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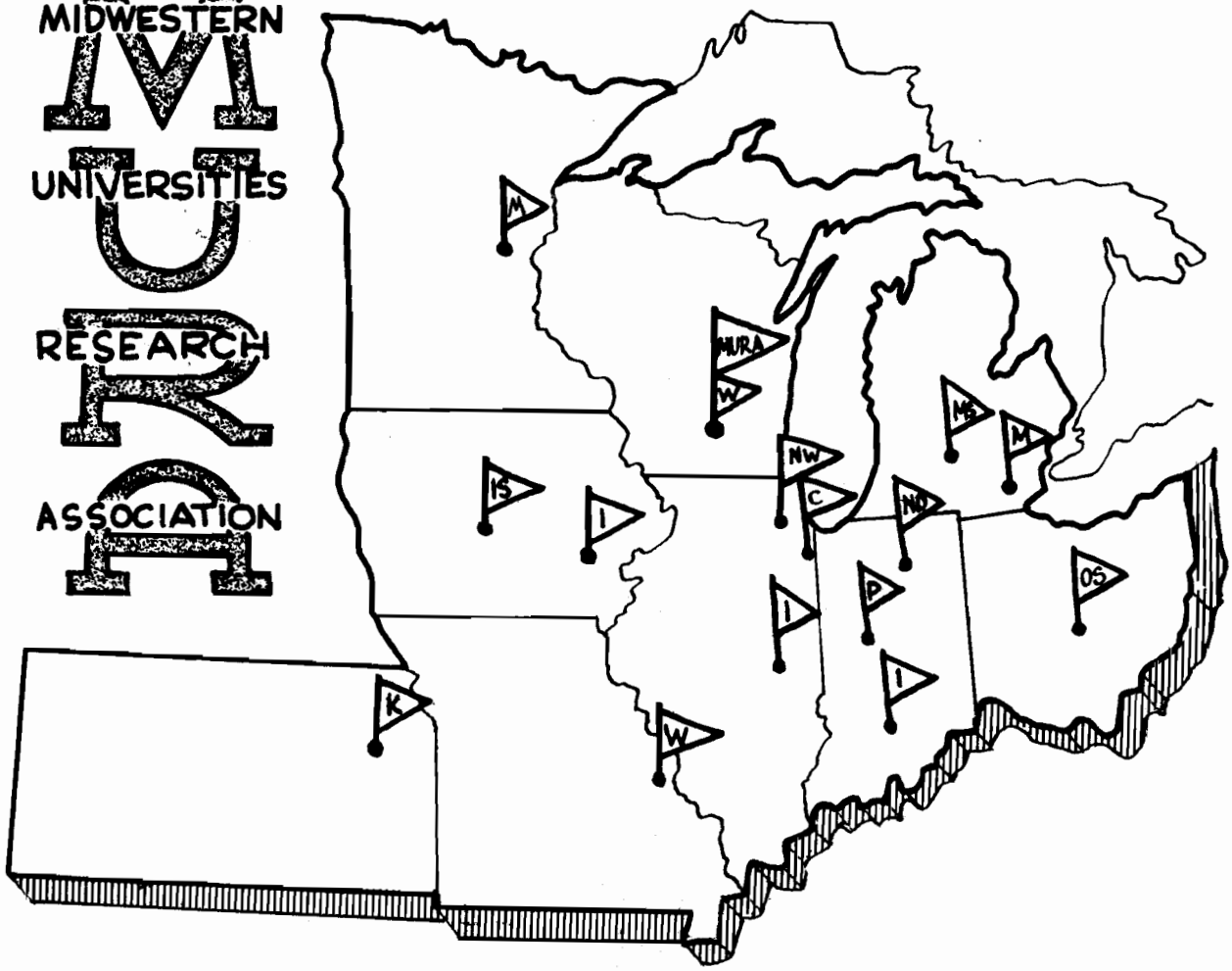

ENERGY LOSS EFFECTS AND THE INSTABILITY OF A COASTING BEAM

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ENERGY LOSS EFFECTS AND THE INSTABILITY OF A COASTING BEAM**

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ABSTRACT

It is shown that an energy loss mechanism which results in a uniform rate of loss independent of velocity has no effect on the criterion for instability of a coasting beam.

