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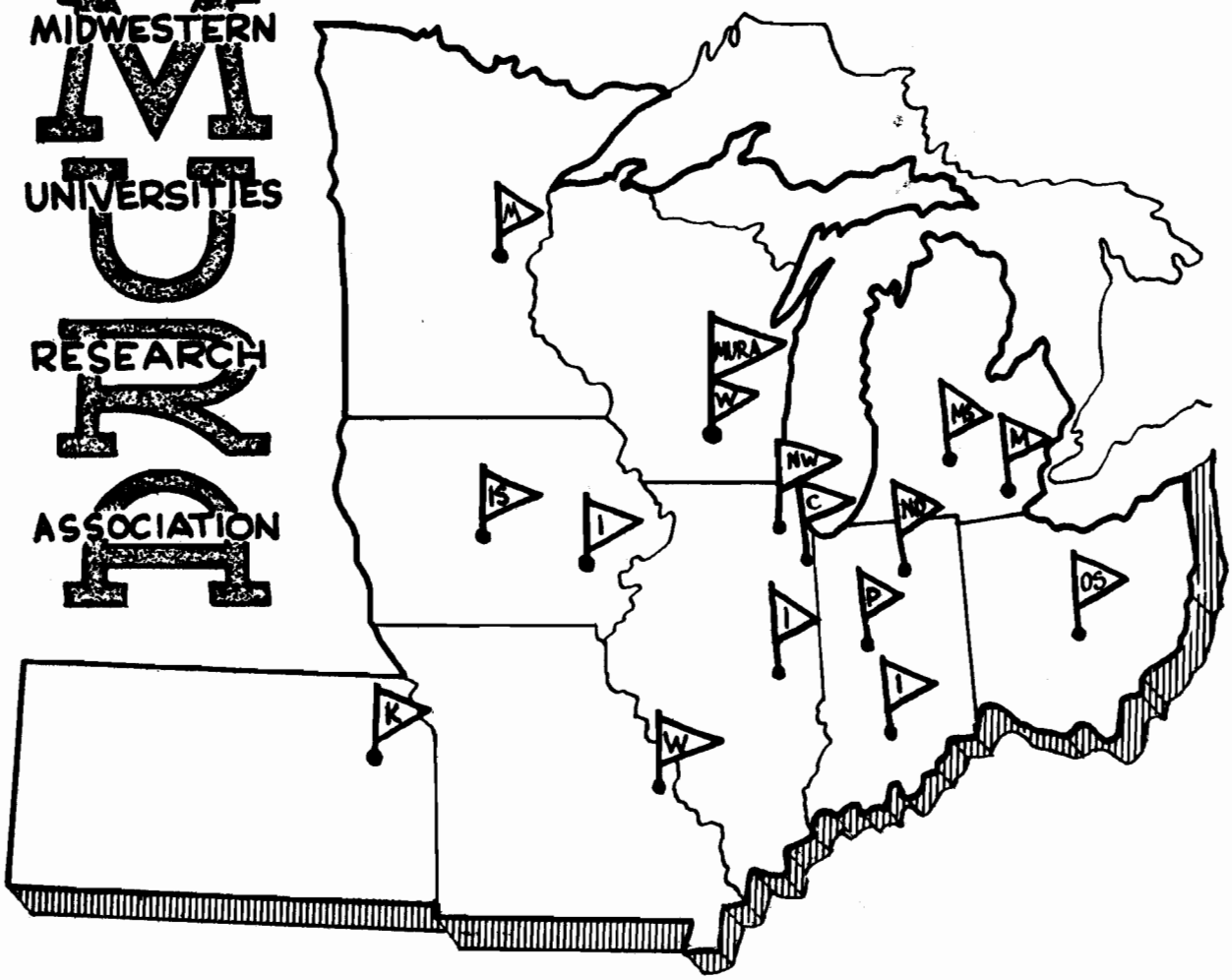

REPORT SQUAVER II
Program (246)
August, 1959

NUMBER 501
Computer Program
Internal

SQUAVER II
Program (246)
Brandt Kehoe

This program is a generalization of SQUAVER (or SQUAVE, the original problem), program 190, to include motion off the median plane. It was suggested by George Parzen in a memorandum dated April 30, 1959.

