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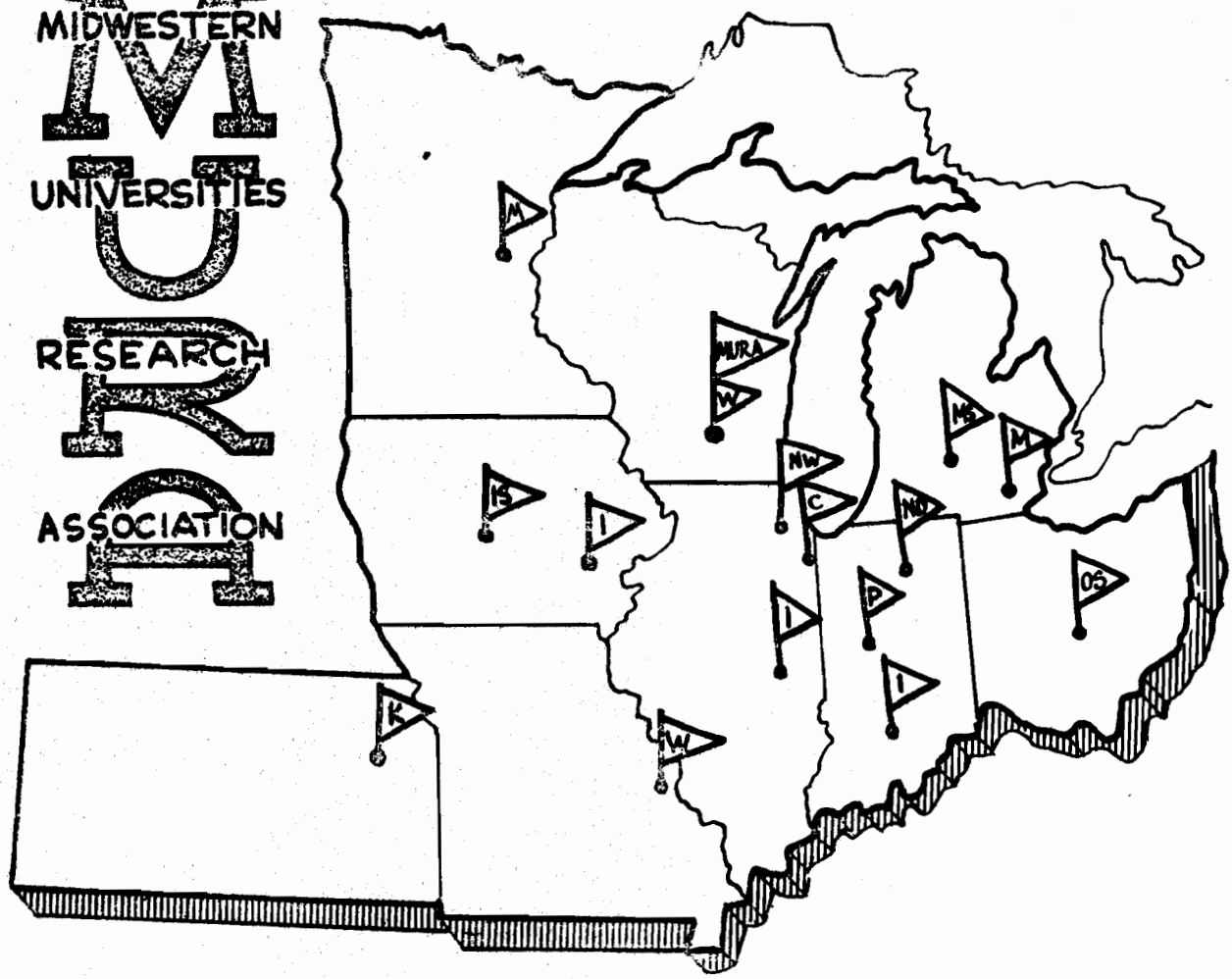

COMPUTER STUDIES OF BEAM STACKING EFFECTS

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ABSTRACT

The process of stacking beams at a certain energy causes an energy spread in beams already stacked at that energy or at other energies. A statistical study of this effect was made using the digital computer. RMS energy scattering and average displacements of the particles from their initial energies are given for $\Gamma = \sin \phi_s = 0.55$, for various initial energies near the turn off point of the r.f. oscillator.