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I. INTRODUCTION

The instability of a coasting beam of particles has been studied[1] under the assumption that the beam is completely unneutralized and non-radiating. A partially or completely neutralized beam will exhibit two-stream processes which have been shown not to be serious for MURA accelerators.[2] Neutralizing particles will have the effect of creating an energy loss mechanism for the coasting beam. This report constitutes an investigation of the effect of energy loss on the instability of a coasting beam. It is assumed that the rate of energy loss is constant per unit time, independent of the velocity of the particles in the beam. This assumption would appear to be approximately valid for relativistic particles. It is shown that such an energy loss mechanism in no way alters the criterion for instability or changes the rate of growth for unstable situations. The more general case in which the energy loss is a function of the energy of the particle will be discussed in a future report.

II. FORMULATION

Using the notation of Ref. 1, we let $\psi(\varphi, W, t)$ be the distribution function, which must satisfy the Boltzmann equation in μ-space:

$$\frac{\partial \psi}{\partial t} = -\dot{\varphi}\,\frac{\partial \psi}{\partial \varphi} - \dot{W}\,\frac{\partial \psi}{\partial W} \quad . \tag{1}$$

The Hamiltonian for the system may be written

$$H = \left(\pi f \frac{df}{dE}\right) W^2 + 2\pi R e V(\varphi) \quad , \tag{2}$$

where the longitudinal electric field $\mathcal{E} = -\frac{\partial V}{\partial \varphi}$.

In the previous investigation $\mathcal{E}$ arose from the distribution function ψ itself. We want now to include the energy loss mechanisms. This we can do approximately by simply including in $V(\varphi)$ a term corresponding to an energy (i.e. W) independent reversely directed electric field. Thus we take

$$V(\varphi) = V(\psi, \varphi) + k e \varphi , \tag{3}$$

corresponding to a retarding electric field $\mathcal{E} = - k e$ and hence an energy loss of

$$\left.\frac{dE}{dt}\right|_o = - 2\pi R f e^2 k \quad . \tag{4}$$

We may use Eq. (4) to express k in terms of the energy loss per unit time, $\left.\frac{dE}{dt}\right|_o$.

We now seek a stationary solution ψ_o to Eq. (1) which becomes, if we assume the stationary solution is φ independent

$$\frac{\partial \psi_o}{\partial t} = 2\pi R e^2 k \frac{\partial \psi_o}{\partial W} \quad . \tag{5}$$

The solution to this equation is

$$\begin{aligned} \psi_o(W, t) &= \psi_o(2\pi R e^2 k t + W) \\ &= \psi_o\left(W - \left.\frac{1}{f}\frac{dE}{dt}\right|_o t\right) \end{aligned} \tag{6}$$

where $\psi_o(x)$ is an arbitrary function. Clearly this is correct, simply corresponding to a uniform beam being degraded in energy at a constant rate.

We now make a canonical transformation to a new system of coordinates in which $\psi_o(W, t)$ is time independent. This can be accomplished by the

generating function

$$S = -\left(W - \frac{1}{f}\frac{dE}{dt}\bigg|_o t\right)\varphi^* \tag{7}$$

which yields for the new canonical coordinates and momenta φ^* and W^*

$$W^* = -\frac{\partial S}{\partial \varphi^*} = W - \frac{1}{f}\frac{dE}{dt}\bigg|_o t$$

$$\varphi = -\frac{\partial S}{\partial W} = \varphi^* \quad . \tag{8}$$

The Hamiltonian (Eq. (2)) becomes

$$H^*(W^*, \varphi^*) = H(W, \varphi) + \frac{\partial S}{\partial t}$$

$$H^*(W^*, \varphi^*) = \pi f \frac{df}{dE}\left(W^* + \frac{1}{f}\frac{dE}{dt}\bigg|_o t\right)^2 + 2\pi R\, e\, V(\varphi^*) + \frac{1}{f}\frac{dE}{dt}\bigg|_o \varphi^* \, . \tag{9}$$

We can now conveniently seek to determine the stability of the solution $\psi_o(W, t)$. For this purpose we work in the new variables φ^* - W^*, and recall that the Boltzmann equation can be written in the same form as Eq. (1) in terms of any canonical coordinates and momenta. Thus $\psi(\varphi^*, W^*, t)$ satisfies

$$\frac{\partial \psi}{\partial t} = -\dot{\varphi}^* \frac{\partial \psi}{\partial \varphi^*} - \dot{W}^* \frac{\partial \psi}{\partial W^*} \tag{10}$$