SQUAVER II calculates the orbit of a particle in a magnetic field H_x, H_y, H_θ. The differential equations describing the motion and integrated by this program are:

$$\frac{dx}{d\theta} = (A + x)\frac{p_x}{\sqrt{1 - p_x^2 - p_y^2}}$$

$$\frac{dy}{d\theta} = (A + x)\frac{p_y}{\sqrt{1 - p_x^2 - p_y^2}}$$

$$\frac{dp_x}{d\theta} = \sqrt{1 - p_x^2 - p_y^2} - \Lambda (A + x)\left[H_y - \frac{p_y}{\sqrt{1 - p_x^2 - p_y^2}} H_\theta\right]$$

$$\frac{dp_y}{d\theta} = \Lambda (A + x)\left[H_x - \frac{p_x}{\sqrt{1 - p_x^2 - p_y^2}} H_\theta\right],$$

Magnetic Field

The field is specified on the median plane (y - component only) in a set of intervals in θ. The number and width of these intervals are to be determined by the user within the following framework:

1) The magnetic field around the whole circle is divided into S superperiods, all identical;
2) Angular widths are expressed as fractions of the "natural sector width", $\frac{2\pi}{N}$;
3) The set of intervals described by the input must span one superperiod, thus the sum of the interval widths, I_j, must equal N/S;
4) The total number of intervals in a superperiod must be ≤ 64.

In each of these intervals, for example the Jth interval, the field is specified in the following form:

$$(B_y)_j = \lambda_j (A + x)^k \left\{A_j(x) \sin \eta_j + B_j(x) \sin X_j + C_j(x)\right\}$$

$$\text{where } \eta_j = 2\pi\left[n_j \frac{N\varphi}{2\pi} - \alpha_j\right]$$

$$X_j = 2\pi\left[3 n_j \frac{N\varphi}{2\pi} - \beta_j\right]$$

$$A_j(x) = \sum_{i=0}^{6} a_i^j x^i \qquad B_j(x) = \sum_{i=0}^{6} b_i^j x^i \qquad C_j(x) = \sum_{i=0}^{6} c_i^j x^i$$

and

$$\varphi = \theta - \frac{1}{\omega N} \ln (A + x).$$

The field off the median plane is given by:

$$H_x = \left(\frac{\partial B_y}{\partial x}\right) y \quad ,$$

$$H_\theta = \frac{1}{A + x} \left(\frac{\partial B_y}{\partial \theta}\right) y \text{, and}$$

$$H_y = B_y - \frac{y^2}{2} \quad \nabla^2 B_y.$$

These fields are correct to second order in y. The derivatives indicated are calculated numerically according to the following formulae:

$$\frac{\partial B_y}{\partial x} = B_{y,x} = \frac{1}{2\epsilon} \left\{ B_y(\theta, x + \epsilon) - B_y(\theta, x - \epsilon) \right\}$$

$$\frac{\partial B_y}{\partial \theta} = B_{y,\theta} = \frac{1}{2\left(\frac{2\pi\delta}{N}\right)} \left\{ B_y\left(\theta + \frac{2\pi\delta}{N}, x\right) - B_y\left(\theta - \frac{2\pi\delta}{N}, x\right) \right\}$$

$$\nabla^2 B_y = \left(\frac{\partial^2 B_y}{\partial x^2} + \frac{1}{A+x} B_{y,x} + \frac{1}{(A+x)^2} \frac{\partial^2 B_y}{\partial \theta^2} \right)$$

$$= \frac{1}{\epsilon^2} \left\{ B_y(\theta, x+\epsilon) + B_y(\theta, x-\epsilon) - 2B_y(\theta, x) \right\}$$

$$+ \frac{1}{A+x} B_{y,x}$$

$$+ \frac{1}{(A+x)^2} \frac{1}{\left(\frac{2\pi\delta}{N}\right)^2} \left\{ B_y\left(\theta + \frac{2\pi\delta}{N}, x\right) + B_y\left(\theta - \frac{2\pi\delta}{N}, x\right) - 2B_y(\theta, x) \right\}.$$

Discontinuities in the Field

If B_y is discontinuous as a function of θ at an interval boundary, the momenta P_y and P_x will change discontinuously across this boundary. The change in P_x, ΔP_x is of second order in y and is not computed by this program. The discontinuity in P_y, ΔP_y to first order in y, is given by

$$\Delta P_y = -\Lambda \frac{xy}{A+x} (\Delta B_y) = -\Lambda \frac{p_x}{\sqrt{1 - p_x^2 - p_y^2}} \, y (\Delta B_y).$$

This change is calculated by SQUAVER II.

