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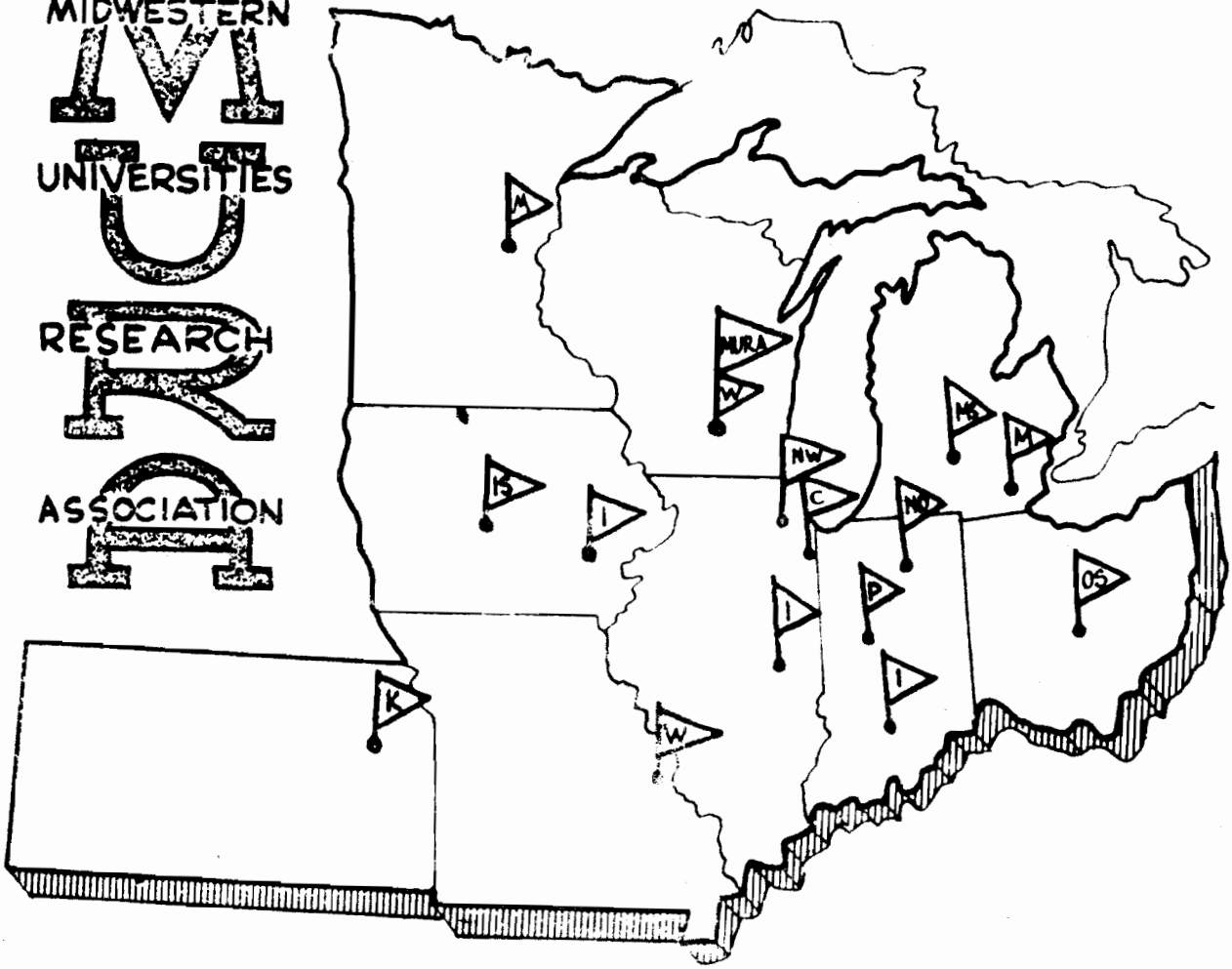

IMAGE FORCES IN THE THIRD MURA MODEL

J. Van Bladel

REPORT NUMBER 466

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IMAGE FORCES IN THE THIRD MURA MODEL

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June 12, 1959

ABSTRACT

The electric and magnetic fields which exist around the 50A beam are investigated. These fields are strongly influenced by the presence of the vacuum tank and of the magnetic circuit.

*AEC Research and Development Report. Research supported by the Atomic Energy Commission, Contract No. AT (11-1) 384.