Now an equilibrium distribution is described by an arbitrary function $\psi_o(W^*)$ since now

$$\frac{\partial \psi}{\partial t} = 0 \qquad \frac{\partial \psi}{\partial \varphi^*} = 0$$

$$\dot{W}^* = 0.$$

The last follows from

$$\dot{W}^* = - \frac{\partial H^*}{\partial \varphi^*} = - 2\pi R\, e \frac{\partial V}{\partial \varphi^*} - \frac{1}{f}\frac{dE}{dt}\Big|_o$$

which by Eq. (3) is

$$\dot{W}^* = - 2\pi R\, e^2 k - \frac{1}{f}\frac{dE}{dt}\Big|_o ,$$

and by Eq. (4) yields $\dot{W}^* = 0$.

We now look for small variations from this solution by letting

$$\psi(\varphi^*, W^*, t) = \psi_o(W^*) + \psi_1(\varphi^*, W^*, t) \tag{11}$$

and inserting this into Eq. (10) and linearizing the resulting equation to obtain

$$\frac{\partial \psi_1}{\partial t} = - \dot{\varphi}^* \frac{\partial \psi_1}{\partial \varphi^*} - \dot{W}^* \frac{\partial \psi_o}{\partial W^*} \tag{12}$$

since the term in $\dot{W}^*$ is linear in ψ_1 . From the Hamiltonian of Eq. (9) we obtain

$$\frac{\partial \psi_1}{\partial t} = - 2\pi f \frac{df}{dE}\left(W^* + \frac{1}{f}\frac{dE}{dt}\Big|_o t\right) \frac{\partial \psi_1}{\partial \varphi^*} + \\ + \frac{2\pi R\, e \left(\frac{\partial \psi_o}{\partial W}\right) e\, g}{2\pi \gamma^2 R^2} \frac{\partial}{\partial \varphi^*} \int \psi_1(\varphi^*, W^*, t)\, d W^*, \tag{13}$$

where the usual expression for the electric field (Eq. (25) of Ref. 1) has been used. We may obtain a solution to Eq. (13) which is periodic in space if we assume that

$$\psi_1(\varphi^*, W^*, t) = \overline{\psi}_1(W^*)\, e^{i(n\varphi^* - \omega t - \lambda t^2)}, \tag{14}$$

where $\overline{\psi}_1(W^*)$, ω, and λ must yet be determined. Inserting Eq. (14) into Eq. (13) we find:

$$- i(\omega + 2\lambda t)\ \overline{\psi}_1(W^*) = -2\pi f \frac{df}{dE}\left(W^* + \frac{1}{f}\frac{dE}{dt}\bigg|_o t\right) i n\ \overline{\psi}_1(W^*)$$

$$+ \frac{e^2 g}{\gamma^2 R}\ i n \int \overline{\psi}_1(W^*)\, d W^* \left(\frac{\partial \psi_o}{\partial W^*}\right),$$

which implies that

$$\lambda = \pi \frac{df}{dE} \frac{dE}{dt}\bigg|_o , \tag{15}$$

and further that

$$\overline{\psi}_1(W^*)\left(n\, 2\pi f \frac{df}{dE} W^* - \omega\right) = -\frac{e^2 g \left(\frac{\partial \psi_o}{\partial W^*}\right) n}{\gamma^2 R} \int \overline{\psi}_1(W^*)\, d W^* . \tag{16}$$

This last equation, upon substituting $\dot{\varphi}^* = 2\pi f \frac{df}{dE} W^*$ is exactly Eq. (29) of Ref. 1. Hence, since $\psi_o(W^*)$ may reasonably be expected to be of the same form as in Ref. 1, the solutions to Eq. (16) will be of the same nature as those considered previously.

Thus the general solution is of the form of Eq. (14) with ω related to n by the same dispersion relation as in the absence of energy loss. Now however, there is an exponential dependence on the square of the time. By Eq. (15) it is clear that λ is real and thus this additional time dependence leads to an altered frequency of the perturbation, but has no effect on the stability, or rate of instability. That an energy loss of the type we assumed, namely constant loss per unit time independent of velocity, should lead to this result is not unexpected.

FOOTNOTES

1. C. E. Nielsen and A. M. Sessler, MURA-441 (1959).
2. A. M. Sessler and K. R. Symon, Bull. Am. Phys. Soc. II 4, 55 (1959).