{r_m}\right)^{2k+2}$$

gives the radiation per turn of an electron moving in a circle of radius r_o in a magnetic field $B_z = B_o \cos N \theta$, where B_o is the field at $r = r_o$ and $\theta = 0$ (center of a positive magnet). The factor $(\sum_n g_n^2)$ gives the effects of the higher ($n > 1$) Fourier components of the magnetic field. The factor F gives the effects of orbit scalloping.

For the MURA model operated as a two-way accelerator, the appropriate accelerator parameters are[2] $C = 8.04$, $k = 9.3$, $r_o = 198$ cm., $\frac{r_m - r_o}{r_o} = 0.0282$, $\left(\frac{r_o}{r_m}\right)^{2k+2} = 0.56$, $\sum_n g_n^2 = 1.21$, and $F = 1.11$. Thus

$$W_1 = 7.4 \times 10^{-8} \gamma^4 \quad \text{e.v./turn} \tag{9}$$

For a kinetic energy of 36 Mev, $W_1 = 1.92$ e.v./turn.

III. RADIATION DAMPING AND ANTI-DAMPING OF THE BETATRON OSCILLATIONS

The effects of radiation on the betatron oscillations in AG and FFAG accelerators have been considered by Robinson[3] and by Kolomenski and Lebedev.[4] In general, radiation losses give rise to three kinds of forces

which perturb the betatron oscillations. These are: (1) a damping force due to the radiation reaction, (2) a driving force due to coupling between betatron and synchrotron oscillations, and (3) a driving force due to the quantum emission of radiation. The quantum emission effects are negligible in the MURA model.

If the beam reaches full energy with vertical and radial betatron oscillation amplitudes A_{vo} and A_{ro}, respectively, the amplitudes at time t are given by

$$A_v = A_{vo}\, e^{\alpha_v t}$$
$$A_r = A_{ro}\, e^{\alpha_r t} \tag{10}$$

The damping rates (α_v and α_r) have been calculated by Robinson[3] who obtains

$$\alpha_v = -\frac{P_\gamma}{2E}$$
$$\alpha_r = \frac{P_\gamma}{2E}(1-\alpha) \tag{11}$$

where P_γ is the radiation loss per second, E is the total energy of a beam electron, and α is the momentum compaction factor ($\alpha = \frac{1}{k+1}$ for the MURA accelerator). The damping of the vertical betatron oscillations is due directly to the radiation reaction force. The radial anti-damping is due to a combination of a damping force resulting directly from radiation reaction and a driving force due to coupling of betatron and synchrotron oscillations. In Robinson's derivation of the damping rates given by Eq. (11), it is assumed that the vertical betatron oscillations are decoupled from the radial betatron

oscillations and synchrotron oscillations. Using $k = 9.3$, a kinetic energy of 36 Mev, and a frequency of revolution of 23.2 Mc/sec., Eqs. (9) and (11) yield $P_\gamma \cong 44.5$ Mev/sec. and

$$\alpha_v = -0.61 \text{ sec}^{-1}$$

$$\alpha_r = +0.55 \text{ sec}^{-1}$$

The radial oscillations are strongly anti-damped. Taking the initial amplitude of the radial betatron oscillations to be $A_{ro} = 1$ mm. and the radial stability limit to be 0.019 (3.8 cm.), the stability limit will be reached in about 6.5 seconds.

Clearly, continuous injection of electrons will result in the presence of radial betatron oscillations of large amplitude (up to about 3.8 cm.). In the "two-way" operation of the accelerator, these large amplitude radial betatron oscillations will cause a large spread in the intersection angle of the colliding beams; this spread may lead to ambiguities in the interpretation of experimental results (e.g., in Møller scattering). Since the amplitude of the radial betatron oscillations grows exponentially with time, it may be feasible to use pulsed injection and to use the beam only for a short time (one or two seconds) after stacking.

IV. THE GROWTH OF BETATRON OSCILLATIONS DUE TO MULTIPLE SCATTERING

The effects of multiple scattering on the betatron oscillations have been considered by Blachman and Courant.[5] The multiple scattering of beam electrons on residual gas atoms produces an increase in the betatron oscillation amplitudes.