I. FREE SPACE FIELDS

The beam current in the new model will hopefully reach intensities of 50 Amp. At the velocity of light, this corresponds to a linear charge density

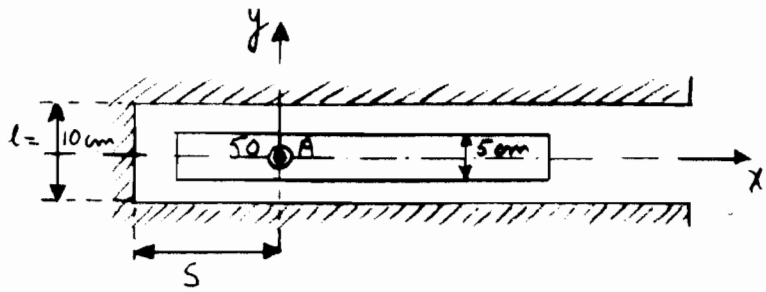

Figure 1

$$\Lambda = 0.166 \ \mu\text{Cb/m}$$

These currents and charge densities will create sizeable electric and magnetic fields in the vicinity of the beam, and it is desirable to have at least a qualitative idea of their importance, in particular to investigate the phenomena which occur when a ring of injected particles approaches the large coasting beam from the inside. To simplify the analysis, it will be assumed that phenomena are two-dimensional, and that both the main beam and the approaching beam travel in parallel directions. The effect of the ripple due to betatron oscillations will be investigated in a future report, and is neglected here. Both beams are composed of electrons. The total force on an approaching electron is

$$\bar{F} = \varepsilon \left[\bar{E} + \bar{v} \times \bar{B} \right]$$

where $\bar{E}$ and $\bar{B}$ are the fields due to the main beam, and $\bar{v}$ the velocity of the electron. We shall restrict our attention to electrons moving in the equatorial plane y = 0. In that plane, $\bar{B}$ is along the y axis, $\bar{E}$ along the x axis, so that the total

force is directed along the x axis, and is composed of a repulsive part due to the electric field and an attractive part due to the magnetic field.

$$\bar{F} = \varepsilon(E - vB)\bar{u}_x = \varepsilon(E - \beta c B)\bar{u}_x$$

If the main beam were isolated in space, both $\bar{E}$ and $\bar{B}$ would vary in a simple fashion, as evidenced in Figures 2 and 3. When the beam is assumed to be concentrated on a line, the variation is simply of the form $\frac{1}{x}$. When the beam is a cylindrical column with circular cross-section (radius a), filled with constant charge density, the $\frac{1}{x}$ law is valid up to the outer radius, and is replaced by a linear variation within the beam (Figure 4). From now on, it will be assumed that the beam is infinitely thin. This does not alter the image forces in any marked way and the free-space forces can always be modified as in Figure 4.

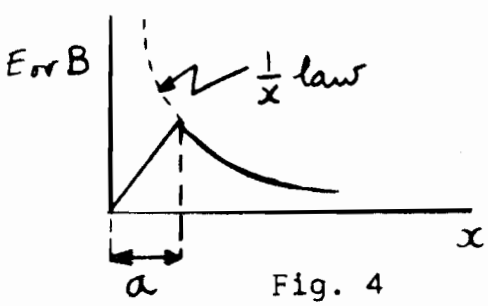

Fig. 4

When the approaching beam moves at the velocity of light, electric and magnetic forces cancel each other. When the velocity is lower, the electric repulsion dominates. Figure 5 indicates how much repulsion is experienced as a function of x for β =0.9. Forces are expressed in equivalent gauss at the velocity of light, i.e. in units of ε/c

$$\bar{F} = \frac{\varepsilon}{c}\left[cE - \beta B\right]\bar{u}_x$$

The forces are strongly modified by the structure surrounding the beam. We now proceed to investigate these additional electric and magnetic forces.

II. IMAGE ELECTRIC FIELD

Assuming the beam to be homogeneous, the physical situation is that of a charged line inside a rectangular metallic enclosure (the vacuum tank). The charged negative line will induce positive charges on the walls of the tank; these charges are the source of the image field, and the latter will clearly be attractive. Formulas for the potential inside the rectangle can be found in the literature.* In the equatorial plane, they reduce to

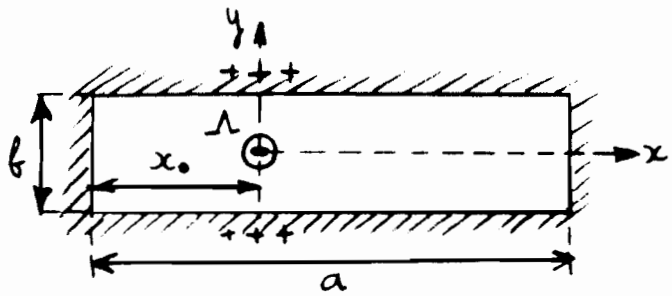

Fig. 7

$$\phi = \frac{2\Lambda}{\varepsilon_0 \pi} \sum_{n=1,3}^{\infty} \frac{1}{n} \cdot \frac{\sinh n\pi x_0/b \cdot \sinh n\pi(a-x-x_0)/b}{\sinh n\pi a/b}$$

Important simplifications occur because the tank is very flat $(a/b > 10)$ and the beam is well inside the tank $(x/b \gg 1\ ,\ a-x_0/b \gg 1)$ In other words, the small sides of the rectangle are so far away that their influence is negligible, and all the induced charges are very close to the beam. Finally

$$\phi = \frac{\Lambda}{\varepsilon_0 \pi} \sum_{n=1,3}^{\infty} \frac{1}{n} \cdot e^{-\frac{n\pi x}{b}}$$

and the electric field has a magnitude

$$E = -\frac{d\phi}{dx} = \frac{\Lambda}{\varepsilon_0 b} \sum_{n=1,3}^{\infty} e^{-\frac{n\pi x}{b}}$$

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*W. R. Smythe: "Static and Dynamic Electricity" 2nd edition, p. 107, Problem 51. McGraw Hill Book Company, 1950.