Interval Boundaries

The magnetic field may be of a radial or spiral nature, depending upon whether $1/\omega$ is zero or non-zero. Either condition may exist with radial interval boundaries. Spiral fields with radial boundaries correspond to radial straight sections in a spiral field accelerator. The boundary of a spiral field interval may, however, also be spiral. This last condition is indicated to the program by setting μ non-zero. The boundary is then given by:

$$\theta - \frac{1}{\omega N} \ln (A + x) = \text{constant.}$$

Integration Procedure

Each interval is divided into NRK equal steps, and except for the special cases listed below, integration is from the beginning of one of these steps to the end. Integration is by the Gill-Kutta method using MU RKY3.

Exceptions to equal steps:

a) Initial step: The first step integrated by the program is from the initial position, $\frac{N\theta_o}{2\pi}$ to the end of the step in which it is located. See Fig. 1 below, steps i) and ii).

b) Final step in an interval: For a non-spiraling machine this step should be the same as all others except that it prevents the accumulation of round-off error. The length of the step is calculated to end at the boundary of the interval. For a spiraling boundary machine, this step is calculated to take into account the fact that the angular positions of the boundary is a function of x, the radius. Again, the length of the step is designed to complete the integration at the boundary. This calculation includes a first order approximation to the value of x at the end of the step. See step iii) on Fig. 1.

c) First step in an interval: In order to put the integration back in step with the fixed RK steps, this change is introduced. See step iv) on Fig. 1.

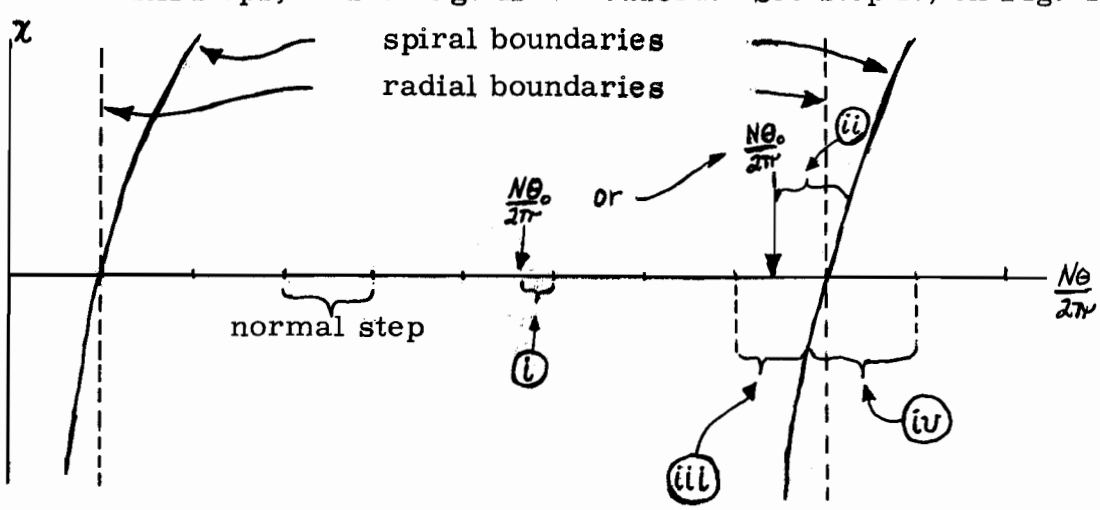

Figure 1

Input

The agendum sheet is largely self explanatory. However, the values of NPO and $(N\theta/2\pi)_o$ deserve some consideration at this point. Because of the starting procedure, see point (a) in the section above, it is best to give $(N\theta/2\pi)_o$ a value of $\sim 10^{-6}$ less than the point at which printout is desired, and set NPO = 1 (=NPLTO if plotting is desired). It is important that $(N\theta/2\pi)_o$ be greater than -N/S.

Repeat cards are used to load intervals with parameters identical to those of another interval which has been entered conventionally. For example, if intervals 1 and 2 have been specified on the top of page 2 of the SQUAVER II AGENDUM, repeat cards of the form:

3	1
5	1
4	2
6	2

will give a six interval superperiod with odd intervals identical and equal to interval 1, and even intervals identical and equal to interval 2. Here, as mentioned above, 3 times the width of interval 1 (contents of 1002) plus 3 times the width of interval 2 (contents of 1030) should equal N/S.

If there are three or more different intervals, the addresses for non-zero parameters should be entered in the last ADDRESS column on the agendum sheet.

Output

The only unlabeled numbers printed on the output are the interval data-blocks of 4 lines consisting of a line label and 7 numbers just below the line giving K, $1/W$, etc. The label of the first line is the interval number j. It is followed by H_j, NRK_j, I_j, λ_j, n_j, α_j, and β_j. The other three lines contain A, a_o^j, a_1^j, . . . a_6^j; B, b_o^j, b_1^j, . . . b_6^j; C, c_o^j, c_1^j, . . . c_6^j.