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In MURA Report 106 the methods of radio-frequency (r - f) acceleration were presented and the detailed theory worked out. An oscillator supplying an r - f voltage to an accelerating gap is used to accelerate the particles. As the frequency of the oscillator is modulated, the buckets move up or down accordingly. As explained in 106, a bucket is a region in phase space of energies and phases within which the particles execute stable phase oscillations about a synchronous energy. This bucket is produced near the energy for which the revolution frequency of the particles is equal to the oscillator frequency.

Using this method, particles can be accelerated from injection to the output energy or stacked at an intermediate energy. This paper is concerned with the energy spread of a stacked beam caused by the stacking process. According to Liouville's Theorem, as a bucket moves upward, particles outside the bucket are forced downward in energy. Theoretically, particles outside the bucket are displaced downward an average amount equal to the average height b of the bucket ($b = \text{area}/2\pi$). A beam of particles which is approached by but not passed by a bucket will be scattered in energy also, but the average displacement will be zero.

This paper gives results of computer studies of the effects described above. A moving bucket is illustrated in Figure 1. For reasons to be explained later, good statistics can be obtained only if the beams are chosen initially to be spread uniformly throughout a small phase area $2\pi\Delta y$ (see top of Fig. 1).

A description of the project is given in Figure 2. An r - f oscillator is turned on at a frequency corresponding to $E_s (E_s \ll E_1)$ and modulated to a frequency corresponding to $E_f = E_3$ where it is turned off. The stable phase

angle ϕ_s is given by $\Gamma = \sin\phi_s = 0.55$. Six stacked beams, $E_1 - E_6$, separated by approximately half the height of the bucket, were investigated. The average displacements of the particles from their original energies and the RMS deviation from the average were calculated for each of the six beams. The beam at E_1 was studied much more thoroughly than the rest and its deviations are presumably more accurate. The numerical results in units of b are given in Table 1.

As predicted, the beams at E_4, E_5, and E_6 have an average displacement close to zero with fairly small RMS deviation. E_1, which should be the most accurate, shows a displacement which is within statistical error of its theoretical value (unity). The anomalous large mean displacement for E_2 and E_3 will be explained later.

Figure 1 is a plot of the curves of constant hamiltonian which in y, ϕ space are given by

$$\frac{1}{2}y^2 + \cos\phi + \Gamma\phi = C, \tag{1}$$

where

$$y = \left[\frac{2\pi h\,|f_s'|}{V}\right]^{1/2}(W - W_s), \tag{2}$$

$$= \left[\frac{2\pi h}{fV}\frac{df}{dE}\right]^{1/2}(E - E_s),$$

$$W = \int\frac{dE}{f(E)}, \tag{3}$$

$$f_s' = \frac{df}{dw} = f\left.\frac{df}{dE}\right)_{E = E_s}, \tag{4}$$

and h is the harmonic number, V is the gap voltage, f is the revolution frequency of the particles, ϕ is the phase of the particles relative to the oscillator, E_s is the synchronous energy, and

$$\Gamma = \frac{\dot{\nu}\,E_s}{h\,K_s f_s^2\,V} = \sin\phi_s, \tag{5}$$

where $\dot{\nu}$ is the frequency modulation rate of the oscillator and $\chi_s = Edf/fdE$. In y - units, the maximum total height of the bucket is 2.4 and the mean height b is 0.85.

As mentioned above certain conditions must be met to obtain good statistics. For large y in Eq. (1), a change of 2π in ϕ produces a change $\Delta y = 2\pi\Gamma/y$ in y. Δy has been magnified at the top of Figure 1 to illustrate what the curves of constant C look like for large y. If a beam of particles is represented by a line across phase space at one energy, it may cross the curves of constant C in as many as 3 places and as few as 1. Particles that lie on or near the particular curve which corresponds to the separatrix spend a long time near the unstable fixed point as the bucket ascends, and consequently are carried along to high energies. Thus, incorrect statistics would be obtained if the particles lie initially on a line at one energy and for proper statistics the particles initially should be spread uniformly throughout an area $2\pi\Delta y$.

On the basis of the above argument, it is conceivable that had the bucket been allowed to continue upward in energy to $E_f \gg E_1$, the final energies of the particles near the separatrix would have been much greater. This would decrease the average displacement for E_1, E_2, E_3 and would presumably bring them to the theoretical mean displacement (unity).