This variation is plotted in Figure 2 for the parameters adopted in the model. To check how concentrated the induced charges are around the locations $x = 0$, $y = \pm b/2$ (i.e. directly on top of the beam on the walls), the field due to positive line charges $\Lambda/2$ placed in free space at those locations has been added in Figure 2. As expected, the field is stronger close to the beam location than with the actual images, but weaker at large distances. The attractive force due to the image field is shown on Figure 5.

III. IMAGE MAGNETIC FIELDS

The magnetic circuit represented in Fig. 1 is an idealization of the actual profile, but the results obtained with that configuration should give good orders of magnitude. The presence of the vacuum tank, in D. C. conditions and with the $K_m = 1$ of the tank metal, can be ignored, and we are led to an investigation of the magnetic field of a current in a deep rectangular slot. This problem is a classical one in electric machinery, and its solution can be found in several textbooks.* With the coordinates indicated in Figure 1, it is found that

$$\bar{B} = \left[\frac{\sinh \pi (x+2s)/\ell}{\cosh \pi \frac{(x+2s)}{\ell} + 1} + \frac{\sinh \pi \frac{x+2s}{\ell}}{\cosh \pi \frac{x+2s}{\ell} - 1} + \frac{\sinh \pi \frac{x}{\ell}}{\cosh \pi \frac{x}{\ell} + 1} + \frac{\sinh \pi \frac{x}{\ell}}{\cosh \pi \frac{x}{\ell} - 1} \right] \frac{\pi}{2} \bar{u}_y \quad \text{(Gauss)}$$

for a current of 50 Amperes.

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*See e.g. B. Hague "Electromagnetic Problems in Electrical Engineering" p. 187, Oxford University Press, 1929.

There again, considerable simplification is afforded by the geometry, which makes $s/\ell \gg 1$, and one obtains, to an excellent approximation,

$$\bar{B} = \frac{\pi}{2}\left[2 + \sinh \pi \frac{x}{\ell}\left(\frac{1}{\cosh \pi \frac{x}{\ell} + 1} + \frac{1}{\cosh \pi \frac{x}{\ell} - 1}\right)\right]\bar{u}_y$$

for positive x, and not too close to the end of the air gap. The resulting field is depicted in Figure 6, where equal fluxes flow between the drawn lines of force. Clearly, the presence of the iron, which provides a desirable path of low reluctance, reinforces the field to the right of the beam, and weakens it to the left, where the lines of force have a longer path in air. This is again evidenced in Figure 3. In fact, the field extends, with constant value of roughly 6 Gauss, down to the injection region at the end of the air gap. The reinforcement of the magnetic field results in a reinforcement of the attractive force. This is shown in Figure 5. It is quite possible that the image forces will have a detrimental effect on the dynamics of the particles; furthermore, the existence of an additional field of a few Gausses at the injector is undesirable. One might consequently want to try and cancel out the image forces. Inverse currents of intensity 25A distributed on the iron walls on both sides of the beam current will certainly cancel out the field at the injector. To investigate the near field in more detail, we turn back to Hague (loc. cit.)

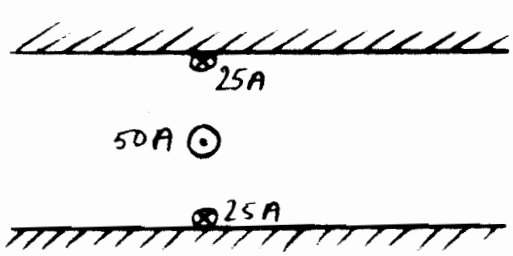

Fig. 8

for formulas giving $\bar{B}$ in the presence of a current lying on an iron surface of infinite permeability. Concentrating the countercurrents right above the beam, i.e. at x=0 y= $\pm \ell/2$, we find that, in the equatorial plane,

$$\bar{B} = -\pi\left[\tanh\frac{\pi(2x+s)}{\ell} + \tanh\frac{\pi x}{\ell}\right]\bar{u}_y \qquad \text{Gauss}$$

which, for positive x and because of the geometry of the system $(s \gg \ell)$, reduces to

$$\bar{B} = -\pi\left[1 + \tanh\frac{\pi x}{\ell}\right]\bar{u}_y \qquad \text{Gauss}$$