The last line of the computed values has as its line label the number of the last RK step.

If NPLTO and DNPLT are not zero, the program plots P_x vs. x and P_y vs. y following the calculation. If fewer than two points have been stored, plotting is suppressed.

Series Runs

When SQUAVER II reads an ID card with NEWSERIES in columns 37-45,

it clears all data and sets A, x_{max}, y_{max}, $(\sqrt{p_x^2 + p_y^2})_{max}$, Λ, all λ, and all n as indicated on the agendum sheet before reading new data. For subsequent runs, the ID card may be blank in 37-45. In this case, existing data is not altered unless new values are read in. To change interval data it will be necessary to alter each interval as desired, even if repeat cards are used in loading them. Specifically, in the example above in the section on REPEAT CARDS, if α in intervals 1, 3, 5 is to be changed to 0.5, the following three cards must be entered:

```
1005    5-1
1063    5-1
1121    5-1.
```

Any number of series - each begun by a run with NEWSERIES on the ID card - may be loaded at one time.

Search Feature

If SENSE SWITCH 6 is up, the program will halt a series as soon as any one of the runs goes NE Runge-Kutta steps. The program will then read cards until either a new series is indicated by the NEWSERIES card or until an ENDINPUT card is encountered.

Appendix

Operating Instructions

Loading order: Binary deck, any number of NEWSERIES decks, ENDINPUT card.

Each NEWSERIES deck consists of a NEWSERIES run followed by any number of series runs. Each NEWSERIES run consists of: ID card with NEW-SERIES; integer cards; END INTEGERS card; floating point data cards; END FP NOS card; repeat cards; and BLANK CARD (blank in 1-36). Each series run consists of: ID card with right field blank; integers, if any; END INTEGERS; floating point numbers; END FP NOS; BLANK CARD.

Sense Switches

SW1: Normally down, up - HPR at $(12510)_8$. Pushing start button causes program to read a new run. Suggested use is stopping for tape dump. At this point no index registers or sense lights need be saved.

SW2: Normally down, up causes transfer to end run and read new run.

SW3: Not used.

SW4: Normally down, up causes debugging print out in the auxiliary program RKAUX. If up RKAUX prints out two lines on each pass

(4 passes per RK step). The first line contains:

$(N\theta/2\pi)$; x; P_x; y; P_y; P_θ; sin η ; H_x.

The second line contains:

B_y; $B_{y,x}$; $B_{y,\theta}$; $\nabla^2 B_y$; $\frac{2\pi}{N} x'$; $\frac{2\pi}{N} P_x'$; $\frac{2\pi}{N} y'$; $\frac{2\pi}{N} P_y'$.

SW5: Not used.
SW6: Normally down. See SEARCH FEATURE.

STOPS

ENDINPUT card produces HTR $(12000)_8$ at $(12003)_8$. Push START button to read new run.

TAPE DUMP STOP HPR at $(12510)_8$. See SW1 discussion.

Other stops probably reflect input difficulties - either user error or program limitations. The chief difficulty of latter sort is referred to above in the first paragraph under INPUT. If $0 \gg (N\theta/2\pi_o) \gg -10^{-6}$ difficulties of a mysterious nature may result. In general, if the difference between the end of the first RK step and $(N\theta/2\pi)_o$ is not significant with respect to the larger of N/S and $(N\theta/2\pi)_o$, trouble may result. For significance, it is required that this difference is not destroyed in round off.

Amendments to the Program

SQUAVER II was programed with some thought to future alteration. For example, the momentum discontinuity calculation may be improved (PBUMP subroutine), field derivation may be made more exact or even analytic, field definition may be changed, etc. These can be done largely independently without altering the program. If more parameters are needed per interval, reassemble with NOPAR and NOPARA changed to reflect the new number of parameters. This will require a new agendum sheet.

Obviously, because of the large quantity of data, the subroutine cannot be totally independent, and care should be exercised in any revision.

Restrictions

This program <u>does not</u> integrate backwards nor can it be made to do so without major revision. A sample agendum form is attached.