Table 1

	Beam position Relative to Energy at Turn-off	Average Displacement	RMS Scattering
E_1	-2.50	-1.05 ± 0.04	1.20 ± 0.04
E_2	-1.25	-1.24 ± 0.09	1.09 ± 0.09
E_3	0.0	-1.41 ± 0.11	1.38 ± 0.11
E_4	1.25	-.24 ± 0.06	.69 ± 0.06
E_5	2.50	-.05 ± 0.03	.35 ± 0.03
E_6	3.75	-.02 ± 0.02	0.239 ± 0.02

Table 1. Average energy displacement and RMS energy scattering for various initial beam energies, relative to turn-off point of bucket. ($\Gamma = 0.55$)
(All energies are in units of expected mean displacement for a beam passed by the bucket.)

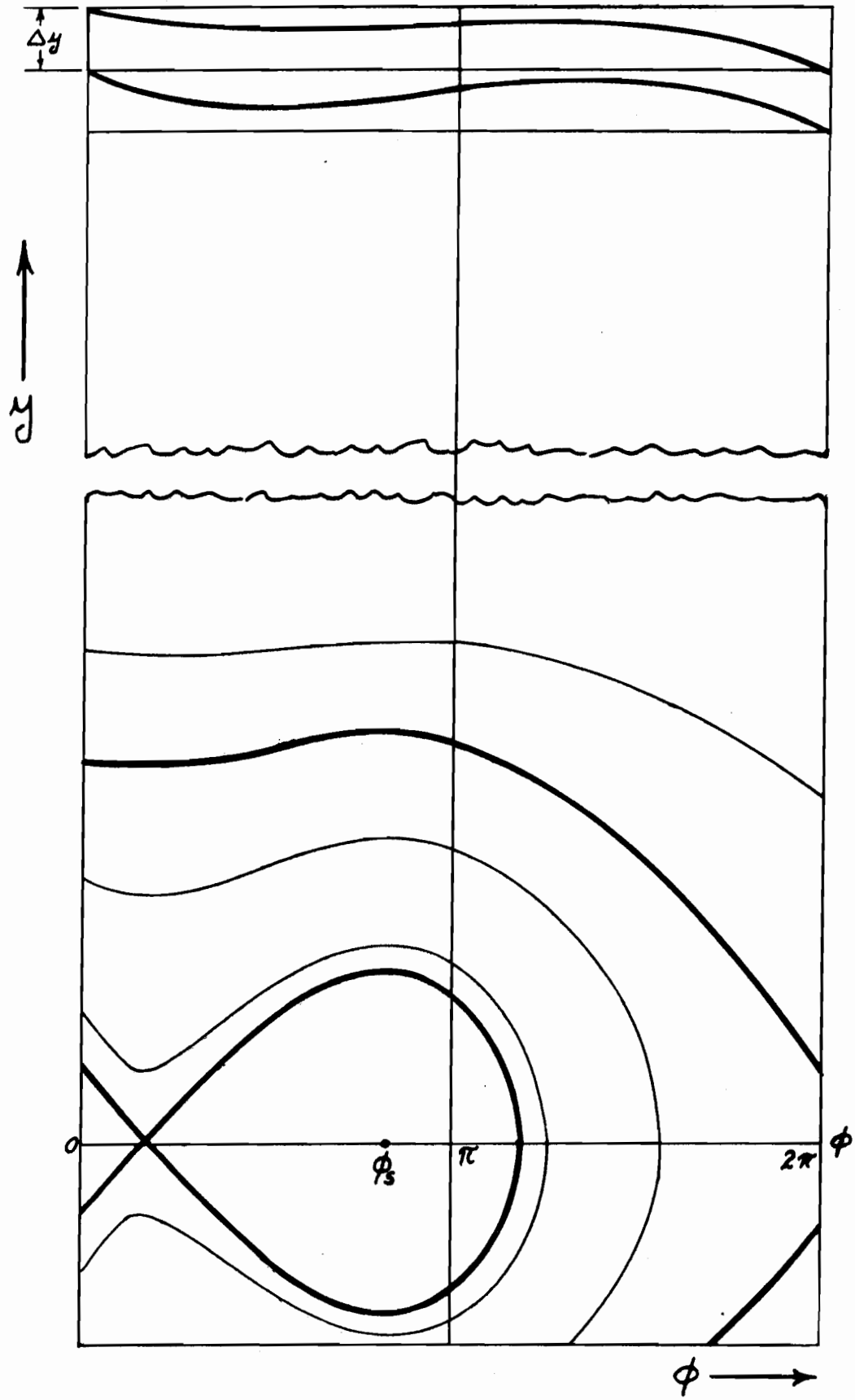

Fig. 1. Moving bucket $\Gamma = .5$ showing condition for good statistics.