The effects of multiple scattering can be evaluated in terms of the increase per second of the mean-square amplitude of the betatron oscillation. We shall designate this rate of increase of the mean-square amplitude (either vertical or radial) by $\langle u^2 \rangle$. We shall make an estimate of the value of $\langle u^2 \rangle$ by treating the (vertical or radial) betatron oscillation as a pure sinusoidal oscillation of frequency $\nu\omega$, where ν is the tune and ω is the angular frequency of revolution. Then it can readily be shown that

$$\langle u^2 \rangle = \frac{r_o^2}{4\nu^2} \frac{\langle \theta^2 \rangle}{\tau_c} \tag{12}$$

where $\langle \theta^2 \rangle$ is the mean-square scattering angle, τ_c is the mean time between successive collisions with gas atoms, and r_o is the average orbit radius. The collision time, τ_c, is approximately independent of the electron's energy and is given by

$$\tau_c \cong \frac{1}{N\sigma_c} \tag{13}$$

where N is the number of residual gas atoms per cm.^{3}, and σ is the total cross section for the scattering of an electron on a gas atom. Using a screened Coulomb potential (screening length = $\frac{a_o}{Z^{1/3}}$, where $a_o = \hbar^2/me^2$ is the first Bohr radius in hydrogen) to determine the scattering, the cross section and mean-square scattering angle are given by

$$\sigma \cong \frac{\pi Z^2}{\rho^2} \left(\frac{e^2}{mc^2}\right)^2 \tag{14}$$

and

$$\langle \theta^2 \rangle \cong \frac{8\rho^2}{\beta^2\gamma^2} \log\left(\frac{\pi\beta\gamma}{\rho}\right) \tag{15}$$

where $\rho = \frac{1}{2} \frac{\hbar}{mc} \frac{Z^{1/3}}{a_o}$. Using $Z^2 = 50$, and taking the electron's kinetic energy to be 36 Mev, we obtain

$$\langle u^2 \rangle = \frac{0.44}{\nu^2} p \quad \frac{cm^2}{sec.} \tag{16}$$

where p is the residual gas pressure in units of 10^{-7} mm. of mercury.

We shall now consider separately the effects of multiple scattering on vertical and radial betatron oscillations.

A. Vertical Betatron Oscillations

The multiple scattering of beam electrons on residual gas atoms causes the mean-square amplitude of the vertical betatron oscillations to grow. However, because of the radiation damping (cf. Eqs. (10), (11)), this growth proceeds only for a time E/P_γ , after which equilibrium is established and the mean-square amplitude remains constant. The equilibrium value of the mean-square amplitude is

$$\langle A_v^2 \rangle = \frac{E}{P_\gamma} \langle u^2 \rangle \tag{17}$$

For a beam energy of 36 Mev and vertical betatron oscillation frequency $\nu_z = 5.32$

$$\langle A_v^2 \rangle = 1.14\, p \times 10^{-2}\ cm.^2$$

This corresponds to a root-mean-square amplitude for the vertical betatron oscillations of about $1.1 \sqrt{p}$ millimeters. This value is not greatly different from the adiabatically damped initial amplitude.

B. Radial Betatron Oscillations

We shall consider now the growth of the amplitude of the radial betatron oscillation due to mulitple scattering only. We consider an electron

which initially has a small radial betatron oscillation amplitude. Because of multiple scattering, this amplitude will grow until it reaches the stability limit; the electron is then lost from the beam. The probability, $P(A_s, t)$ that at time t the radial amplitude of betatron oscillation is less than the stability limit A_s (measured in centimeters) is given by[5]

$$P(A_s, t) \cong e^{-\frac{(2.4)^2 \langle u^2 \rangle}{2 A_s^2} t} \tag{18}$$

Because of multiple scattering, the beam tends to decay exponentially with a lifetime

$$\tau_{m.s.} \cong \frac{2 A_s^2}{(2.4)^2 \langle u^2 \rangle} \tag{19}$$

Using $\nu_r = 6.38$, Eqs. (16) and (19) yield

$$\tau_{m.s.} = \frac{32 A_s^2}{p} \tag{20}$$

Because this multiple scattering lifetime is very much longer than the radiation anti-damping lifetime, the growth of the radial betatron oscillations is due almost entirely to the radiation anti-damping.

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