When plotted, this field happens to practically coincide (in magnitude but not in sign) with the total image field shown in Figure 5, so that almost perfect cancellation occurs. This is a happy accident, due to the fortunate choice of the geometry. It would not hold if the actual profile of the magnetic circuit were taken into account, or if the width of the vacuum tank were changed. The property also depends on the location of the countercurrents, which were situated right on top of the beam. In the machine the beam follows a sinusoidal path with maximum excursions of ± 3 cm on both sides of a straight line. The question is: Should one try to follow the meandering of the beam with the compensating winding, or is an average position at a constant radius good enough? To answer this question, the (now partial) cancellation of the image field has been plotted on Figure 9 for compensating windings situated 2.5 cm to the left and to the right of their optimum position. One feels justified to conclude from the figure that compensating windings with intensity 25A, placed in an average position, will cancel out the image forces at large distances (e.g. at the injector), and will limit the

short distance forces to a few Gauss, located in a region of radial extent roughly equal to the gap width.

As a final application, the force on an electron approaching the beam with velocity o.9c has been plotted in Figure 10. The beam is a cylindrical column of diameter 1 cm, and the compensating windings are located 2.5 cm to the left of their ideal position.

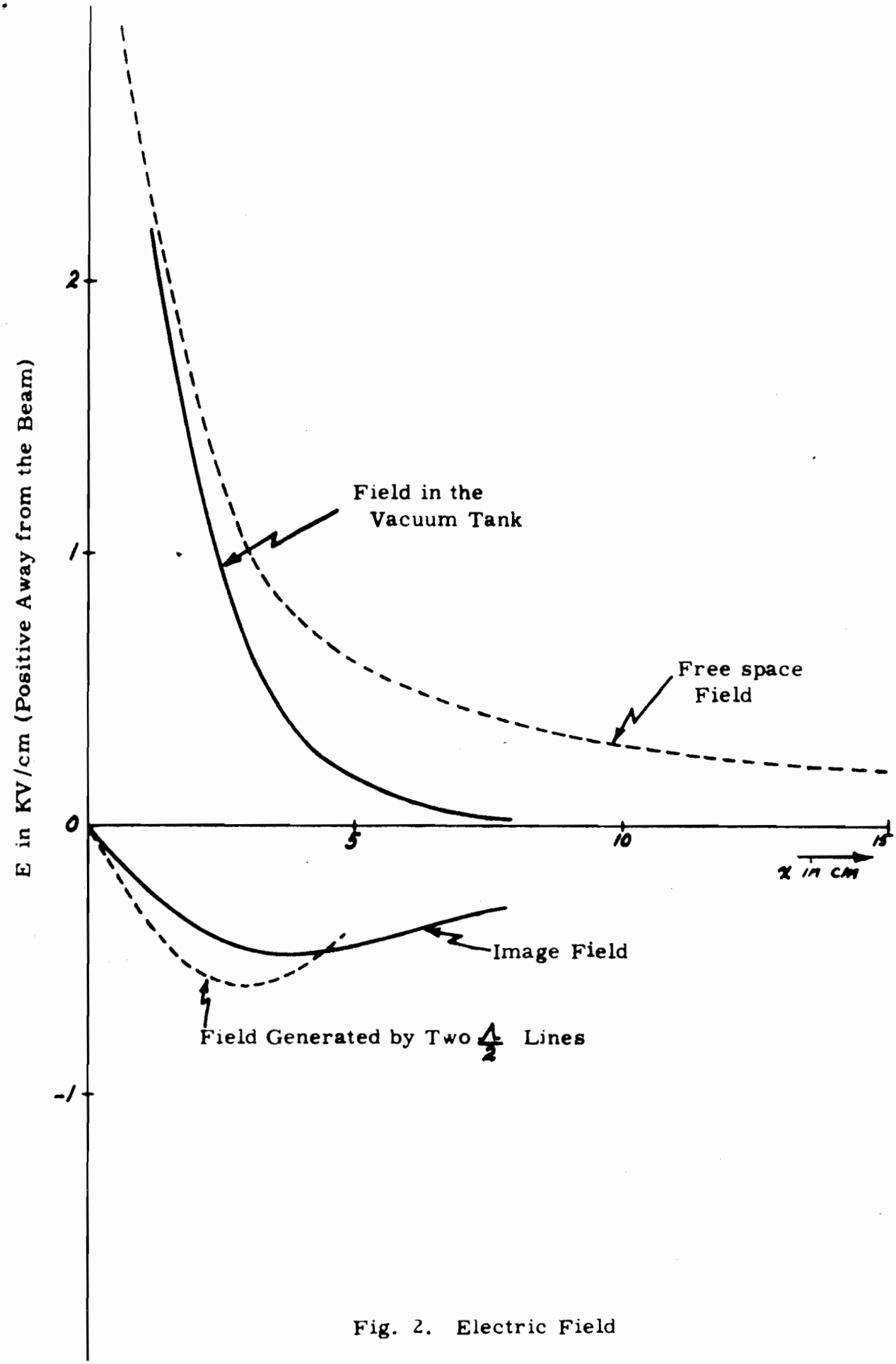

Fig. 2. Electric Field

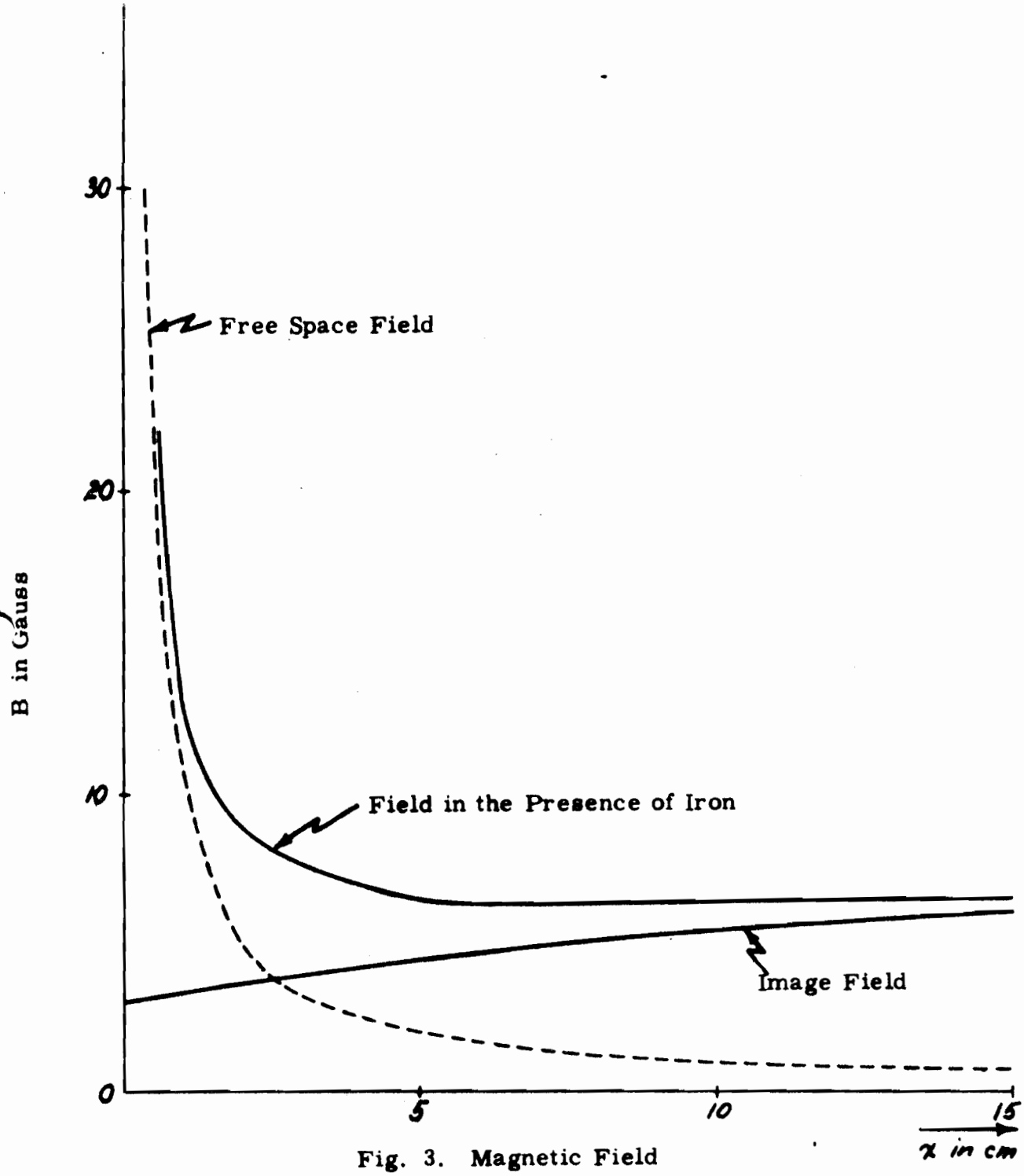

Fig. 3. Magnetic Field

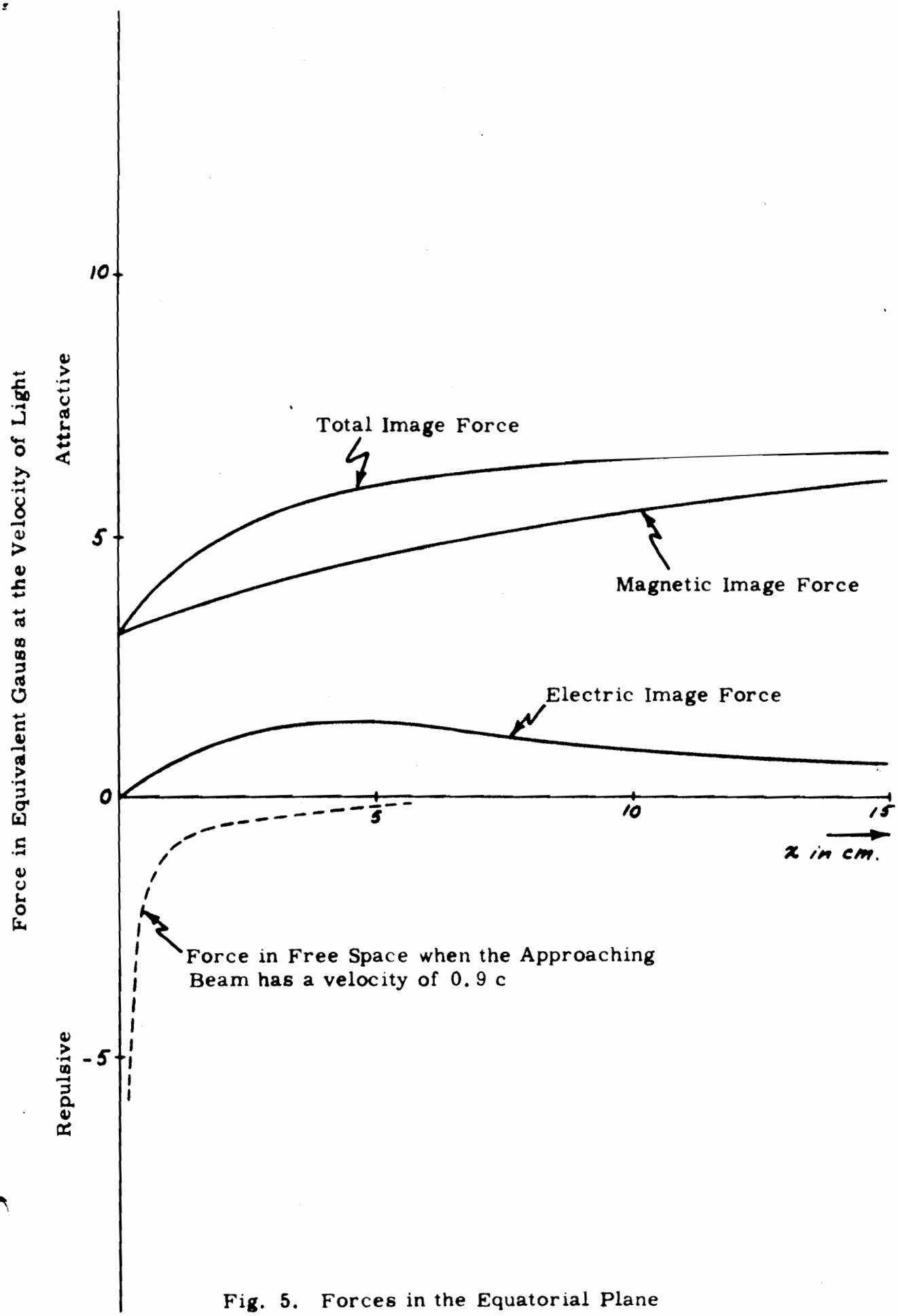