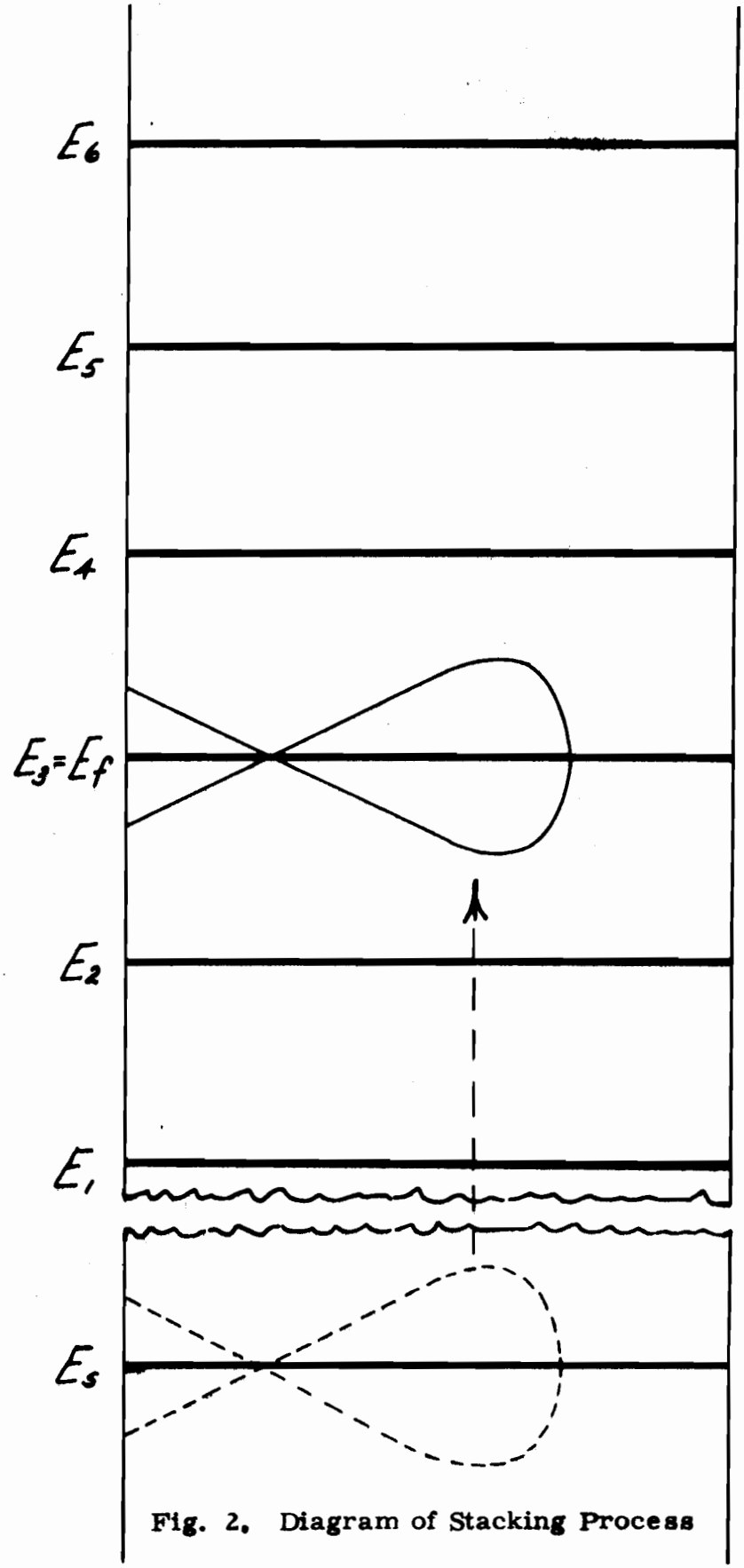

Fig. 2. Diagram of Stacking Process