SQUAVER II AGENDUM

PROGRAM 246

	Parameter	Address	n	Remarks
INTEGERS	ID	899		NEWSERIES
	NE	900		No. of RK steps.
	NPO	901		No. of RK steps to first print.
	ΔNP	902		No. of RK steps between prints.
	NPLTO	903		No. of RK steps to first plot storage.
	ΔNPLT	904		No. of RK steps between plot storage
				END INTEGERS

	Parameter	Address	n	exp	Remarks
FLOATING POINT NUMBERS	X_{max}	905			(= 0.5 if not set)
	Y_{max}	918			(= 0.5 if not set)
	$(\sqrt{P_x^2+P_y^2})_{max}$	906			(= 0.9 if not set)
	K	907			
	S	908			
	A	909			(= 1.0 if not set)
	1/W	910			
	N	911			
	μ	912			=1.0 for spiral sector boundaries =0.0 for radial sector boundaries
	$(N\theta/2\pi)_o$	913			
	X_o	914			
	P_{x_o}	915			
	Y_o	916			
	P_{y_o}	917			
	ϵ	919			Δ X for field derivatives
	δ	920			$\Delta \frac{N\theta}{2\pi}$ for field derivatives
	Λ	928			(= 1.0 if not set)

SS6 UP - CHECK BOX FOR SEARCH FEATURE

INTERVAL DATA

Address for parameter of Jth interval given by the address of that parameter in I_1 plus (J - 1) x 28.

Address		I_1	Address		I_2	Address		I
1001	N_{RK}		1029	N_{RK}				
1002	I		1030	I				
1003	λ		1031	λ				
1004	n		1032	n				
1005	α		1033	α				
1006	β		1034	β				
1007	a_0		1035	a_0				
1008	a_1		1036	a_1				
1009	a_2		1037	a_2				
1010	a_3		1038	a_3				
1011	a_4		1039	a_4				
1012	a_5		1040	a_5				
1013	a_6		1041	a_6				
1014	b_0		1042	b_0				
1015	b_1		1043	b_1				
1016	b_2		1044	b_2				
1017	b_3		1045	b_3				
1018	b_4		1046	b_4				
1019	b_5		1047	b_5				
1020	b_6		1048	b_6				
1021	c_0		1049	c_0				
1022	c_1		1050	c_1				
1023	c_2		1051	c_2				
1024	c_3		1052	c_3				
1025	c_4		1053	c_4				
1026	c_5		1054	c_5				
1027	c_6		1055	c_6				

(λ , n = 1.0 if not set)

END FP NOS

REPEAT CARDS

Interval a = Interval b, where b previously specified.

a	b	a	b	a	b	a	b

BLANK CARD

MEMORANDUM

To: Technical Staff
From: G. A. Westlund
Subject: Changes to SQUAVER II (Program 246)

Certain changes to SQUAVER II have been suggested by G. Parzen.

A phase ripple has been added to make α vary with r.

$$\alpha = \alpha_0 + f \cos \left[2\pi \left(\frac{1}{g} \ln (A + x) + h \right) \right]$$

B_y is now augmented by ΔB_y, where

$$\Delta B_y = \Lambda (A + x)^k \, I \cos \left(\frac{J}{W} \ln (A + x) - 2\pi L \right) \cos (mN\theta - 2\pi q)$$

The new parameters can be entered on the standard SQUAVER II agenda as floating point numbers with the following addresses:

891	f	881	I
892	g	882	J
893	h	883	L
		884	m
		885	q

Note: m = 1 unless otherwise specified.

An additional print-out of these entry parameters has been provided:

PHASE RIPPLE F, G, H (f) (g) (h)

VALUE OF I, J, L, M, Q (I) (J) (L) (m) (q)

If the values for f and/or I are not entered or entered as 0, then the program proceeds as the original SQUAVER II (i.e. as through these modifications were removed even though other overwrite parameters may be present.)

Since these changes are considered permanent, this memo should become a permanent part of the original SQUAVER II report.

MEMO to: Computer Users

From: G. A. Westlund - January 1, 1960

Subject: Squaver IIC and Squaver IIIC (Program 279)

These programs are similar to Squaver II (Program 246) and Squaver III (Program 254) with the following exception requested by George Parzen.

A new magnetic field component, B_y^*, is calculated and is related to the former B_y as $B_y^* = B_y\left\{1+\left[\cos(\frac{j}{w}\ln(A+x)-2\pi e)\right]\left[d_o+d_1\cos(mN\theta-2\pi f)\right]\right\}$

Data parameters are entered via the standard Squaver II agendum sheets and additional parameters are specified on a supplemental agendum form. The symbols and corresponding addresses of the extra quantities are:

881	j	884	d_o
882	e	885	d_1
883	f	886	m

It is suggested that this memo become a permanent part of the Squaver II and Squaver III writeups.