Fig. 5. Forces in the Equatorial Plane

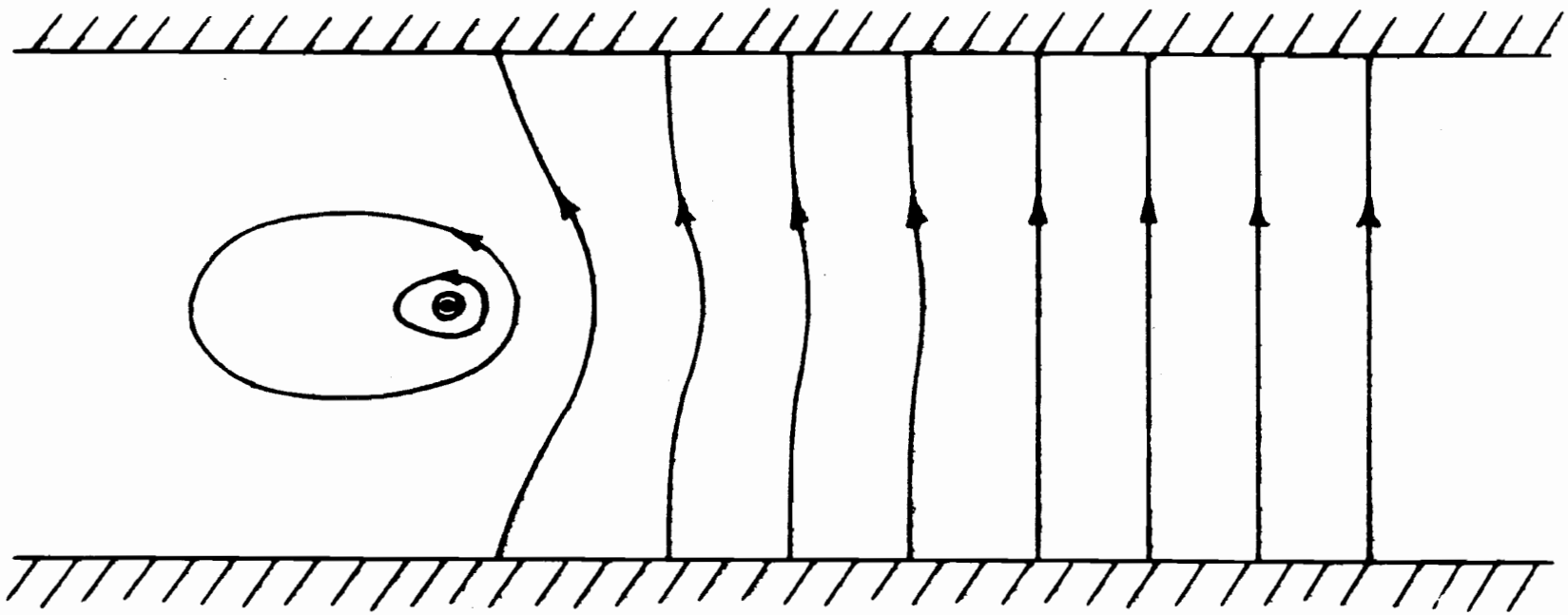

Fig. 6. Magnetic Field Around the Beam

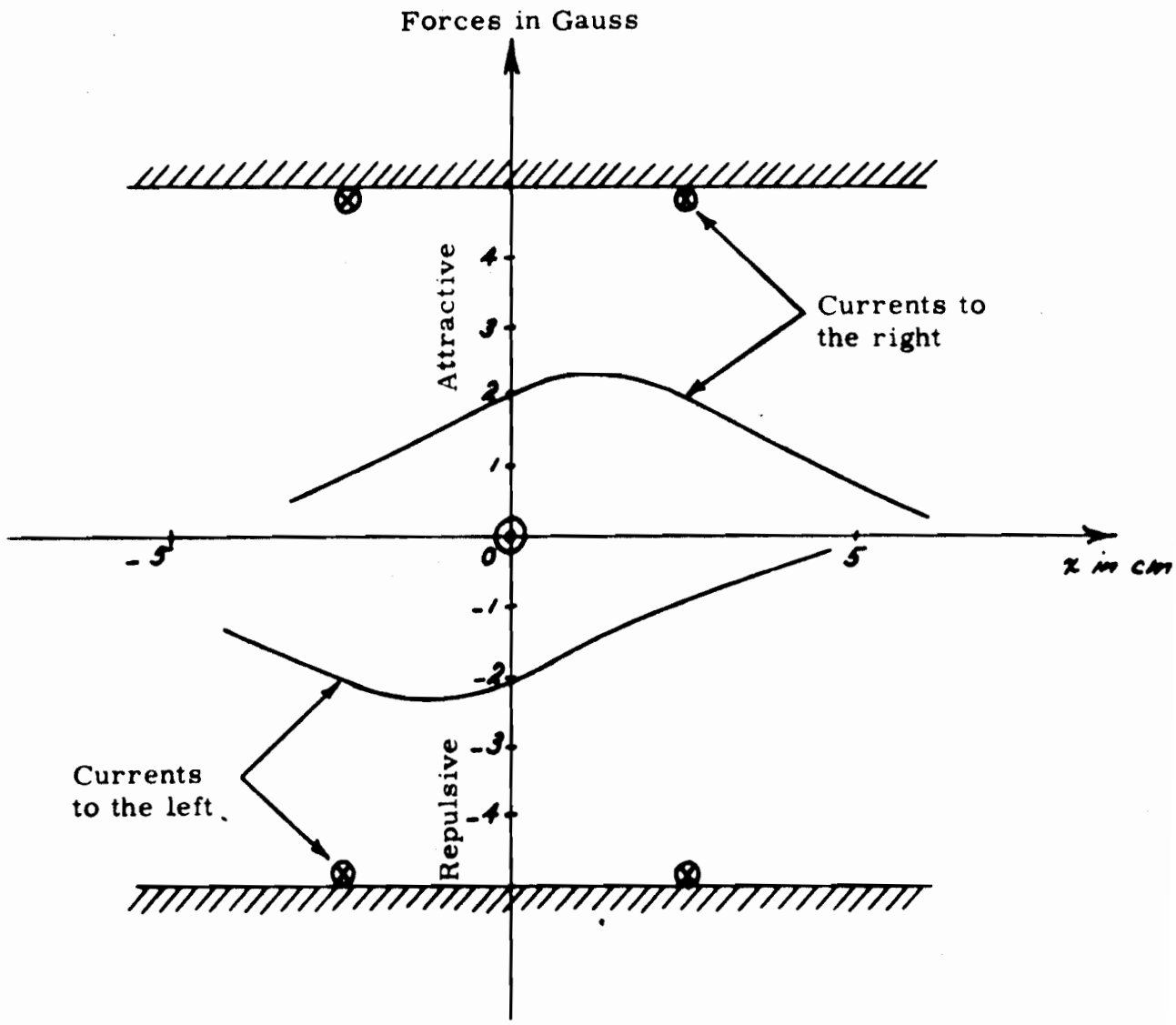

Fig. 9. Partial Cancellation of the Image Forces

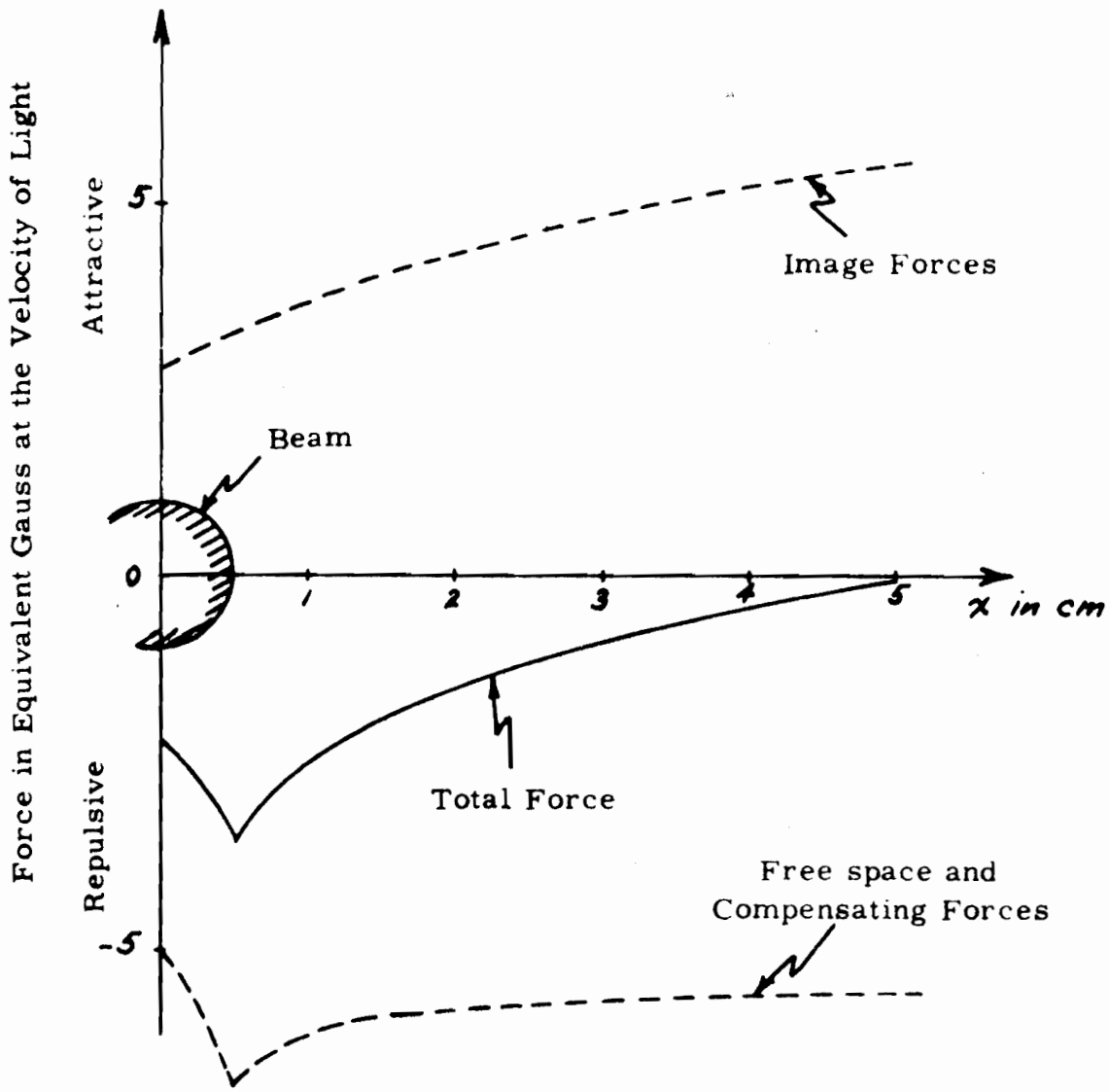
Force in Equivalent Gauss at the Velocity of Light
Attractive
Repulsive
5
-5
0
1
2
3
4
5
x in cm
Image Forces
Beam
Total Force
Free space and
Compensating Forces

Fig. 10